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Stress fields due to dislocation arrays at interfaces

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Abstract

The stress formulas for dislocation arrays located at the interface between two different materials are derived and shown to be simple generalizations of the well-known formulas for arrays in homogeneous bodies. The results are given for arrays that consist of discrete distribution of edge dislocations, and for those modeled by a continuous distribution of infinitesimal dislocations. The far field stresses exerted by a discrete array represent the stress field of the corresponding continuous array. The stress distribution due to an infinite dislocation array beneath the free surface of a semi-infinite body is derived and used in conjunction with an energy criterion to discuss a condition for spontaneous nucleation of the dislocation array.

1. Introduction

A study of dislocation arrays is an important issue in modeling various features of material behavior. Dislocation wall structures have been extensively used to model the grain boundaries, persistent slip bands, polygonalization, etc. (Nabarro, 1967; Hirth and Lothe, 1968). The arrays of dislocations along the common slip plane have been successfully utilized in the thin film and interface studies to accommodate for the misfit between two adjacent crystals (Matthews, 1979; Jesser and van der Merwe, 1989; Freund, 1993). They have also been used to interpret various results of the mathematical theory of plasticity (Mura, 1987).

A detailed study of stress fields produced by finite, infinite, and semi-infinite walls of edge dislocations in homogeneous infinite and semi-infinite bodies has been presented in a preceding paper (Lubarda and Kouris, 1996). Here, we extend that analysis by considering dislocation walls and other arrays at the interface between two different materials. This is accomplished by using a solution for the stress field due to a single dislocation at the interface (Head, 1953; Dundurs and Mura, 1964; Dundurs, 1969). Simple closed form expressions are derived, which represent generalizations of the well-known formulas for homogeneous bodies (Hirth and Lothe, 1968). The arrays that consist of a discrete distribution of edge dislocations, as well as the arrays modeled by a continuous distribution of infinitesimal dislocations are analyzed. The stress distribution due to an infinite array of edge dislocations beneath the free surface of a semi-infinite body is then derived. This is used to discuss a condition for spontaneous formation of the dislocation array, based on an energy criterion proposed by Herring (1951).

Elastic isotropy is assumed throughout the paper. Dislocation walls in a homogeneous anisotropic material were considered by Chang (1962). Dislocation

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walls at the interface between two different orthotropic materials were studied by Chou et al. (1975). The relationship between the stress field of dislocation walls at interfaces and the Frank formula for a grain boundary was discussed by Hirth et al. (1979).

2. Dislocation walls at interface of two joined half-spaces

Consider an edge dislocation located at the interface of two joined elastic half spaces, with its Burgers vector normal to the interface (Fig. 1a). Let μ_1 and ν_1 be the shear modulus and the Poisson ratio of the material ①, and μ_2 and ν_2 of the material ②. The Airy stress function for this problem is given by Eqs. (28) and (29) of [8], which can be rewritten in a compact form as

$$\Phi = -k_0 b[r \ln r \sin\theta + (1-\omega) r\theta \cos\theta]. \quad (1)$$

In Eq. (1), r and θ are the polar coordinates, b is the magnitude of the dislocation Burgers vector, and $\omega = 1 \mp \beta$, where the minus sign corresponds to material ① and the plus sign to material ②. The constant k_0 is defined by

$$k_0 = \frac{\mu_1^*}{2\pi(1-\nu_1)}, \quad \mu_1^* = \frac{1+\alpha}{1-\beta^2}\mu_1. \quad (2)$$

The Dundurs parameters

$$\alpha = \frac{\Gamma(\kappa_1+1) - \kappa_2 - 1}{\Gamma(\kappa_1+1) + \kappa_2 + 1},$$

$$\beta = \frac{\Gamma(\kappa_1-1) - \kappa_2 + 1}{\Gamma(\kappa_1+1) + \kappa_2 + 1} \quad (3)$$

are conveniently employed, where $\Gamma = \mu_2/\mu_1$, while $\kappa_1 = 3 - 4\nu_1$ and $\kappa_2 = 3 - 4\nu_2$ are the Kolosov elastic constants for plane strain. For two identical homogeneous materials, $\alpha = \beta = 0$.

It is observed that for each material the Airy stress function (1) consists of two parts, one corresponding to a dislocation in an infinite homogeneous body with elastic properties (μ_1^*, ν_1), and the other corresponding to a Flamant concentrated force of magnitude $\beta k_0 b\pi$, tangent to the free surface of a semi-infinite body. The stress components are obtained from (1) by the appropriate differentiation, which gives

$$\sigma_r = -k_0 b(2\omega - 1)\frac{\sin\theta}{r}, \quad \sigma_\theta = -k_0 b\frac{\sin\theta}{r},$$

$$\tau_{r\theta} = k_0 b\frac{\cos\theta}{r}. \quad (4)$$

The principal stress directions relative to the radial direction are inclined at an angle α defined by $\tan 2\alpha = \cot\theta/(1-\omega)$. If the two half spaces are of identical material ($\omega = 1$), Eq. (4) reduces to the well-known expressions for a dislocation in an infinite homogeneous medium (Nabarro (1967), Eq. (2.12), p. 56), where the principal stress directions are at 45° relative to the radial direction. The same is also the case if the two materials are different, but both incompressible, since then $\kappa_1 = \kappa_2 = 1$, and $\beta = 0$.

The Cartesian in-plane stress components at an arbitrary point (x, y) are likewise

$$\sigma_x = -k_0 b\frac{y[(2\omega+1)x^2 + y^2]}{(x^2+y^2)^2} \quad (5)$$

$$\sigma_y = k_0 b\frac{y[x^2 - (2\omega-1)y^2]}{(x^2+y^2)^2} \quad (6)$$

$$\tau_{xy} = k_0 b\frac{x[x^2 - (2\omega-1)y^2]}{(x^2+y^2)^2}. \quad (7)$$

Note that the shear stress variation τ_{xy} along the slip plane of the dislocation $(y=0)$ is the same in both materials. Furthermore, since the solution contains a contribution from the Flamant concentrated force, the integral of the shear stress along any line parallel to the y-axis is equal to $\beta k_0 b\pi$ (Dundurs and Sendeckyj, 1965; Comninou, 1977). If the two half spaces are of identical materials, Eqs. (5)–(7) reduce to the stress expressions due to dislocation in an infinite homogeneous medium. However, an edge dislocation at the interface of two inhomogeneous materials is a non-equilibrium, high energy configuration which is relaxed by pushing dislocation out of the interface, toward a softer material. This has been discussed by Hirth et al. (1979), Gutkin et al. (1989), Gutkin and Romanov (1994), and Lubarda (1996). Eqs. (5)–(7) are simple generalizations of the corresponding formulas for a homogeneous infinite body, obtained when $\omega = 1$. Nakahara et al. (1972) used equivalent stress expressions, which were written in a less compact form separately for each half-space.

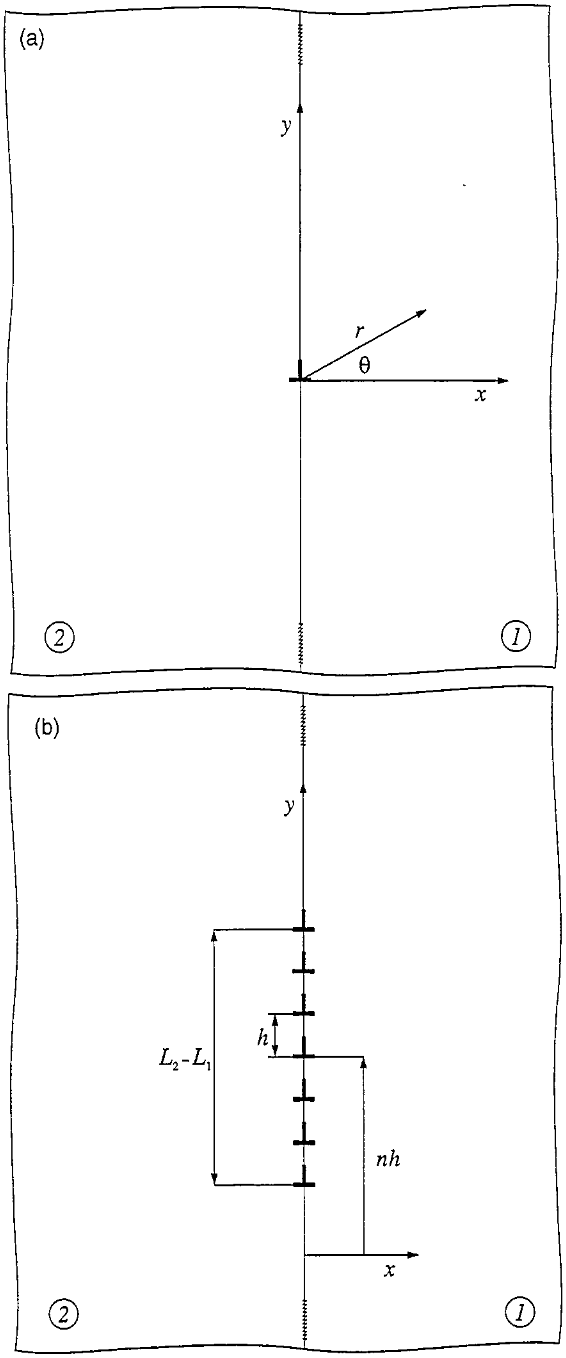

Fig. 1. (a) An edge dislocation at the interface between two different semi-infinite bodies. The Burgers vector of dislocation is normal to the interface; (b) Dislocation wall of identical edge dislocation along the interface between two semi-infinite bodies.

Consider a dislocation wall at the interface of the two joined half spaces consisting of identical, uniformly spaced edge dislocations along the y axis, from $y = L_1$ to $y = L_2$ (Fig. 1b). Let $L_1 = N_1 h$ and $L_2 = N_2 h$, where h is the uniform dislocation spacing. The in-plane stresses at an arbitrary point (x, y) are obtained by adding the contributions from all dislocations in the wall, which gives

$$\sigma_x = -k_0 b \sum_{n=N_1}^{N_2} \times \frac{(y-nh)\left[(2\omega+1)x^2 + (y-nh)^2\right]}{\left[x^2+(y-nh)^2\right]^2} \tag{8}$$

$$\sigma_y = k_0 b \sum_{n=N_1}^{N_2} \frac{(y-nh)\left[x^2 - (2\omega-1)(y-nh)^2\right]}{\left[x^2+(y-nh)^2\right]^2} \tag{9}$$

$$\tau_{xy} = k_0 b \sum_{n=N_1}^{N_2} \frac{x\left[x^2 - (2\omega-1)(y-nh)^2\right]}{\left[x^2+(y-nh)^2\right]^2}. \tag{10}$$

If $N_1 = -\infty$ and $N_2 = \infty$, an infinitely long wall in both directions is obtained. The sums appearing in Eqs. (8)–(10) can be performed analytically, following an analogous procedure used by Hirth and Lothe (1968) in the case of a homogeneous infinite body. The resulting stresses are

$$\sigma_x = -k\pi \frac{\sin(2\pi\eta)}{A^2}\left[A + 2\omega\pi\xi \sinh(2\pi\xi)\right] \tag{11}$$

$$\sigma_y = -k\pi \frac{\sin(2\pi\eta)}{A^2}\left[(2\omega-1)A - 2\omega\pi\xi \times \sinh(2\pi\xi)\right] \tag{12}$$

$$\tau_{xy} = k\pi \frac{2\pi\xi}{A^2}\left\{\omega\left[\cosh(2\pi\xi)\cos(2\pi\eta) - 1\right] + (1-\omega) A \frac{\sinh(2\pi\xi)}{2\pi\xi}\right\}, \tag{13}$$

where $k = k_0 b/h$, and $A = \cosh(2\pi\xi) - \cos(2\pi\eta)$. The non-dimensional variables $\xi = x/h$ and $\eta = y/h$ are conveniently employed. If two half spaces are of identical material (or both incompressible), $\omega = 1$

and Eqs. (11)–(13) reduce to Eqs. (19–75)–(19–77) of Hirth and Lothe (1968), p. 670. Similar, but separate expressions for the two half-spaces, were also derived by Chou and Lin (1975), although their Eqs. (9a) and (9b) contain a missprint, due to omitted $\sinh(2\pi\xi)$ term.

In the limit as $x \to \pm\infty$, both normal stresses vanish, but a far field constant shear stress exists, equal to $\beta k\pi$. The three stress components in (11)–(13) are periodic functions of the coordinate η, with a period equal to one, so that along the slip plane of any dislocation in the wall $\sigma_x = \sigma_y = 0$, and

$$\tau_{xy} = \frac{k\pi^2\xi}{\sinh^2(\pi\xi)}\left[\omega + (1-\omega)\frac{\sinh(2\pi\xi)}{2\pi\xi}\right]. \tag{14}$$

For $\omega = 1$, Eq. (14) reduces to the well-known expression given, for example, by Nabarro (1967) (Eq. (2.91), p. 96). It should be noted, however, that a dislocation wall at the interface between two different materials is not an equilibrium dislocation configuration, and the wall is subject to forces which tend to move it toward the softer material. This instability has been discussed by Chou and Lin (1975), and Hirth et al. (1979).

A semi-infinite wall is obtained if $N_1 = 0$ and $N_2 = \infty$. The sums appearing in the expressions for the normal stresses σ_x and σ_y diverge. The shear stress along the slip plane of the leading dislocation in the wall (i.e., along the x-axis), is given by

$$\tau_{xy} = k\left[\frac{1}{\xi} + \xi\sum_{n=1}^{\infty}\frac{\xi^2 - (2\omega - 1)n^2}{(\xi^2 + n^2)^2}\right], \tag{15}$$

which can be summed to give

$$\tau_{xy} = k\left\{\frac{1}{2\xi} + \frac{\pi^2\xi}{2\sinh^2(\pi\xi)} \times \left[\omega + (1-\omega)\frac{\sinh(2\pi\xi)}{2\pi\xi}\right]\right\}. \tag{16}$$

This also can be obtained directly from the infinite wall solution (14) by an appropriate superposition.

If a dislocation wall is modeled by a continuous distribution of infinitesimal dislocations with density $1/h$, the stresses are obtained by an appropriate

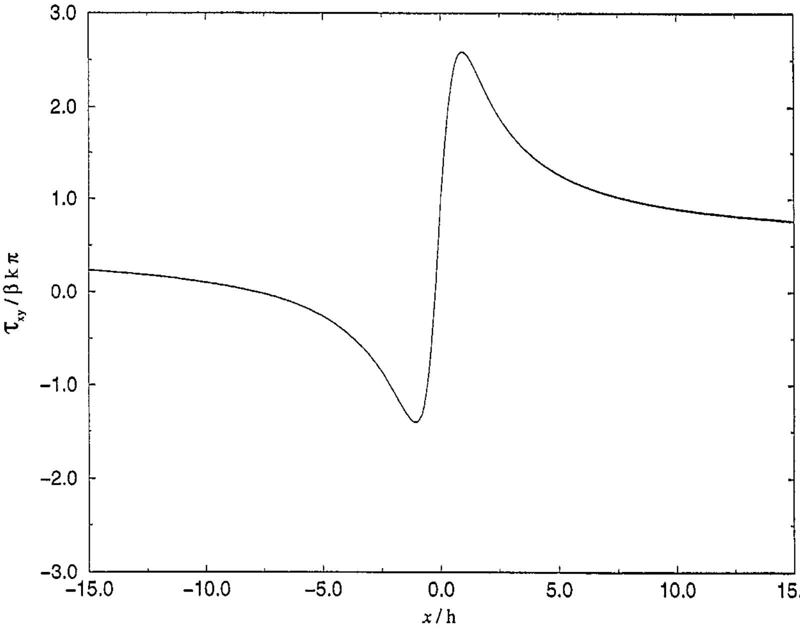

Fig. 2. The shear stress variation along the line at distance h above the leading dislocation of a semi-infinite dislocation wall at the interface between two different materials. As $x \to \pm\infty$, the shear stress approaches the constant value of $\beta k\pi/2$. If two materials are the same, the shear stress distribution becomes antisymmetric, with a zero far-away value as x approaches to $\pm\infty$.

integration. For the wall along the y-axis extending from L_1 to L_2, this gives

$$\sigma_x = k\left\{\frac{1}{2}\ln\frac{x^2+(y-v)^2}{x^2} - \omega\frac{x^2}{x^2+(y-v)^2}\right\}_{L_1}^{L_2} \tag{17}$$

$$\sigma_y = k\left\{\frac{1}{2}(2\omega-1)\ \ln\frac{x^2+(y-v)^2}{x^2} + \omega\frac{x^2}{x^2+(y-v)^2}\right\}_{L_1}^{L_2} \tag{18}$$

$$\tau_{xy} = k\left\{(\omega-1)\ \tan^{-1}\frac{y-v}{x} - \omega\frac{x(y-v)}{x^2+(y-v)^2}\right\}_{L_1}^{L_2}. \tag{19}$$

In the case of a homogeneous infinite body ($\omega = 1$), the above expressions reduce to Eqs. (9)–(11) of Lubarda and Kouris (1996).

An infinite wall of continuously distributed dislocations is obtained if $L_1 = -\infty$ and $L_2 = \infty$. When the infinite wall is at the interface of two different half spaces, from Eqs. (17)–(19) it follows that the normal stresses are zero, but the shear stress is constant throughout the body and equal to $\beta k\pi$. For the wall in a homogeneous infinite body, all stress components vanish. The existence of a narrow longitudinal layer around the wall with the significant stresses in the case of a discrete dislocation distribution is, therefore, lost when the wall is modeled by a continuous distribution of infinitesimal dislocations.

If $L_1 = 0$ and $L_2 = \infty$ are substituted in Eqs. (17)–(19), the stress distribution for a semi-infinite wall is obtained. In this case both normal stresses diverge, while the shear stress becomes

$$\tau_{xy} = k\left[\beta\frac{\pi}{2} + (1-\omega)\ \tan^{-1}\frac{y}{x} + \omega\frac{xy}{x^2+y^2}\right]. \tag{20}$$

Note that τ_{xy} is constant and equal to $\beta k\pi/2$ along the x-axis. The variation of τ_{xy} along the line $y = h$ is shown in Fig. 2. As $x \to \pm\infty$, τ_{xy} approaches the far away value of $\beta k\pi/2$. For $\beta = 0$ and $\omega = 1$, Eq. (20) reduces to the expression (21–50) of Hirth and Lothe (1968), p. 711.

3. Dislocation array with the Burgers vector parallel to the interface

If a dislocation at the interface has its Burgers vector parallel to the interface (Fig. 3a), the Airy stress function is

$$\Phi = -k_0 b[r\ \ln\ r\ \cos\ \theta - (1-\omega)r\theta\ \sin\ \theta]. \tag{21}$$

The stress components in polar coordinates are

$$\sigma_r = -k_0 b(2\omega-1)\frac{\cos\ \theta}{r},\ \sigma_\theta = -k_0 b\frac{\cos\ \theta}{r},$$

$$\tau_{r\theta} = -k_0 b\frac{\sin\ \theta}{r}, \tag{22}$$

while the corresponding Cartesian components are

$$\sigma_x = k_0 b\frac{x[y^2-(2\omega-1)x^2]}{(x^2+y^2)^2} \tag{23}$$

$$\sigma_y = -k_0 b\frac{x[(2\omega+1)y^2+x^2]}{(x^2+y^2)^2} \tag{24}$$

$$\tau_{xy} = k_0 b\frac{y[y^2-(2\omega-1)x^2]}{(x^2+y^2)^2}. \tag{25}$$

Since the solution contains a contribution from the Flamant concentrated force, the integral of the normal stress σ_x along any line parallel to the y-axis is equal to $\beta k_0 b\pi$. (An extension of this result, originally observed by Dundurs and Sendeckyj (1965), to the case of anisotropic bicrystals was given by Barnett and Hirth (1974)). If the two half spaces are of identical materials, Eqs. (23)–(25) reduce to the stress expressions due to dislocation in an infinite homogeneous medium, in which case there is no net force along any line parallel to the y-axis.

Consider an infinite dislocation array at the interface of the two joined half spaces consisting of identical, uniformly spaced edge dislocations of spacing h (Fig. 3b). The in-plane stresses at an arbitrary point (x, y) are obtained by adding the contributions from all dislocations in the array. Using the expressions

(23)–(25) for the stresses due to a single dislocation at the point (0, nh), one obtains the stresses for the dislocation array as

$$\sigma_x = k_0 bx \sum_{n=-\infty}^{\infty} \frac{(y-nh)^2-(2\omega-1)x^2}{\left[x^2+(y-nh)^2\right]^2} \tag{26}$$

$$\sigma_y = -k_0 bx \sum_{n=-\infty}^{\infty} \frac{(2\omega+1)(y-nh)^2+x^2}{\left[x^2+(y-nh)^2\right]^2} \tag{27}$$

$$\tau_{xy} = k_0 bx \sum_{n=-\infty}^{\infty} \times \frac{(y-nh)\left[(y-nh)^2-(2\omega-1)x^2\right]}{\left[x^2+(y-nh)^2\right]^2}. \tag{28}$$

The sums appearing in Eqs. (26)–(28) can be performed analytically, with the resulting stress distribution

$$\sigma_x = -k\pi\frac{2\pi\xi}{A^2}\left\{\omega\left[\cosh(2\pi\xi)\cos(2\pi\eta)-1\right] + (\omega-1)A\frac{\sinh(2\pi\xi)}{2\pi\xi}\right\} \tag{29}$$

$$\sigma_y = k\pi\frac{2\pi\xi}{A^2}\left\{\omega\left[\cosh(2\pi\xi)\cos(2\pi\eta)-1\right] - (\omega+1)A\frac{\sinh(2\pi\xi)}{2\pi\xi}\right\} \tag{30}$$

$$\tau_{xy} = k\pi\frac{\sin(2\pi\eta)}{A^2}\left\{A-2\omega\pi\xi\sinh(2\pi\xi)\right\}, \tag{31}$$

where again $k = k_0 b/h$, and $A = \cosh(2\pi\xi) - \cos(2\pi\eta)$. If two half spaces are of identical material (or both incompressible), $\omega = 1$ and Eqs. (29)–(31) reduce to Eqs. (19–79) of Hirth and Lothe (1968), p. 671 (apart from a printing error in the last equation of (19–79) in the first edition of the mentioned reference, where σ_0 should be replaced by $-\sigma_0$).
In the limit as $x \to \pm\infty$, the shear stress goes to zero, but a non-vanishing normal stress field exists, given by $\sigma_x = \beta k\pi$, and $\sigma_y = (\beta \mp 2)k\pi$. If two materials are identical ($\omega = 1$), only $\sigma_y = \mp 2k\pi$ is non-zero. As an illustration, consider a dislocation array at the interface between aluminum ($\mu_1 =$ 26GPa, $\nu_1 = 0.33$) and copper ($\mu_2 =$ 45GPa, $\nu_2 =$

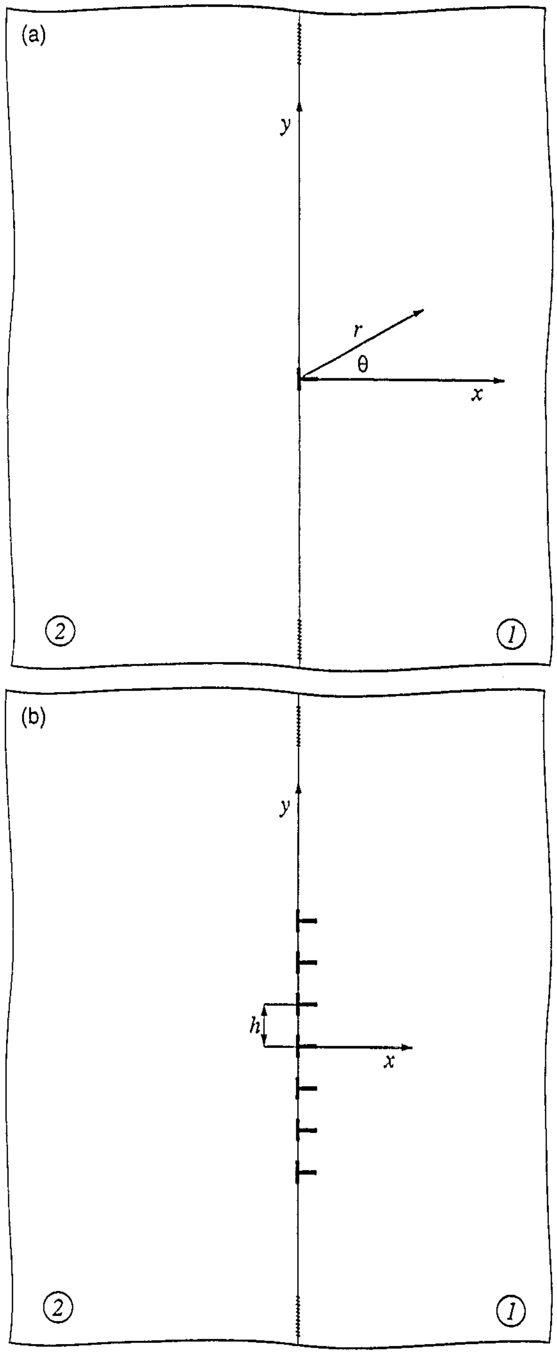

Fig. 3. (a) An edge dislocation at the interface between two different semi-infinite bodies. The Burgers vector of dislocation is parallel to the interface; (b) An infinite array of edge dislocation along the interface between two semi-infinite bodies.

0.35), so that from (3) the Dundurs parameters are $\alpha = 0.28$ and $\beta = 0.08$. The variations of σ_x along the lines $x = \pm 0.1h$ and $x = \pm h$ are shown in Fig. 4. The integral of the normal stress σ_x along any segment of length h is equal to $h\beta k\pi$. Along the lines $x = \pm 2h$, σ_x is essentially constant and equal to $\beta k\pi$. Along the interface line (y-axis), the normal stress σ_x is zero, except at the locations of the

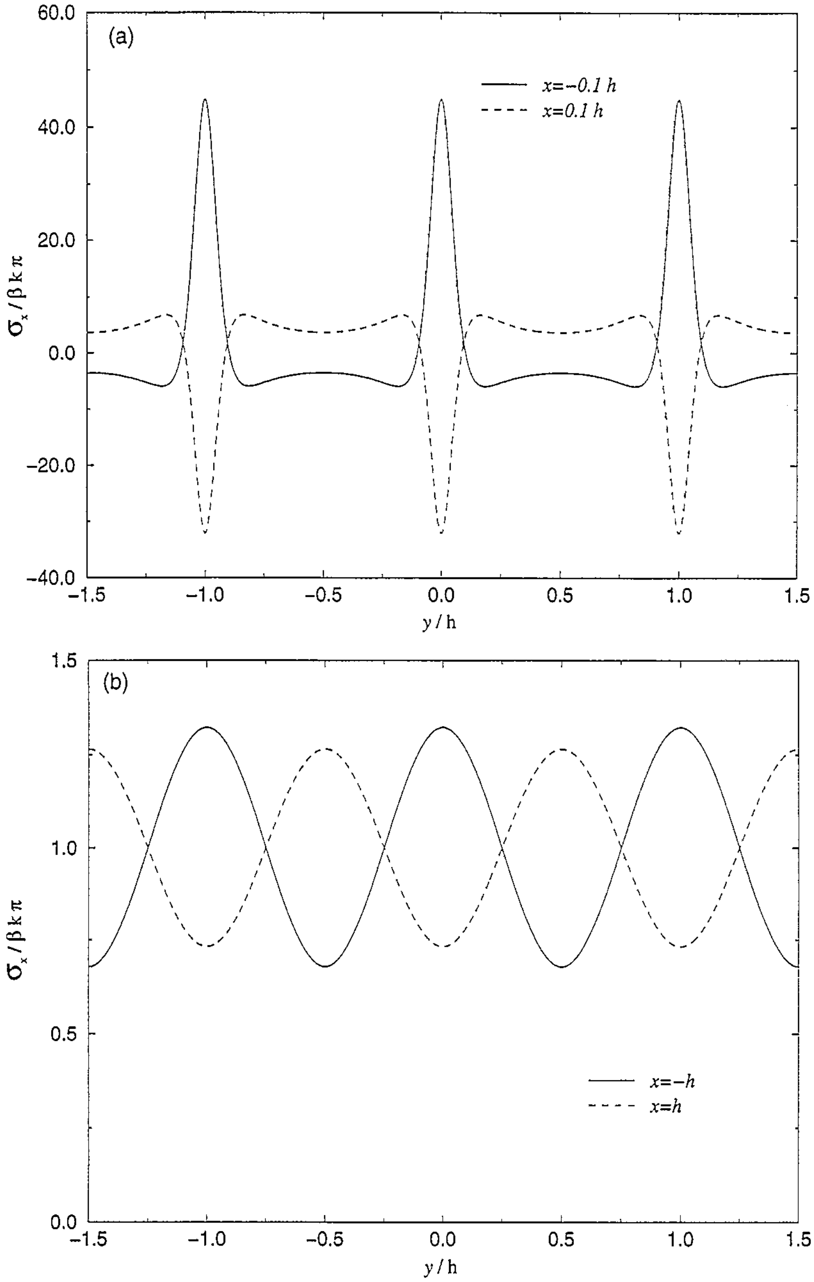

Fig. 4. (a) The variation of the normal stress σ_x along the lines $x = \pm 0.1h$ produced by the infinite dislocation array from Fig. 3b, along the interface between two different materials with $\beta = 0.08$. The integral along the segment of length h is in both cases the same and equal to $h\beta k\pi$; (b) The same as in (a), along the lines $x = \pm h$.

dislocation centers, where the concentrated forces of magnitude $h\beta k\pi$ exist. These forces equilibrate the far away stress $\sigma_x = \beta k\pi$.

The three stress components in (29)–(31) are periodic functions of the coordinate η, with a period equal to one, so that along any line parallel to the

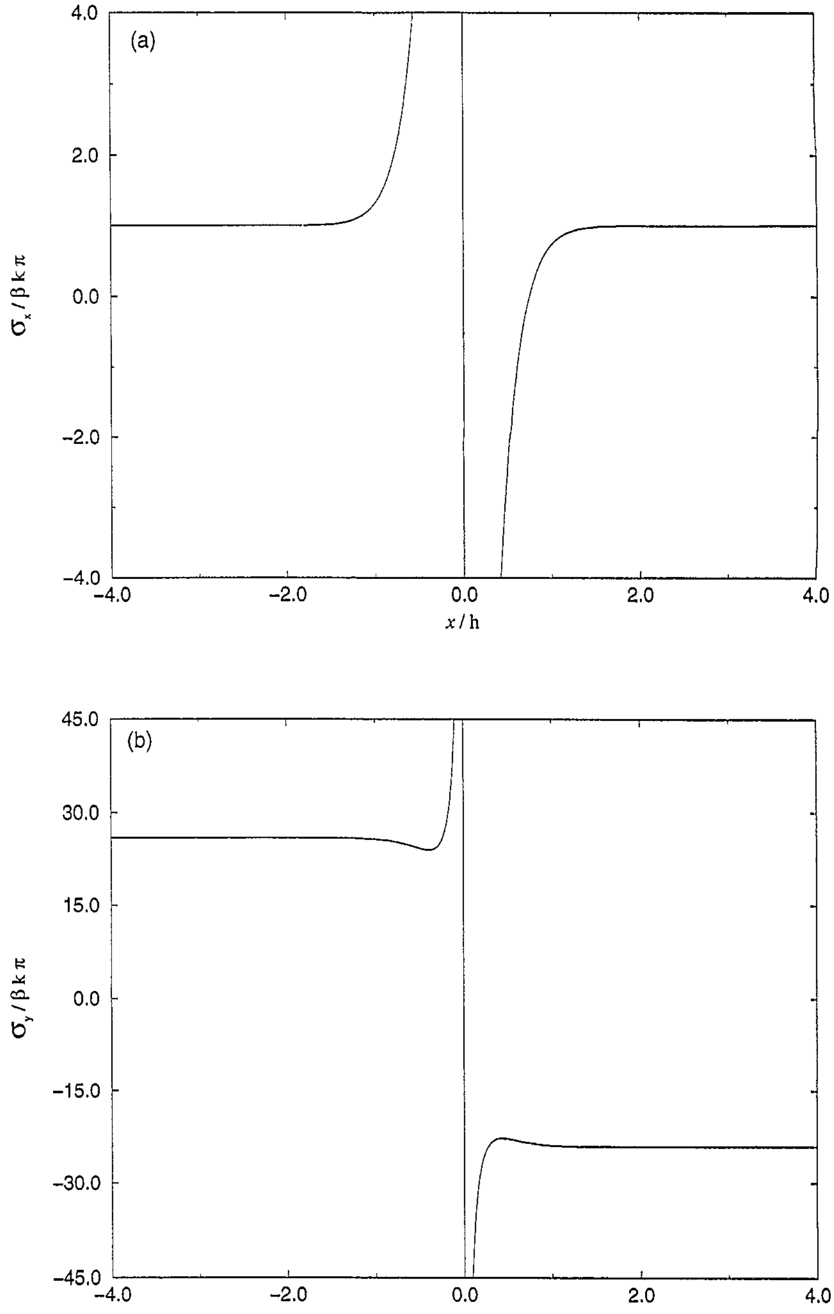

Fig. 5. (a) The variation of the normal stress σ_x along the line $y = 0$ produced by the infinite dislocation array from Fig. 3b, along the interface between two different materials with $\beta = 0.08$. The stress σ_x tends to constant value of $\beta k\pi$, as $x \to \pm\infty$,; (b) The same as in (a), for the normal stress σ_y. Away from the dislocation array, the stress quickly sets in to the value $(\beta - 2)k\pi$ in the material ①, and to the value $(\beta + 2)k\pi$ in the material ②.

x-axis and passing through the dislocation center, $\tau_{xy}=0$ and

$$\sigma_x=-\frac{2k\pi^2\xi}{\cosh(2\pi\xi)-1}\left[\omega+(\omega-1)\frac{\sinh(2\pi\xi)}{2\pi\xi}\right]. \tag{32}$$

$$\sigma_y=\frac{2k\pi^2\xi}{\cosh(2\pi\xi)-1}\left[\omega-(\omega+1)\frac{\sinh(2\pi\xi)}{2\pi\xi}\right]. \tag{33}$$

For a dislocation array at the interface between the aluminum and copper ($\beta=0.08$), the plots of (32) and (33) are shown in Fig. 5.

When a dislocation array is modeled by a continuous distribution of infinitesimal dislocations with density $1/h$ and, therefore, with the specific Burgers vector of magnitude b/h, the stresses are obtained by an appropriate integration. The result is

$$\sigma_x=\beta k\pi,\ \sigma_y=(\beta\mp 2)k\pi,\ \tau_{xy}=0. \tag{34}$$

These are, in fact, the far field ($x\to\pm\infty$) stresses of a discrete dislocation array (Fig. 5). In the case of a homogeneous infinite body, $\sigma_x=\tau_{xy}=0$ and $\sigma_y=\mp 2k\pi$, with a specific strain energy equal to one fourth of $\mu(b/h)^2/(1-\nu)$.

4. Dislocation array beneath the free surface of a semi-infinite body

Consider an infinite dislocation array parallel to the free surface of a semi-infinite body, and at distance l from it (Fig. 6). The array consists of identical, uniformly spaced edge dislocations of spacing h. Such a dislocation array was considered by Herring (1951), who discussed its relation to the surface stress and energy, and by Hartley (1969) who derived the stress distribution for a more general case of the Burgers vector inclined to the free surface. The stress distribution due to dislocation wall near the free surface and normal to it was derived by Lubarda and Kouris (1996). The in-plane stresses at an arbitrary point $(x,\ y)$ in Fig. 6 are obtained by adding the contributions from all dislocations in the array. The stresses due to a single dislocation at the point $(u,-l)$ can be derived using an image proce-

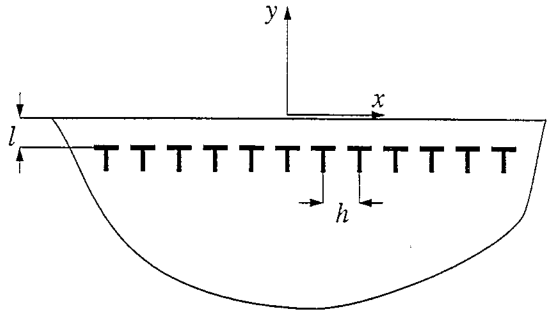

Fig. 6. An infinite dislocation array of identical edge dislocation with uniform spacing h, at the distance l beneath the free surface of a semi-infinite body.

dure (Head, 1953), which gives the stresses as the sum of two stress fields:

$$\sigma_x=k_0 b\left\{\frac{(y+l)\left[3(x-u)^2+(y+l)^2\right]}{\left[(x-u)^2+(y+l)^2\right]^2}-\frac{(y-l)\left[3(x-u)^2+(y-l)^2\right]}{\left[(x-u)^2+(y-l)^2\right]^2}\right\} \tag{35}$$

$$\sigma_y=-k_0 b\left\{\frac{(y+l)\left[(x-u)^2-(y+l)^2\right]}{\left[(x-u)^2+(y+l)^2\right]^2}-\frac{(y-l)\left[(x-u)^2-(y-l)^2\right]}{\left[(x-u)^2+(y-l)^2\right]^2}\right\} \tag{36}$$

$$\tau_{xy}=-k_0 b\left\{\frac{(x-u)\left[(x-u)^2-(y+l)^2\right]}{\left[(x-u)^2+(y+l)^2\right]^2}-\frac{(x-u)\left[(x-u)^2-(y-l)^2\right]}{\left[(x-u)^2+(y-l)^2\right]^2}\right\}, \tag{37}$$

and

$$\sigma_x=2k_0 bl\left\{(x-u)^4-6(x-u)^2 y(y-l)+2y(y-l)^3-(y-l)^4\right\}\times\left[(x-u)^2+(y-l)^2\right]^{-3} \tag{38}$$

$$\sigma_y=2k_0 bl\left\{(x-u)^4+6(x-u)^2 y(y-l)-2y(y-l)^3-(y-l)^4\right\}\times\left[(x-u)^2+(y-l)^2\right]^{-3} \tag{39}$$

$$\tau_{xy} = 4k_0 bl \frac{(x-u)y\left[(x-u)^2 - 3(y-l)^2\right]}{\left[(x-u)^2 + (y-l)^2\right]^3}, \tag{40}$$

where $k_0 = \mu/2\pi(1-\nu)$. The corresponding Airy stress function for the total stress distribution is

$$\Phi = k_0 b\left[\frac{1}{2}(y+l)\ \ln\frac{(x-u)^2 + (y+l)^2}{(x-u)^2 + (y-l)^2} + 2l\frac{y(y-l)}{(x-u)^2 + (y-l)^2}\right]. \tag{41}$$

The stress field due to an infinite dislocation array of identical, uniformly spaced edge dislocations of spacing h is obtained by adding the contributions from all dislocations in the array. Although the sums can be performed in a closed form (Hartley, 1969; Lubarda, 1996), the resulting formulas are somewhat lengthy. Consequently, consider a particularly important feature of the stress distribution, i.e., the normal stress σ_x variation along the free surface. From (35) and (38), with $u = nh$ and $y = 0$, it follows that

$$\sigma_x(\xi, 0) = 8kl_0 \sum_{n=-\infty}^{\infty} \frac{(\xi - n)^2}{\left[(\xi - n)^2 + l_0^2\right]^2}, \tag{42}$$

where $\xi = x/h$, $l_0 = l/h$, and $k = k_0 b/h$. This can be summed to give

$$\sigma_x(\xi, 0) = 4\pi k\left\{\frac{\sinh(2\pi l_0)}{\cosh(2\pi l_0) - \cos(2\pi\xi)} - 2\pi l_0 \frac{\cosh(2\pi l_0)\cos(2\pi\xi) - 1}{\left[\cosh(2\pi l_0) - \cos(2\pi\xi)\right]^2}\right\}, \tag{43}$$

which is in agreement with Eq. (2) of Hartley (1969), apart from a printing error in the fourth line of that equation, where the plus sign preceding the term $2(\alpha + \beta)$ should be replaced by the minus sign.
The plots are shown in Fig. 7 for several values of l_0. The oscillatory behavior of $\sigma_x(\xi, 0)$ is more pronounced for smaller values of l. For $l > h$, the

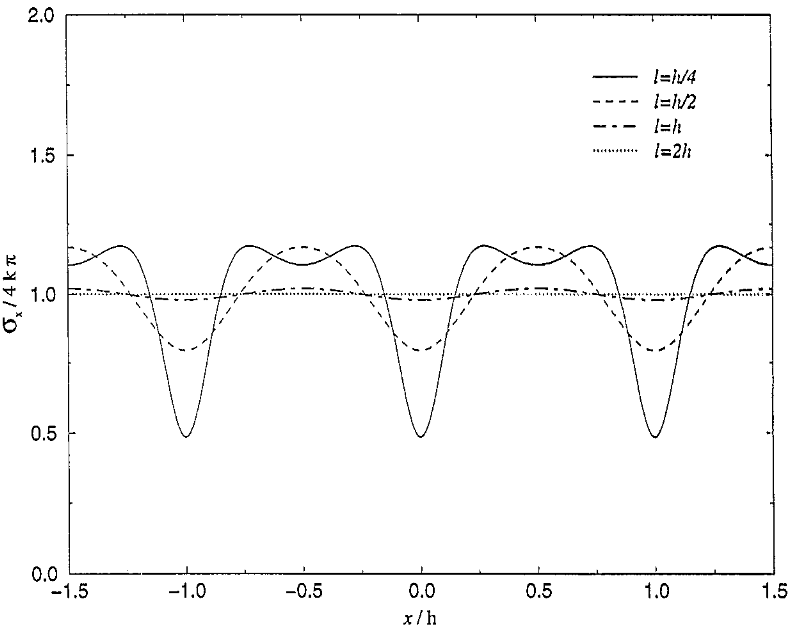

Fig. 7. The variation of the normal stress $\sigma_x(\xi, 0)$ along the surface $y = 0$ caused by infinite dislocation array from Fig. 6. A pronounced oscillatory stress behavior at small values of l disappears for $l > h$, when the stress becomes essentially constant and equal to $4\pi k$.

stress $\sigma_x(\xi, 0)$ is essentially constant and equal to $4\pi k$.

A simple result about the nature of the stress distribution in a semi-infinite body is discovered when the dislocation array is modeled by a continuous distribution of infinitesimal dislocations of density $1/h$. By appropriate integration of (35)–(37) and (38)–(40), it follows that the normal stress σ_y and the shear stress τ_{xy} are identically equal to zero throughout the body, while the normal stress σ_x is constant and equal to $4\pi k$ above the dislocation array ($y > -l$), and zero everywhere below the dislocation array ($y < -l$). The considered dislocation array, therefore, sets in a constant surface tension in a thin layer adjacent to the free surface, and the zero stress below it. This produces a constant misfit strain (of amount b/h), well-known from the thin film and the misfit dislocation studies. Note that the specific strain energy in the layer is equal to $\mu(b/h)^2/(1-\nu)$. In this expression the spacing h should be less or at most equal to the layer thickness l, so that the stress in the corresponding problem with a discrete dislocation distribution becomes approximately constant throughout the layer. Indeed, from the formulas given in the Appendix for the array with a discrete dislocation distribution, Fig. 8 shows the normal stress $\sigma_x(\pm h/2, \eta)$ along the vertical lines at distance $h/2$ from the dislocation center, i.e. exactly in-between the two adjacent dislocations from the array. Clearly, for l greater than h the results of a continuous distribution are nearly coincident with those of a discrete distribution (except near the dislocation cores, along the vertical lines passing through the dislocation centers).

It is not hard to see that a uniform continuous dislocation distribution of density $1/h^2$, everywhere in the layer $y > -l$ beneath the free surface, produces a linear stress distribution $\sigma_x = 4\pi k(l+y)/h$ within the layer, and zero stress outside the layer. This result by itself is of interest, since it can be conveniently utilized, in the spirit of Mura (1987), to interpret various results of the mathematical theory of plasticity.

According to Herring (1951), dislocation array just beneath the free surface will spontaneously form if the lowering of the surface energy caused by the array is greater than the strain energy of the array itself. The change of a specific surface energy γ associated with a surface strain of amount $\epsilon = b/h$ is $(\partial\gamma/\partial\epsilon)b/h$. Since $\partial\gamma/\partial\epsilon = f - \gamma$, where f is the

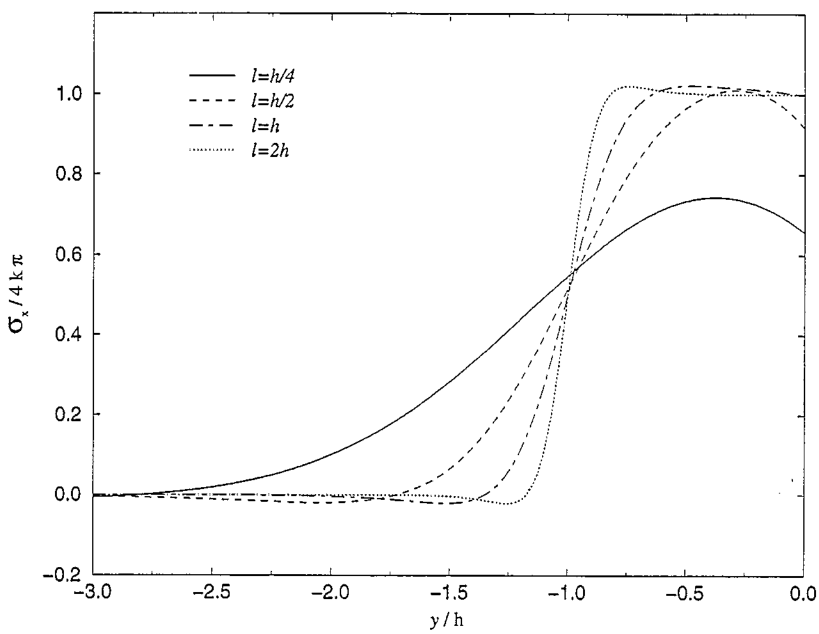

Fig. 8. The variation of the normal stress $\sigma_x(\pm h/2, \eta)$ produced by the infinite dislocation array from Fig. 6. For $l > h$, the stress becomes nearly constant and equal to $4\pi k$ in the subsurface layer $-l < y < 0$, and equal to zero in the rest of the body ($y < -l$).

surface stress (Herring, 1951), the nucleation condition becomes $|f-\gamma| > (l/h)\mu b/(1-\nu)$. For most materials the condition is not satisfied, although for some materials it could be, particularly for those with a compressive rather than tensile surface stress ($f<0$). It should be also pointed out that an estimate of the specific strain energy due to dislocation array used by Herring (1951) and Cammarata (1994) is based on an approximate result for a single dislocation near the free surface, which underestimates the actual strain energy. An exact expression for the energy of the array has been derived by Willis et al. (1990). An alternative derivation was given by Lubarda (1996).

5. Conclusion

The closed form solutions for the stress fields due to various dislocation arrays at the interface between two different materials are derived. The well-known results for dislocation arrays in a homogeneous body follow as a special case from these more general expressions. The arrays with discrete and continuous dislocation distributions are both analyzed. The stress field of the arrays with a continuous distribution of infinitesimal dislocations is identical to the far-away stress field of the arrays with a discrete dislocation distribution. The stress field due to an infinite array of edge dislocations beneath the free surface of a semi-infinite body shows that this dislocation array can produce a constant surface stress in a thin layer adjacent to the free surface, with no stress below it. The solution is used to discuss under what condition is a spontaneous formation of the dislocation array a likely event.

Appendix A

An expression for the normal stress σ_x due to dislocation array in Fig. 6, along the vertical line passing through the dislocation center, is obtained as the sum of Eqs. (35) and (38) at $x=0$, i.e.

$$\sigma_x = k\sum_{n=-\infty}^{\infty}\left\{(\eta+l_0)\frac{3n^2+(\eta+l_0)^2}{\left[n^2+(\eta+l_0)^2\right]^2} - (\eta-l_0)\frac{3n^2+(\eta-l_0)^2}{\left[n^2+(\eta-l_0)^2\right]^2} + 2l_0\frac{n^2-(\eta-l_0)^2}{\left[n^2+(\eta-l_0)^2\right]^2} - 4l_0\eta(\eta-l_0)\frac{3n^2-(\eta-l_0)^2}{\left[n^2+(\eta-l_0)^2\right]^3}\right\}, \tag{A.1}$$

where $\eta=y/h$, $l_0=l/h$ and $k=k_0 b/h$. By using the well-known sum

$$\sum_{n=-\infty}^{\infty}\frac{1}{z^2+n^2}=\frac{\pi}{z}\coth(\pi z), \tag{A.2}$$

Eq. (A.1) can be cast in the following compact form

$$\sigma_x = k\pi\left[2(\coth\varphi_1-\coth\varphi_2)-\frac{\varphi_1}{\sinh^2\varphi_1} + \frac{\varphi_2}{\sinh^2\varphi_2}-\frac{2\pi l_0}{\sinh^2\varphi_2}(1-2\pi\eta\coth\varphi_2)\right], \tag{A.3}$$

with $\varphi_1=\pi(y+l)/h$, and $\varphi_2=\pi(y-l)/h$.

An expression for the normal stress σ_x along the vertical line at distance $h/2$ from the dislocation center is obtained from (A.1) by replacing n with $2n+1$ and by introducing the appropriate definitions of η and l_0. Then, since

$$\sum_{n=-\infty}^{\infty}\frac{1}{z^2+(2n+1)^2}=\frac{\pi}{2z}\tanh\frac{\pi z}{2}, \tag{A.4}$$

it follows that

$$\sigma_x = k\pi\left[2(\tanh\varphi_1-\tanh\varphi_2)+\frac{\varphi_1}{\cosh^2\varphi_1} - \frac{\varphi_2}{\cosh^2\varphi_2}+\frac{2\pi l_0}{\cosh^2\varphi_2}(1-2\pi\eta\tanh\varphi_2)\right]. \tag{A.5}$$

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