Problem 1

A vortex sheet is defined as a distribution of vortices in the limit where we have infinite point vortices that are infinitesimally close to each other, and their strength becomes infinitesimally small. This concept can be understood following these steps:

1. Show that the velocity field generated by a vortex sheet of uniform intensity per unit length, $\gamma(s) = U$, located at $y = 0, 0 \leq x \leq a$ (see Fig 1-1) is given by:

\[
\begin{align*}
  u &= \frac{U}{2\pi} \arctan\left(\frac{x}{y}\right) - \frac{U}{4\pi} \arctan\left(\frac{x-a}{y}\right), \\
  v &= \frac{U}{4\pi} \ln\left(\frac{(x-a)^2 + y^2}{x^2 + y^2}\right). 
\end{align*}
\]

Calculate the two velocity components $u$ and $v$ at $(x,y) = (a/2, a/2)$.

2. Consider the velocity field generated by two point-vortex of intensity $\Gamma = Ua/2$ located at $(x,y) = (0,0)$ and $(x,y) = (a,0)$. Calculate $(u,v)$ generated by this configuration at $(x,y) = (a/2, a/2)$.
3. Consider the velocity field generated by three point-vortex of intensity $\Gamma = Ua/3$ located at $(x, y) = (0, 0)$, $(x, y) = (a/2, 0)$ and $(x, y) = (a, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (a/2, a/2)$.

4. Consider the velocity field generated by four point-vortex of intensity $\Gamma = Ua/4$ located at $(x, y) = (0, 0)$, $(x, y) = (a/3, 0)$, $(x, y) = (2a/3, 0)$ and $(x, y) = (a, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (a/2, a/2)$.

5. Consider the velocity field generated by five point-vortex of intensity $\Gamma = Ua/5$ located at $(x, y) = (0, 0)$, $(x, y) = (a/4, 0)$, $(x, y) = (a/2, 0)$, $(x, y) = (3a/4, 0)$ and $(x, y) = (a, 0)$. Calculate $(u, v)$ generated by this configuration at $(x, y) = (a/2, a/2)$.

6. How does the velocity generated by the vortex sheet at $(x, y) = (a/2, a/2)$ compare to the velocity generated by the point vortex distribution as we progressively increase the number of vortices and decrease their intensity? What do you expect to happen when the number of vortices $N \to \infty$ if their intensity $\Gamma$ goes to zero as $Ua/N$?

**Problem 2**

You are sailing with *del Alamo The Pirate* on the sailboat *DanielII* equipped with a fin underneath the hull and a big sail mounted (see Fig 2-1a). When you are heading across the wind, the propelling force is given by the lift of the sail, while the fin acts as a stabilizer. We can model the effects of the sail and the fin as two thin, symmetric airfoils with planform area $S_{sail}$ and $S_{fin}$, respectively. The angle of attack of the sail is $\alpha$, and the angle of attack of the fin is $\beta$ ($1 \gg \alpha \gg \beta > 0$). By considering the force balance equations from the provided diagram Fig 2-1b and the designated coordinate system, calculate the following:

1. The angle $\beta$.

2. The velocity you are sailing ($v_{water}$).

Use the following parameters: $\rho_{air} = 1.2 (kg/m^3), \rho_{water} = 1200 (kg/m^3), v_{air} = 20 (knots) = 10.29 (m/s), \alpha = 0.25 (rad), C_{d,hull} = 0.05$. Scale the force using the effective chord length $c_{sail} = 192.3 (m), c_{hull} = 1 (m), c_{fin} = 0.8 (m)$. Assume that 1) all the forces are acting on the same horizontal plane, and model the system as 2-dimensional correspondingly; 2) Drag forces from the fin and the sail are negligible.
Fig 2-1: Diagrams for problem 2.