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Plastic Spin and Related Issues in Phenomenological Plasticity

The role of plastic spin within the framework of a phenomenological polycrystalline plasticity is examined. We show that if elastic and plastic strain rates are properly identified, partitioning of total spin and identification of its "plastic" part is not required in the elastoplastic constitutive analysis of elastically isotropic materials.

1 Introduction

In recent years the subject of the partitioning of the total spin into its elastic and plastic parts has received a considerable attention within the framework of polycrystalline phenomenological plasticity theory. We will show that such partitioning is arbitrary in the absence of a physical structure that describes the process of the plastic flow as in single crystal plasticity. In fact, within a phenomenological plasticity theory, there is no need to partition the total spin into its constituents nor make any reference to them.

To introduce the so-called plastic spin within a polycrystalline, phenomenological plasticity theory is a consequence of the improper identification of elastic and plastic parts of the total strain rate. This led to the statement in the literature that a tacit assumption of the Prandtl-Reuss equations is that the plastic spin is zero. Furthermore, it has been interpreted by some that a more complete plasticity theory can be obtained by including a constitutive law for plastic spin. Advocates of such an approach have made a superficial parallel between polycrystalline plasticity and single crystal plasticity, in which an additional physical structure is introduced to describe material flowing through the crystal lattice.

Within a phenomenological plasticity of polycrystalline materials, plastic spin has been introduced to describe certain features of anisotropic hardening (Dafalias, 1983; Loret, 1983). These attempts were motivated by the observation that spurious oscillations of shear stress occurred in the analysis of monotonically increasing simple shear (Nagtegaal and de Jong, 1982). Various plastic spin measures were subsequently introduced and discussed by many authors; i.e., Dafalias (1985, 1987), Dashner (1986), Zbib and Aifantis (1988), Aravas and Aifantis (1991), and others. Earlier work on the subject of plastic spin and its significance also includes Mandel (1973) and Kratochvil (1973). Since most of the work on plastic spin utilizes the multiplicative decomposition of deformation gradient into its elastic and plastic parts, introduced by Lee (1969), the same is done here. Elastoplastic constitutive analysis based on this decomposition has been a topic of active research in the last several decades (Willis, 1969; Freund, 1970; Mandel, 1973; Sidoroff, 1975; Kleiber 1975; Lubliner 1978; Lubarda and Lee 1981; Needleman 1985; Agah-Tehrani et al. 1987; Moran et al., 1990; Lubarda, 1991a, 1991b, etc.). Various issues related to applicability of the decomposition in elastoplasticity were discussed by Green and Naghdi (1971), Nemat-Nasser (1979, 1982), Casey and Naghdi (1980), and Naghdi (1990). In the context of single crystal plasticity, the multiplicative decomposition was used by Asaro and Rice (1977), Hill and Havner (1982), Asaro (1983a, 1983b), Mohan et al. (1992), and others.

2 Partition of Elastoplastic Strain Rate and Spin into Their Constituents

To elucidate our ideas we restrict attention to elastically isotropic polycrystalline materials that can harden in an arbitrary manner during the course of elastoplastic deformation. Let $\mathbf{F}$ be the deformation gradient that deforms the body to its current configuration. Following Lee (1969), we introduce an intermediate configuration which is obtained from the current configuration by (possibly virtual) elastic distressing to zero stress. The multiplicative decomposition then holds

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p, \tag{1}$$

where $\mathbf{F}_e$ is the elastic and $\mathbf{F}_p$ the plastic part of deformation gradient $\mathbf{F}$. The intermediate configuration is not uniquely defined since it remains unstressed after any superposed rigid-body rotation. Alternatively, the intermediate configuration can be obtained by the mapping $\mathbf{F}_p$ applied to the initial configuration. To the extent that $\mathbf{F}_p$ contains an unspecified rotation the intermediate configuration is also nonunique.

We adopt the assumption that plastic deformation does not affect the elastic characteristics of the material. In this case, the response of elastically isotropic material does not depend on any rotation superposed to the intermediate configuration.

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The hyperelasticity law defined with respect to any intermediate configuration is (Truesdell and Noll, 1965)

$$\tau = 2\mathbf{F}_e \frac{\partial \psi_e(\mathbf{C}_e)}{\partial \mathbf{C}_e} \mathbf{F}_e^T, \tag{2}$$

Here ψ_e, the strain energy per unit unstressed volume, is an isotropic function of the right Cauchy-Green deformation tensor $\mathbf{C}_e = \mathbf{F}_e^T\mathbf{F}_e$ and $\tau = (\det\mathbf{F}_e)\sigma$ is the Cauchy stress σ weighted by $\det\mathbf{F}_e$. If plastic deformation is taken to be incompressible, $\det\mathbf{F}_e = \det\mathbf{F}$ and τ is then the Kirchhoff stress.

Using (1) in the velocity gradient expression $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$, we obtain

$$\mathbf{L} = \dot{\mathbf{F}}_e\mathbf{F}_e^{-1} + \mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}, \tag{3}$$

where superimposed dot denotes the material time derivative. The strain rate $\mathbf{D}$ and spin $\mathbf{W}$ are given by the symmetric and antisymmetric parts of $\mathbf{L}$:

$$\mathbf{D} = (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s + [\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_s \tag{4}$$

$$\mathbf{W} = (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a + [\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a. \tag{5}$$

The first term on the right-hand side of (4) has often been identified as the elastic strain rates and the second term as the plastic strain rate. Other definitions of elastic and plastic strain rates have also been proposed. These identifications are made without taking full use of the kinetics of elastoplastic deformation. Similarly, the first term on the right-hand side of (5) has often been identified as the elastic spin and the second term as the plastic spin. However, the later partitioning is arbitrary in the absence of a relevant physical structure upon which spin can be defined. We shall show that the need for the partitioning $\mathbf{W}$ arises from an improper identification of the elastic and plastic strain rates in (4). When the elastic and plastic strain rates are properly identified, only the total spin enters the elastoplastic constitutive formulation. Thus the partitioning of $\mathbf{W}$ is unnecessary.

We begin with partitioning $\mathbf{D}$ into $\mathbf{D}_e$ and $\mathbf{D}_p$. We note that the elastic and plastic strain rate cannot be identified from (4) and the corresponding kinematics, since $\mathbf{F}_e$ or $\mathbf{F}_p$ are not uniquely defined. Furthermore, any proposed partitioning of $\mathbf{D}$ which does not take into considerations the kinetics of elastoplastic deformation is insufficiently specified and arbitrary.

To develop this line of reasoning, we begin by differentiating the hyperelasticity relation (2) to obtain

$$\dot{\tau} - (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})\tau - \tau(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})^T = \hat{\mathbf{A}}_e : (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s. \tag{6}$$

Here : denotes the trace product and $\hat{\mathbf{A}}_e$ is the corresponding tensor of elastic moduli, with the components

$$\hat{A}^e_{ijkl} = 4F^e_{im}F^e_{jn}\frac{\partial^2\psi_e(\mathbf{C}_e)}{\partial C^e_{mn}\partial C^e_{pq}}F^e_{kp}F^e_{lq}. \tag{7}$$

The left-hand side of (6) is the Lie (or convected) derivative of stress τ, with respect to $\mathbf{F}_e$ (e.g., Simo and Ortiz, 1985). Equation (6) can be equivalently rewritten in terms of the Jaumann derivative with respect to the spin $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a$, as

$$\dot{\tau} - (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a\tau + \tau(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a = \mathbf{A}_e : (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s. \tag{8}$$

The elastic moduli tensor $\mathbf{A}_e$ has the components

$$A^e_{ijkl} = \frac{1}{2}(\delta_{ik}\tau_{jl} + \delta_{il}\tau_{jk} + \tau_{ik}\delta_{jl} + \tau_{il}\delta_{jk}) + \hat{A}^e_{ijkl}, \tag{9}$$

where δ_{ij} is the Kronecker delta.

If deformation is purely elastic, i.e., $\mathbf{F} = \mathbf{F}_e$, then clearly $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$ in (8) is the elastic strain rate. In that case, $\mathbf{F}_e$ is defined relative to a nonchanging (initial) configuration. However, if deformation is elastoplastic, $\mathbf{F}_e$ is defined relative to an intermediate configuration, which is changing during elastoplastic deformation. Consequently, two difficulties can arise in the identification of the elastic strain rate. First, since $\mathbf{F}_e$ and $\mathbf{F}_p$ are specified only to within an arbitrary rotation, $\mathbf{F}_e\mathbf{F}_e^{-1}$ and its symmetric and antisymmetric parts are nonunique. Therefore, a precise physical interpretation of strain rate measure $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$ and the assertion that it represents the elastic part of the total strain rate cannot be made. Secondly, the deforming intermediate configuration also contributes to the elastic strain rate so that it is not in general given by $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$. These difficulties are resolved in the treatment presented below.

The elastic strain rate is defined by the kinetic relation $\mathbf{D}_e = \mathbf{A}_e^{-1} : \overset{\circ}{\tau}$, where $\mathbf{D}_e$ represents the reversible part of $\mathbf{D}$ in the sense that the strain increment $\mathbf{D}_e dt$ is recovered upon loading-unloading cycle of the stress increment $\overset{\circ}{\tau}dt$. Here, $\overset{\circ}{\tau} = \dot{\tau} - \mathbf{W}\tau + \tau\mathbf{W}$ is the Jaumann derivative relative to the material spin $\mathbf{W}$ of the Kirchhoff stress $\tau = (\det\mathbf{F})\sigma$. With $\mathbf{D}_e$ defined in this way, the remaining part of the total strain rate, $\mathbf{D}_p = \mathbf{D} - \mathbf{D}_e$, is plastic (residual) contribution. This $\mathbf{D}_p$ can be shown to be governed by plastic potential and codirectional with the outward normal to current yield surface in the Cauchy stress space (Hill and Rice, 1973). When the reference state is taken to momentarily coincide with the current state, these strain rates correspond to the choice of the logarithmic strain and its conjugate stress, as discussed in Hill (1978).

To obtain the desired form, $\mathbf{D}_e = \mathbf{A}_e^{-1} : \overset{\circ}{\tau}$, plastic incompressibility is assumed so that τ appearing in (8) and (9) is the Kirchhoff stress. We then use (5) to eliminate $(\mathbf{F}_e\mathbf{F}_e^{-1})_a$ from (8) to get

$$\dot{\tau} = \mathbf{A}_e : \{(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s - \mathbf{A}_e^{-1} : \{[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\}\}. \tag{10}$$

Therefore, with the elastic strain rate given by

$$\mathbf{D}_e = (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s - \mathbf{A}_e^{-1} : \{[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\}, \tag{11}$$

the inversion of (10) provides the desired relation

$$\mathbf{D}_e = \mathbf{A}_e^{-1} : \overset{\circ}{\tau}. \tag{12}$$

Note from (7) and (9) that the elastic compliance tensor $\mathbf{A}_e^{-1}$ possesses the following symmetry and reciprocity properties:

$$(A_e^{-1})_{ijkl} = (A_e^{-1})_{ijlk} = (A_e^{-1})_{jikl} = (A_e^{-1})_{klij}. \tag{13}$$

As a consequence $\mathbf{A}_e^{-1} : \overset{\circ}{\tau}$ is derivable from elastic rate potential $\phi_e = 1/2\,\mathbf{A}_e^{-1} : (\overset{\circ}{\tau} \otimes \overset{\circ}{\tau})$ as its gradient $\partial\phi_e/\partial\overset{\circ}{\tau}$ ($\otimes$ denotes the tensor product) and, therefore, $\mathbf{D}_e$, defined by (12), gives the reversible strain increment that is recovered upon unloading of the Jaumann stress increment associated with $\overset{\circ}{\tau}$.

We note that the second part of the strain rate $\mathbf{D}_e$ in its representation (11) makes no contribution to elastic work, which follows by observing that, in view of elastic isotropy, the strain rate $\mathbf{A}_e^{-1} : \{[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\,\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\}$ has parallel principal directions to those of the associated stress rate $[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\,\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a$. Since the direction of the above stress rate is normal to τ, their trace is zero. Hence, $\dot{\psi}_e = \tau : \mathbf{D}_e \equiv \tau : (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$.

After identifying elastic strain rate, as given by (11) and (12), the remaining part of the total strain rate is the plastic (residual) part, which is governed by plastic potential and normal to the corresponding yield surface. From (4) and (11), therefore,

$$\mathbf{D}_p = [\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_s + \mathbf{A}_e^{-1} : \{[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\}. \tag{14}$$

Observe that $\mathbf{A}_e^{-1}$ and $\overset{\circ}{\tau}$, appearing in (12), are independent of any superposed rotation on the intermediate configuration. *Therefore* (12) *specifies* $\mathbf{D}_e$ *uniquely*. In contrast, the constituents of $\mathbf{D}_e$, namely $(\mathbf{F}_e\mathbf{F}_e^{-1})_s$ and $[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a$, do depend on the choice of an intermediate configuration. Similar remarks apply to $\mathbf{D}_p$ and its representation (14).

We have shown that the right-hand side of (11) is the correct relation for the elastic strain rate, and not $(\mathbf{F}_e\mathbf{F}_e^{-1})_s$ alone. Nevertheless, the latter has been taken to be the elastic strain

rate in many studies. The following question arises naturally. Is there a choice of intermediate configuration for which $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$ is exactly $\mathbf{D}_e$? From (12) it can be seen that this is the case when the intermediate configuration is chosen such that the so-called "plastic spin," $[\dot{\mathbf{F}}_e(\mathbf{F}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a$, vanishes identically.

Within the framework under discussion, the above choice of spin represents a geometric (kinematic) specification of the intermediate configuration. It is not a constitutive assumption and has no consequences on (12). For any other choice of intermediate configuration, $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s$ is not the elastic strain rate $\mathbf{D}_e$ $(= \mathbf{A}_e^{-1} : \overset{\circ}{\tau})$, as can be seen from (11).

We could just as well define an intermediate configuration by requiring that the "elastic spin," $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a$ vanish identically. In this case the plastic spin is exactly equal to total spin, $[\dot{\mathbf{F}}_e(\mathbf{F}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a = \mathbf{W}$. The end result is still Eq. (12), as can be checked by inspection. Therefore, the partitioning of the total spin into elastic and plastic parts by direct identification of the terms in (5), or more elaborate kinematical considerations, is superficial and unnecessary. These aspects appear to be overlooked in much of the related work on this subject.

3 An Alternative Derivation

We now present an alternative derivation of the expression for $\mathbf{D}_e$, which gives additional insight in the kinematics of elastoplastic deformation and partitioning of the strain rate. To that end and in light of the assumed isotropy of the strain energy function ψ_e, we rewrite the hyperelasticity law (2) in the form

$$\tau = 2\mathbf{V}_e \frac{\partial \psi_e(\mathbf{B}_e)}{\partial \mathbf{B}_e}\mathbf{V}_e, \tag{15}$$

where $\tau = (\det \mathbf{V}_e)\sigma$ is the Kirchhoff stress since plastic incompressibility is assumed. Note that only the elastic stretch tensor $\mathbf{V}_e$ and the deformation tensor $\mathbf{B}_e = \mathbf{V}_e^2$ appear in this representation, since the stress response does not depend on rotation $\mathbf{R}_e$ associated with $\mathbf{F}_e$ in its stretch-rotation decomposition $\mathbf{F}_e = \mathbf{V}_e\mathbf{R}_e$.

Introduce the spin Ω, which is defined as the solution of the following equation (Lubarda, 1991a):

$$\mathbf{W} = (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_a + (\mathbf{V}_e\Omega\mathbf{V}_e^{-1})_a. \tag{16}$$

Ω is an invariant quantity, independent of the choice of intermediate configuration, as neither $\mathbf{W}$ nor $\mathbf{V}_e$ and its material derivative depend on the chosen intermediate configuration. It then follows by applying to (15) the Jaumann derivative with respect to Ω

$$\overset{*}{\tau} = (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})\tau + \tau(\mathbf{V}_e^{-1}\dot{\mathbf{V}}_e) + 2\mathbf{V}_e\left[\frac{\partial^2\psi_e(\mathbf{B}_e)}{\partial\mathbf{B}_e\otimes\partial\mathbf{B}_e} : \overset{*}{\mathbf{B}}_e\right]\mathbf{V}_e, \tag{17}$$

where the superposed $*$ is the Jaumann derivative with respect to the spin Ω; i.e., $(^*) = (\dot{}) - \Omega(\,) + (\,)\Omega$. To obtain the desired form in terms of $\overset{\circ}{\tau}$, we rearrange (16) to get

$$(\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_a = \mathbf{W} - \Omega, \tag{18}$$

and substitute (18) in (17) to reach the form

$$\overset{\circ}{\tau} = (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s\tau + \tau(\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s + 4\mathbf{V}_e\left\{\frac{\partial^2\psi_e(\mathbf{B}_e)}{\partial\mathbf{B}_e\otimes\partial\mathbf{B}_e} : [\mathbf{V}_e(\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s\mathbf{V}_e]\right\}\mathbf{V}_e. \tag{19}$$

Therefore, Eq. (19) can be written in the form

$$\overset{\circ}{\tau} = \mathbf{A}_e : \mathbf{D}_e, \tag{20}$$

where $\mathbf{A}_e$ is the well-known elastic moduli tensor with components

$$A^e_{ijkl} = \frac{1}{2}(\delta_{ik}\tau_{jl} + \delta_{il}\tau_{jk} + \tau_{ik}\delta_{jl} + \tau_{il}\delta_{jk}) + 4V^e_{im}V^e_{jn}\frac{\partial^2\psi_e(\mathbf{B}_e)}{\partial B^e_{mn}\partial B^e_{pq}}V^e_{pk}V^e_{ql}, \tag{21}$$

provided that the elastic strain rate is defined by

$$\mathbf{D}_e = (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s. \tag{22}$$

This alternative representation of the elastic strain rate involves only kinematic quantities ($\mathbf{V}_e$ and Ω), while the previous representation (11) involves both, kinematic and kinetic quantities. Of course, the two representations are equivalent, as has been established by the preceding derivation. Note also that elastic moduli tensor (21) exactly coincides with that in (9), due to isotropy of the strain energy function ψ_e. Inversion of (20) therefore again gives the elastic strain rate expression (12).

Kinematic interpretation of (22) is more clear when (22) is written in the form

$$\mathbf{D}_e = (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s + (\mathbf{V}_e\Omega\mathbf{V}_e^{-1})_s. \tag{23}$$

The above shows that $\mathbf{D}_e$ has two contributions. One contribution is from the elastic stretching rate $(\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s$ and the other from the spin Ω. This contribution is in general related to both, $\dot{\mathbf{R}}_e\mathbf{R}_e^{-1}$ and $\dot{\mathbf{F}}_p\mathbf{F}_p^{-1}$, and represents the effect of deforming and rotating intermediate configuration, as well as of the rotation due to $\mathbf{R}_e$. Note that this term has no contribution to the work, i.e., the rate of work (per unit initial volume) expended in elastic deformation is $\dot{\psi}_e = \tau : \mathbf{D}_e = \tau : (\dot{\mathbf{V}}_e\mathbf{V}_e^{-1})_s$, since in view of elastic isotropy, $\mathbf{V}_e^{-1}\tau\mathbf{V}_e = \tau$, and $\tau : \mathbf{V}_e\Omega\mathbf{V}_e^{-1} = \tau : \Omega = 0$.

The derivation presented here, (15)–(23), is analogous to a rate-type finite elasticity formulation. The spin Ω is then simply $\Omega = \dot{\mathbf{R}}\mathbf{R}^{-1}$, where $\mathbf{R}$ is the rotation from the polar decomposition of deformation gradient $\mathbf{F} = \mathbf{V}\mathbf{R}$. In elastoplasticity the spin Ω is uniquely defined by (16), with the total spin on the left-hand side and elastic stretch on the right-hand side.

As we have shown, elastic strain rate $\mathbf{D}_e$ in both representations (11) or (22), does not depend on a superposed rotation on the intermediate configuration. For example, one can choose $\mathbf{R}_e = \mathbf{I}$; i.e., define intermediate configuration to be obtained by distressing without rotation (Lee, 1969), or one can choose distressing program which makes the spin of intermediate configuration to vanish, $(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})_a = \mathbf{0}$, etc. In each case the final constitutive equation is given by (12) and the numerical values of $\mathbf{D}_e$ and Ω are not affected by the choice of an intermediate configuration. However, their form in terms of kinematic quantities $\mathbf{F}_e$, $\mathbf{F}_p$ and their rates do depend on the intermediate configuration. For example, in the case of intermediate configuration defined by $\mathbf{R}_e = \mathbf{I}$, the spin Ω has (for isotropic hardening) a simple, explicit representation $\Omega = (\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})_a$, while the plastic strain rate becomes $\mathbf{D}_p = (\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})_s$. If the intermediate configuration is defined by distressing such that $[\dot{\mathbf{F}}_e(\mathbf{F}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a = 0$, from (5) and polar decomposition of $\mathbf{F}_e$ it follows that $\Omega = \dot{\mathbf{R}}_e\mathbf{R}_e^{-1}$, while $\mathbf{D}_p = \mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}$. Additional kinematic analysis of the elastic strain rate expression and spin Ω is given in Section 6.

A comment should also be made regarding the definition of elastic strain rate used in some related work. In those studies, elastic strain rate is defined by an equation of the type (22), but with the Jaumann rate of $\mathbf{V}_e$ taken relatively to the spin of the director vectors triad, which is introduced to cope with the anisotropic hardening features (referred to as substructural spin). However, for elastically isotropic material the elastic strain rate should not depend on such a spin, nor on any other variable that is introduced to handle the anisotropic hardening. These variables directly effect only the constitutive expression for the plastic strain rate, but should not be present in the definition of the elastic part of the total strain rate. Indeed, in our definition of the elastic strain rate (22), the Jaumann

rate of $\mathbf{V}_e$ is taken relatively to spin Ω which depends only on $\mathbf{V}_e$, $\dot{\mathbf{V}}_e$ and the total material spin $\mathbf{W}$ as shown in (16), regardless of the type of (isotropic or anisotropic) plastic hardening, or any variable that is introduced to describe it.

4 Elastoplastic Constitutive Structure

With the elastic (reversible) part of the strain rate identified by (12), the remaining part, $\mathbf{D}_p = \mathbf{D} - \mathbf{D}_e$, is plastic (residual) contribution. This is governed by the plastic potential and codirectional with the outward normal to current yield surface in the Cauchy stress space (Hill, 1967; Hill and Rice, 1973). It is $\mathbf{D}_p$ (in its entirety) that has to be constructed at the phenomenological level. In the case of isotropic hardening, with the yield function g as an isotropic scalar function of σ, one has the well-known representation

$$\mathbf{D}_p = \frac{1}{h}\left(\frac{\partial g}{\partial \sigma} : \overset{\circ}{\tau}\right)\frac{\partial g}{\partial \sigma}. \tag{24}$$

Here h is a scalar function of plastic deformation history and $\overset{\circ}{\tau}$ is defined for a reference configuration chosen to coincide, instantaneously, with the current configuration. This definition differs from the one used in Sections 2 and 3 by a scalar multiplier $\det\mathbf{F}_e$ which can be absorbed into h. Equations (12) and (24) complete the constitutive description of the considered model of material behavior, giving

$$\mathbf{D} = \mathbf{A}^{-1} : \overset{\circ}{\tau}, \quad \mathbf{A}^{-1} = \mathbf{A}_e^{-1} + \frac{1}{h}\frac{\partial g}{\partial \sigma} \otimes \frac{\partial g}{\partial \sigma}, \tag{25}$$

where $\mathbf{A}^{-1}$ is the instantaneous elastoplastic compliance tensor. The inverted form of (25) is

$$\overset{\circ}{\tau} = \mathbf{A} : \mathbf{D}, \quad \mathbf{A} = \mathbf{A}_e - \frac{1}{H}\mathbf{A}_e : \left(\frac{\partial g}{\partial \sigma} \otimes \frac{\partial g}{\partial \sigma}\right) : \mathbf{A}_e, \tag{26}$$

where $H = h + (\partial g/\partial\sigma) : \mathbf{A}_e : (\partial g/\partial\sigma)$. Transition from this elastoplastic constitutive structure, with the Jaumann derivative of stress $\overset{\circ}{\tau} = \dot{\tau} - \mathbf{W}\sigma + \sigma\mathbf{W}$, to one with the convected derivative $\overset{\circ}{\tau} = \dot{\tau} - \mathbf{L}\sigma - \sigma\mathbf{L}^T$, is direct via $\overset{\circ}{\tau} = \overset{\circ}{\tau} + \mathbf{D}\sigma + \sigma\mathbf{D}$. The corresponding elastoplastic moduli tensors differ by terms of the order of stress.

It is seen that the so-called plastic spin does not appear in the constitutive equations at all. Therefore, the statement recently often made in the literature that isotropic hardening Prandtl-Reuss equations are obtained when the so-called plastic spin is assumed to vanish is incorrect.

If hardening is anisotropic, the constitutive structure of $\mathbf{D}_p$ has to be accordingly constructed. In doing this, one introduces in the analysis some additional ingredients. For example, the back stress and its evolution law have been introduced in the case of the kinematic hardening model. However, the back stress model does not include any statement regarding plastic spin. It can be useful to introduce in the analysis a new spin measure, say the spin of the material line that instantaneously coincides with an eigendirection of the back stress (Lee et al., 1983), but this spin can be directly calculated in terms of the total spin and strain rate. Therefore, an additional constitutive equation for such a back stress (plastic) spin is not required.

As discussed previously, the identification of the constituents of the total spin must be carried out within the framework of a physical structure. This is outlined in the next section.

5 Crystal Plasticity

In single crystal plasticity, a definite additional structure is attached to intermediate configuration: material flows through the crystal lattice via dislocation motion, while the lattice preserves its orientation (Asaro 1983a, 1983b). In that way the intermediate configuration differs from the initial, undeformed configuration by deformation solely due to crystallographic slip, i.e.,

$$\dot{\mathbf{F}}_p\mathbf{F}_p^{-1} = \sum_i \dot{\gamma}^{(i)}\mathbf{s}^{(i)} \otimes \mathbf{m}^{(i)}. \tag{27}$$

Here $\dot{\gamma}^{(i)}$ is the shearing (slipping) rate of the ith active slip system, with the slip direction $\mathbf{s}^{(i)}$ in the slip plane with the normal $\mathbf{m}^{(i)}$. Hence, both symmetric and antisymmetric parts of $\dot{\mathbf{F}}_p\mathbf{F}_p^{-1}$, i.e., the strain rate and spin of the intermediate configuration are prescribed by (27). Integration of (27) leads to a unique $\mathbf{F}_p$. The elastic part of $\mathbf{F}$, $\mathbf{F}_e$, can be obtained from the solution of (1) and is also unique. In the context of crystal plasticity, $\mathbf{F}_e$ represents the stretching and rotation of the lattice with the material embedded on it. The intermediate configuration is therefore uniquely specified and it can be called isoclinic, in the sense of Mandel (1973).

Elastic strain rate can again be expressed by (11), i.e.,

$$\mathbf{D}_e = (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s - \mathbf{A}_e^{-1} : (\mathbf{W}_p\tau - \tau\mathbf{W}_p), \tag{28}$$

with

$$\mathbf{W}_p = [\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a. \tag{29}$$

The first term in the right-hand side of (28) is the lattice strain rate while the remaining term is the contribution from the deforming intermediate configuration induced by crystallographic slip.

Since $(\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a$ represents the lattice spin, from (5) we see that $\mathbf{W}_p = \mathbf{W} - (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a$ is the spin of the material relative to the lattice. In single crystal plasticity, $\mathbf{W}_p$ is customarily called plastic spin, but essential additional structure of the crystal plasticity is all contained in the kinematics of crystallographic slip, attached to intermediate configuration and described by (27), and associated kinetics which specifies the shearing rates $\dot{\gamma}^{(i)}$ in terms of stress and stress rate. From (4) and $\mathbf{D} = \mathbf{D}_e + \mathbf{D}_p$, the plastic strain rate is

$$\mathbf{D}_p = [\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_s + \mathbf{A}_e^{-1} : (\mathbf{W}_p\tau - \tau\mathbf{W}_p). \tag{30}$$

It is $\mathbf{D}_p$ that is normal to the yield surface, and not $[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_s$. In other words, in transforming the slip deformation $\dot{\mathbf{F}}_p\mathbf{F}_p^{-1}$ from intermediate to current configuration by elastic deformation $\mathbf{F}_e$, the corresponding strain rate $[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_s$ is equal to plastic strain rate $\mathbf{D}_p$ with an elastic contribution, due to stress rate $(\mathbf{W}_p\tau - \tau\mathbf{W}_p)$, subtracted off (see also Hill and Havner, 1982). Note that in the foregoing derivation, the lattice elasticity does not have to be isotropic (Lubarda, 1991b).

6 General Expression for Elastic Strain Rate

We show here that the elastic strain rate can also be represented in terms of an arbitrary $\mathbf{F}_e$, in the form

$$\mathbf{D}_e = (\overset{\bullet}{\mathbf{F}}_e\mathbf{F}_e^{-1})_s, \tag{31}$$

where $\overset{\bullet}{\mathbf{F}}_e = \dot{\mathbf{F}}_e - \Omega\mathbf{F}_e + \mathbf{F}_e\Omega_p$ is the Jaumann derivative of $\mathbf{F}_e$, taken relatively to spins Ω and Ω_p, which are associated with the current and intermediate states, respectively. Physically, so defined $\overset{\bullet}{\mathbf{F}}_e$ gives the change of elastic deformation gradient $\mathbf{F}_e$ observed in the coordinate systems that rotate with spin Ω in the current, and spin Ω_p in the intermediate configuration. Spin Ω is independent of the choice of intermediate configuration, whereas spin Ω_p does depend on that choice, as will be seen as follows. By equating (31) and (22),

$$(\overset{\bullet}{\mathbf{F}}_e\mathbf{F}_e^{-1})_s = (\overset{\bullet}{\mathbf{V}}_e\mathbf{V}_e^{-1})_s, \tag{32}$$

we therefore have

$$[\mathbf{V}_e(\dot{\mathbf{R}}_e\mathbf{R}_e^{-1} + \mathbf{R}_e\Omega_p\mathbf{R}_e^T)\mathbf{V}_e^{-1}]_s = (\mathbf{V}_e\Omega\mathbf{V}_e^{-1})_s, \tag{33}$$

from which, by a simple matrix argument, it follows that

$$\Omega_p = \mathbf{R}_e^T(\Omega - \dot{\mathbf{R}}_e\mathbf{R}_e^{-1})\mathbf{R}_e. \tag{34}$$

This expresses the spin Ω_p in terms of the spin Ω, rotation $\mathbf{R}_e$, and its rate $\dot{\mathbf{R}}_e$. Recall that the spin Ω is uniquely determined

by (16). Clearly from (34), Ω_p depends on a particular choice of intermediate configuration. For example, if the intermediate configuration is defined by distressing without rotation ($\mathbf{R}_e = \mathbf{I}$), $\Omega_p \equiv \Omega$. If the intermediate configuration is defined by $[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a = \mathbf{0}$, we have shown in Section 3 that $\Omega = \mathbf{R}_e\dot{\mathbf{R}}_e^{-1}$, and therefore $\Omega_p \equiv 0$. Of course, in each case the elastic strain rate is the same.

We can also write (31) in the expanded form

$$\mathbf{D}_e = (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_s + (\mathbf{F}_e\Omega_p\mathbf{F}_e^{-1})_s, \tag{35}$$

which shows that besides the usual contribution given by the first term on the right-hand side, the elastic strain rate $\mathbf{D}_e$, has an additional contribution, given by the second term, which is caused by the spin Ω_p due to deforming and rotating intermediate configuration. If intermediate configuration is defined by distressing without rotation ($\mathbf{F}_e = \mathbf{V}_e$), above coincides with (23). Comparing (35) with (11), we in addition have

$$(\mathbf{F}_e\Omega_p\mathbf{F}_e^{-1})_s = -\mathbf{A}_e^{-1} : \{[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\tau - \tau[\mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1}]_a\}. \tag{36}$$

We note that the representation (31) can be established directly, by applying the Jaumann derivative $\circ$ to both sides of the hyperelasticity law (2). By demanding $\Omega + (\dot{\mathbf{F}}_e\mathbf{F}_e^{-1})_a = \mathbf{W}$ in an analogous procedure to that described in Section 3, we again arrive at (31) and (34).

Finally, if we rewrite (34) in the form

$$\Omega = \dot{\mathbf{R}}_e\mathbf{R}_e^{-1} + \mathbf{R}_e\Omega_p\mathbf{R}_e^T, \tag{37}$$

we see that the invariant spin Ω has two contributions: one from spin $\dot{\mathbf{R}}_e\mathbf{R}_e^{-1}$, and another which is induced from spin Ω_p and which represents the effect of deforming and rotating intermediate configuration. Both of these contributions depend on the choice of an intermediate configuration, but their combination giving Ω does not.

Results presented in this and previous sections should remove disagreements that appeared in the literature over a long period of time regarding the invariance requirements and independence of the final constitutive equations of the selected intermediate configuration used in developing the theory.

7 Concluding Remarks

We have shown in this paper that, within the framework of a phenomenological plasticity theory, there is no basis and no necessity for partitioning the total spin into its elastic and plastic constituents. Statements to the contrary in literature are the result of improper partitioning of total strain rate and subsequent misinterpretations. The introduction of various ingredients to treat anisotropic hardening, such as back stress or similar internal variables, only affects the structure of the constitutive equation for the plastic strain rate, but does not provide a basis for the identification of the plastic spin. Indeed, we have shown that such identification in the context of a phenomenological theory is unnecessary.

The situation is quite different in single crystal plasticity, where a specific additional structure is introduced in the model (that of material flowing through the crystal lattice) while the lattice preserves its orientation. Material and lattice then have different spins, and it is this difference that is identified as the plastic spin.

A remark should be also made regarding the elastically anisotropic, say persistently orthotropic materials. In this case the rotation of the principal axes of orthotropy is not given by the overall deformation gradient but must be independently determined. This can be accomplished by introducing a constitutive equation for an associated spin, but a reliable procedure for doing this is yet to be proposed. In analogy with single crystal plasticity, one could define the intermediate configuration to be isoclinic, i.e., material "flows" through the structure with fixed oriented axes of orthotropy (director vectors). However, the kinematic and kinetic description of such a process for the polycrystalline material is currently not available. In single crystal plasticity this process is crystallographic slip, i.e., the kinematics of simple shearing and kinetic relationships specifying the shearing rates in terms of stress and stress rate.

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