Problem 1
Consider a wing with elliptical planform shape and aspect ratio $\Lambda = 8$, formed by equal airfoils (with $\frac{\partial c_l}{\partial \alpha} = 2\pi$). Its non-dimensional circulation distribution is:

$$G_b(\theta) = \frac{\Gamma}{bU_\infty} = -\frac{\varepsilon}{\pi \Lambda} \sin(3\theta)$$

where $y/b = \frac{1}{2} \cos \theta$.

1. Calculate the lift coefficient, $c_L$, of the wing when the unperturbed free-stream is parallel to the zero-lift direction ($\alpha_{L=0}$) of the central airfoil.

2. Calculate the induced drag coefficient of the wing, $c_{Di}$.

3. Calculate the angle that the zero-lift direction of the wing forms with the zero-lift direction of the central airfoil. Make a sketch indicating clearly their relative position.

4. Knowing that the maximum lift coefficient of each airfoil, for the considered Reynolds number, is $c_L = 1.4$, determine the airfoil in which the stall begins and the maximum $c_L$ of the wing.
Problem 2

An aircraft of mass $M$ is provided with a wing of rectangular planform shape, span $b$ and large aspect ratio $\Lambda > 1$. It is flying with velocity $U_\infty$ in uniform, rectilinear motion. Both the leading and trailing edge of the airfoils of the wing are located in the plane $z = 0$, and the surface formed by the camber lines of the wing is given by the equation:

$$
\frac{z_c(x, y)}{c} = f \left( \frac{y}{b} \right) \left[ 1 - 4 \left( \frac{x}{c} - \frac{1}{4} \right)^2 \right]
$$

with $-\frac{1}{2} \leq \frac{y}{b} \leq \frac{1}{2}$ and $-\frac{1}{4} \leq \frac{x}{c} \leq \frac{3}{4}$, as seen in Figure 1.

Figure 1: Sketch of the wing.

1. Determine the aerodynamic twist of the wing as a function of $f \left( \frac{y}{b} \right)$.

2. Determine $f \left( \frac{y}{b} \right)$ such that the wing has minimum induced drag. In this case, calculate the angle of attack of the central airfoil and the moment coefficients around the three axis. **Note:** Assume that the free-stream is parallel to the direction of the airfoil chords.
Problem 3

A flock of birds is flying in formation, as seen in Figure 2. Consider that their wings have a span $b$, aspect ratio $\Lambda \gg 1$ and they are flying with velocity $U_\infty$ in uniform, rectilinear motion.

Figure 2: Flock of birds flying in formation.

Under these conditions, the velocity induced on the wing of one of them by the others is $w_f(\theta) = BU_\infty \sin(2\theta)$, where $B$ is a known constant and the angle $\theta$ is defined as usual. We have determined that, under these conditions, the non-dimensional circulation distribution appearing on that wing is $G(\theta) = A_1 \sin \theta + A_2 \sin 2\theta$, where $A_1$ and $A_2$ are known constants. Calculate the force, in the direction of flight, that appears on the wing under consideration. Compare it to the self induced drag of the wing when the bird flies alone.
Problem 4

The Cessna Citation Jet aircraft (figure 3) has a tapered wing that uses the NASA NLF-0213 airfoil from root to tip with $\alpha_{L=0} = -4^\circ$. Use a collocation method with 10 stations to determine the lift coefficient of the whole wing in the following situations.

1. When flying at zero angle of attack.
2. When flying at $\alpha = 5^\circ$.
3. When flying at $\alpha = 5^\circ$ with the flaps deployed ($\alpha_{L=0} = -10^\circ$ for those spanwise sections of the wing where the flaps are located).

Figure 3: Top CAD view of a Cessna Citation CJ13 Aircraft. Units are in feet.