MAE 104, FALL 2011
HOMEWORK 3
Due Thursday 11-03-2011 in class

Guidelines:
Please turn in a neat homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.

Problem 1
A thin flat plate is flying with velocity $V_\infty$ and angle of attack $\alpha$, as shown in figure 1. When $\alpha$ is small, the components of the velocity parallel and perpendicular to the plate are, for a region near the plate, $y << c$:

$$u(x, y) = V_\infty$$
$$v(x, y) = V_\infty \alpha + w$$

where $w$ is the self induced velocity. The velocity field can be calculated by using Thin Airfoil Theory.

![Figure 1: Thin plate.](image)

1. First, substitute the thin plate by a vortex sheet such that the chord is a streamline. Calculate the intensity of the vortex sheet, $\gamma(\theta)$. Plot $\gamma(x)$.
2. Using the previously calculated $\gamma(\theta)$, calculate the vertical self-induced velocity $w$.

3. The flat plate has a chord $c = 3m$. It is flying with velocity $V_\infty = 100m/s$ through air of density $\rho = 1.23kg/m^3$, and it generates a lift per unit span $L' = 10kN/m$. Calculate its angle of attack.

4. The flat plate is now flying with the same velocity and different angle of attack. It feels a moment (per unit span) around the leading edge $M'_{L.E.} = 15kN$. Calculate the new angle of attack.

**Problem 2**

A hydrofoil is a special type of boat. It is built on top of a wing. When in motion, only the wing is submerged. This way, the weight is balanced with the lift (instead of buoyancy), and the drag is much smaller than that of a normal boat. In Figure 2, we sketch a hydrofoil with two wings. They are two flat plates of chord $c_1$ and $c_2$ and angle of attack $\alpha$ and $\beta$ respectively.

![Figure 2: Bidimensional representation of a hydrofoil.](image)

1. Calculate the total lift on the boat.

2. Calculate the total moment around the center of mass of the boat. Assume that both wings are attached to the boat at their middle point, the attachment is at a distance $x_1$ and $x_2$ respectively from the center of mass and the drag is balanced and aligned with the thrust. **Note:** the center of mass is located between the wings.

3. The hydrofoil is sailing under the next conditions: the seawater density is $\rho = 1025kg/m^3$, the velocity of the hydrofoil is $U_\infty = 36km/h$, the total weight of the boat is $W = 20,000kg$, the chords of the wings are $c_1 = 2m.$ and $c_2 = 1m$. and the center of mass is located $x_1 = 1m.$ behind the front wing and $x_2 = 2m.$ ahead of the rear wing. If the boat is in equilibrium, calculate the angle of attack of both wings.
Problem 3

The camber line of an airfoil of chord $c$ is defined as:

$$\frac{y_c(x)}{c} = \sum_{n=0}^{m} a_n \left(\frac{x}{c}\right)^n$$

where $-\frac{c}{2} \leq x \leq \frac{c}{2}$. It satisfies the next conditions:

a) When the angle of attack is $\alpha = 0$, the lift coefficient is $c_l = \pi/24$.

b) When the angle of attack is $\alpha = \alpha_i$, the lift coefficient is $c_l = \pi/8$. We define the ideal angle of attack, $\alpha_i$, as the angle for which $A_0 = 0$.

c) When the angle of attack is $\alpha = \alpha_i$, the coefficient of moment around the central point is $c_{m0} = \pi/32$.

Calculate:

1. Minimum value of $m$ compatible with the previous conditions.

2. Equation of the camber line.

3. Ideal angle of attack.