Guidelines:
Please turn in a neat homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.

Problem 1
The Thin Airfoil Theory is based on linear equations. When a problem is linear, we can apply the principle of linear superposition to divide it into a number of simpler problems, solve them, and add their contributions. In the case of the thin airfoil in Figure 1, it can be decomposed (as seen in Figure 2) into an angle of attack problem (a), a thickness problem (b) and a camber line problem (c). Note that the thickness problem (b) is symmetric: the upper surface is equal to the lower surface.

One of the results of Thin Airfoil Theory is that the pressure coefficient of an airfoil is
calculated as:

\[ c_p^+(x) = -2 \frac{u^+(x)}{U_\infty} \]
\[ c_p^-(x) = -2 \frac{u^-(x)}{U_\infty} \]

where + indicates the upper surface, and - the lower surface of the airfoil. In this problem, we will calculate the pressure coefficient of a cambered airfoil.

We know that the upper surface of the airfoil depicted in Figure 1 has the equation:

\[
\frac{z^+}{c} = \frac{1}{c} \left[ 1 - 2 \frac{x}{c} \right] \sqrt{1 - 4 \left( \frac{x}{c} \right)^2} + \varepsilon_2 \left[ 1 - 4 \left( \frac{x}{c} \right)^2 \right] - \alpha \left[ \frac{x}{c} - \frac{1}{2} \right]
\]

and the lower surface:

\[
\frac{z^-}{c} = -\varepsilon_1 \left[ 1 - 2 \frac{x}{c} \right] \sqrt{1 - 4 \left( \frac{x}{c} \right)^2} + \varepsilon_2 \left[ 1 - 4 \left( \frac{x}{c} \right)^2 \right] - \alpha \left[ \frac{x}{c} - \frac{1}{2} \right]
\]

where \( \alpha \) is the angle of attack, \( \varepsilon_1 \) and \( \varepsilon_2 \) are very small dimensionless geometrical parameters, and \( -\frac{c}{2} \leq x \leq \frac{c}{2} \). Following the illustration in Figure 2, we subdivide them into the three smaller subproblems:

\[
\frac{z^+}{c} = \frac{z_\alpha}{c} + \frac{z^+(x)}{c} + \frac{z_c(x)}{c}
\]
\[
\frac{z^-}{c} = \frac{z_\alpha}{c} + \frac{z^-(x)}{c} + \frac{z_c(x)}{c}
\]

1. Calculate the equation for the components: (a) angle of attack \( \frac{z_\alpha(x)}{c} \), (b) thickness distribution \( \frac{z^+(x)}{c} \) and \( \frac{z^-(x)}{c} \), and (c) the camber line of the airfoil \( \frac{z_c(x)}{c} \).

2. We know that, from Symmetric Thin Airfoil Theory, the horizontal velocity in the upper and lower surfaces of a symmetric airfoil is calculated as:

\[
\frac{u^+(\theta)}{U_\infty} = \frac{u^-\left(\theta\right)}{U_\infty} = \frac{1}{\pi} \int_{0}^{\theta_0=\pi} \frac{dz^+(\theta_0)}{dx} \sin \theta_0 d\theta_0
\]

Figure 2: Decomposition of a thin airfoil into a flat plate, a symmetric airfoil and a camber line.
Calculate the horizontal velocity in the upper and lower surfaces of the thickness distribution.

3. Calculate the coefficient of pressure in the upper and the lower surfaces of the thickness distribution. What is the contribution of these coefficients of pressure to the lift coefficient of the airfoil?

4. The angle of attack and the camber line problems are combined together into the *Lifting Problem*. We know that, for a camber line, the horizontal velocity in the upper and lower surfaces is:

\[
\frac{u^+(\theta)}{U_\infty} = \frac{1}{2} \frac{\gamma(x)}{U_\infty} \\
\frac{u^-(-\theta)}{U_\infty} = -\frac{1}{2} \frac{\gamma(x)}{U_\infty}
\]

Calculate the horizontal velocity in the upper and lower surfaces of the camber line. Note that the camber line includes the angle of attack.

5. Calculate the coefficient of pressure in the upper and the lower surfaces of the camber line. What is the contribution of these coefficients of pressure to the lift coefficient of the airfoil?

6. Calculate the coefficient of pressure in the upper and the lower surfaces of the total airfoil.
Problem 2

Consider an airfoil flying with velocity $U_{\infty}$. Its camber line is represented by the equation:

$$
\frac{z_c(x)}{c} = \begin{cases} 
-\varepsilon \left[ 4 \left( \frac{x}{c} \right)^2 + \frac{x}{c} - \frac{1}{2} \right] ; & -\frac{c}{2} \leq x \leq 0 \\
-\varepsilon \left[ \frac{x}{c} - \frac{1}{2} \right] ; & 0 \leq x \leq \frac{c}{2}
\end{cases}
$$

where $\varepsilon$ is a very small dimensionless parameter. Applying Thin Airfoil Theory, calculate:

1. The ideal angle of attack.
2. The angle of zero-lift. Plot the camber line and the line of zero-lift.

Suppose that the airfoil is articulated at the middle point ($x = 0$), and the back part of the airfoil is deflected an angle $\delta \ll 1$ respect to the previous position (it is deflected downwards). For this new configuration, calculate:

3. The ideal angle of attack of the new configuration.
4. The angle of zero-lift of the new configuration. Indicate the angle that forms this zero-lift line with the zero-lift line of part 2.
5. The lift coefficient of the new camber line when the wind is parallel to the chord of part 1.
6. **Extra credit:** The moment coefficient induced in the rod by the aerodynamic forces acting on the control surface.
Problem 3
An airfoil is provided with a flap, as shown in Figure 3. It flies with velocity $U_\infty$, and its front part is aligned with the incident flow. The flap deflexion follows the equation:

$$\delta(t) = \delta_0 \sin \omega t$$

where $\delta_0$ is very small.

![Figure 3: Airfoil with a deflected flap.](image)

1. Assuming that the process is quasi-steady calculate, as a function of time, the ideal angle of attack, the lift coefficient and the moment coefficient around the point $c/4$. 

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Problem 4

Use the method of vortex panels to calculate the lift generated by a triangular airfoil (chord $c$) immersed in a flow $U_\infty$ with angle of attack $\alpha$, as seen in Figure 4. Assume that the vortex strength along each panel is constant.

Figure 4: Vortex panel Method.

1. Calculate the values of the integrals $I_{i,j}$.

2. Write the linear system of equations for the strength of the vortex panels relative to the air speed, $\gamma_j/U_\infty$, in matrix form.

3. Show that the matrix of this system is singular (zero determinant) before applying Kutta’s condition. What’s the meaning of this result?

4. Rewrite the linear system of equations to include Kutta’s condition and solve it for $\gamma_j/U_\infty$ when:
   
   (a) $\alpha = 0$.
   
   (b) $\alpha = \pi/6$.

5. Obtain the lift coefficient $c_l$ induced by the flow around the triangle from the vortex distribution for these two angles of attack.

6. Write the equation of the camber line of the airfoil. Use Thin Airfoil Theory to calculate the lift coefficient $c_l$ around the cambered airfoil.

7. Compare the results obtained with Thin Airfoil Theory and the vortex panel method. Which result is more accurate? Why?