Guidelines

Please turn in a neat homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel. Remember to specify all the units of your results.

Problem 1

A vortex sheet is defined as a distribution of vortices in the limit where we have infinite point vortices that are infinitesimally close to each other, and their strength becomes infinitesimally small. This concept can be understood following these steps:

1. Show that the velocity field generated by a vortex sheet of uniform intensity per unit length, $\gamma(s) = \frac{U}{\pi}$, located at $y = 0$, $0 \leq x \leq c$ (Fig 1) is given by:

$$u(x, y) = \frac{U}{4\pi} \arctan \left( \frac{x}{y} \right) - \frac{U}{8\pi} \arctan \left( \frac{x - c}{y} \right),$$

$$v(x, y) = \frac{U}{8\pi} \ln \left( \frac{(x - c)^2 + y^2}{x^2 + y^2} \right).$$

Calculate the two velocity components at $(x, y) = (c, c/2)$

2. Consider the velocity field generated by two point vortices of intensity $\Gamma = \frac{Ua}{2}$ located at $(x, y) = (0, 0)$ and $(x, y) = (c, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (c, c/2)$.

3. Consider the velocity field generated by three point vortices of intensity $\Gamma = \frac{Ua}{3}$ located at $(x, y) = (0, 0)$, $(x, y) = (c/2, 0)$ and $(x, y) = (c, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (c, c/2)$.

Figure 1: Vortex Sheet
4. Consider the velocity field generated by four point vortices of intensity $\Gamma = Ua/4$ located at $(x, y) = (0, 0)$, $(x, y) = (0, c/3)$, $(x, y) = (2c/3, 0)$ and $(x, y) = (c, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (c, c/2)$.

5. Consider the velocity field generated by five point vortices of intensity $\Gamma = Ua/5$ located at $(x, y) = (0, 0)$, $(x, y) = (c/4, 0)$, $(x, y) = (0, c/2)$, $(x, y) = (3c/4, 0)$ and $(x, y) = (c, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (c, c/2)$.

6. How does the velocity generated by the vortex sheet at at $(x, y) = (c, c/2)$ compare to the velocity generated by the point vortex distribution as we progressively increase the number of vortices and decrease their intensity? What do you expect to happen when the number of vortices $N \to \infty$ if their intensity $\Gamma$ goes to zero as $Ua/N$?

Problem 2

A glider (Fig. 2) is formed by two flat plates, one of chord $c$ for the wing and the other of chord $c/2$ for the tail, forming an angle $\beta$, as shown in figure. The glider is flying with velocity $V_\infty$ and an angle of attack $\alpha$. The center of the tail is located at a distance $d$ from the origin. We can study this aircraft using Thin Airfoil Theory. Assume that both $\alpha$ and $\beta$ are small and that both wing and tail are at a distance such that the wing doesn’t perturb the flow around the tail.

![Figure 2: Vortex Sheet](image)

1. Start considering only the wing. Replace the the plate by a vortex sheet such that the chord is a streamline. What is the intensity of the vortex sheet $\gamma(\theta)$?

2. Calculate the lift generated by the main wing and the lift generated by the tail.

3. Calculate the coefficient of moment around the leading edge of both wing and tail. Calculate the total coefficient of moment of the glider around the leading edge of the wing.

4. Calculate the aerodynamic center of the glider. Which condition must the mass center of the glider fulfill in order for it to be stable?
Problem 3

Consider a NACA airfoil flying in air at velocity $U_\infty$. Its camber line (Fig. 3) is represented by the equation:

$$y_c(x) = \begin{cases} \frac{M}{P^2} (2Px - x^2), & 0 \leq x \leq Pc \\ \frac{M}{(1-P^2)} (1 - 2P + 2Px - x^2), & Pc \leq x \geq c \end{cases}$$

where $P = \frac{1}{3}$ is the position $x/c$ of maximum camber and $M = 0.1$ is the maximum camber in percentage (10%) of chord length.

Applying Thin Airfoil Theory, calculate:

1. The ideal angle of attack.
2. The lift coefficient $c_l$ at this angle of attack.
3. The moment coefficient $c_{m_{L,E}}$ about the leading-edge at this angle.
4. The angle of zero-lift.