Guidelines

Please turn in a neat homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel. Remember to specify all the units of your results.

Problem 1 (15 points)

A little boy is flying a kite shaped as an airfoil. The kite can freely rotate about the end of the rope $P$, and is tethered to the boy’s hand on point $O$ at a certain angle $\beta$ (see figure 1), producing a reaction force $R$ in point $P$. The kite is flying with an angle of attack $\alpha$, with force coefficients parallel and normal to the kite $C_A = \gamma$ and $C_N = \gamma(1 + \alpha)$ respectively and its moment coefficient about the center of pressure $C_{Mc/A} = 17/4\gamma\alpha$.

Figure 1: Kite tethered

Calculate the modulus of the reaction force $R$ and the angle $\beta$ following these steps:

1. Write the moment balance equation about the leading edge $P$ to calculate the angle of attack $\alpha$.
2. Use the angle of attack from previous step to calculate the normal and axial forces, $N$ and $A$.
3. Calculate the lift and drag forces $L$ and $D$ from $N$, $A$ and the angle of attack.
4. Write the force balance between $L$, $D$ and $R$ to calculate $R$ and $\beta$. 
**Problem 2 (15 points)**

It is known that for flow around blunt bodies at very high Reynolds numbers, the pressure distribution at the windward is very similar to the potential solution, while at the leeward, the flow is detached. To evaluate the aerodynamic drag of a cable of circular section of radius \( r = 1 \text{ cm} \) (see figure 2) in a flow of uniform upstream velocity \( V_\infty = 50 \text{ m/s} \), density \( \rho_\infty = 1.2 \text{ kg/m}^3 \) and static pressure \( p_\infty = 100 \text{ kPa} \), it is assumed that the front side of the cable (windward \( 0 < \theta < \pi/2 \)) has a pressure coefficient \( c_p = 1 - 4 \sin^2 \theta \), while in the rear side (leeward \( \pi/2 < \theta < \pi \)) the flow is detached and the pressure coefficient is the value of the \( c_p \) at the detachment point \( (\theta = \pi/2) \).

![Flow around a circular cable](image)

1. Determine the aerodynamic drag coefficient by integrating \( c_p \) along the surface of the body.
2. Determine the drag force acting on the cylinder.

**Problem 3 (20 points)**

A missile model is tested on a wind tunnel (see figure below) to determine its drag, with the following results: the upstream (station 1) velocity is uniform at \( V_\infty = 60 \text{ m/s} \). The static pressures upstream and downstream are \( p_1 = p_2 = 100 \text{ kPa} \). The downstream velocity distribution \( u \) (in m/s) as a function of \( y \) (in m), symmetric about the centerline of the missile, is given by:

\[
\begin{align*}
  u_2 &= \begin{cases} 
    60 - 40(1 - 2|y|) & \text{if } |y| \leq 1/2; \\
    60 & \text{if } |y| > 1/2 
  \end{cases}
\end{align*}
\]

![Downstream velocity distribution](image)

1. Calculate the drag force exerted by the air on the body per unit length normal to the plane of the figure.
2. In qualitative terms, explain how would the result of (a) change if \( p_2 > p_1 \) or if \( p_2 < p_1 \).