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Short communication

Remarks on axially and centrally symmetric elasticity problems

V.A. Lubarda *

Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

A R T I C L E I N F O

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The solutions to axially and centrally symmetric Lamé problems are derived within the displacement, stress and strain-based approach by the specification of the general boundary conditions which encompass all possible combinations of kinematic and kinetic conditions at the inner and outer boundaries. It is shown that the mathematical structure of the governing differential equations in the strain-based approach is identical to that in the stress-based approach. A reduction of the compatibility conditions from general two and three-dimensional elasticity to their form applicable to axially and centrally symmetric problems is also discussed.

1. Introduction

The purpose of this note devoted to classical Lamé elasticity problems of pressurized hollow disk/cylinder or pressurized sphere [1–3] is to indicate the following four results that were not previously reported in the literature:

(a) The solutions of Lamé problems can be derived simultaneously for general boundary conditions, which encompass all possible combinations of kinematic and kinetic conditions at the inner and outer boundaries.
(b) The solutions of Lamé problems can be obtained by using the displacement, stress, or strain-based approach, independently of each other (apart from boundary conditions).
(c) The governing differential equations of the stress and strain-based approach have the identical mathematical structure for axially symmetric plane elasticity problems, and for centrally symmetric three-dimensional problems.
(d) The reduction of the compatibility conditions from the general elasticity to their form applicable to axially or centrally symmetric problems needs to be done with care to avoid a redundant integration constant associated with a second, rather than a first order differential equation expressing the actual compatibility condition for these problems.

2. Axially symmetric Lamé problem

A thin hollow disk (plane stress conditions), or a long hollow cylinder (plane strain conditions) is subjected to axisymmetric displacement or stress boundary conditions $u(a_i) = u_i$ or $\sigma_r(a_i) = -p_i$, where a_1 are a_2 are the inner and outer radii of the hollow disk or cylinder. The Cauchy differential equation of equilibrium, in the absence of body forces, is [3]

* Tel.: +1 858 534 3169; fax: +1 858 534 5698.
E-mail address: vlubarda@ucsd.edu

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$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tag{1}$$

where σ_r is the radial and σ_θ the hoop stress. The corresponding radial and hoop strains are related to the radial displacement $u = u(r)$ by $\epsilon_r = du/dr$ and $\epsilon_\theta = u/r$. From these relations, the compatibility condition for strain components is $\epsilon_r = d(r\epsilon_\theta)/dr$, which can be rewritten as [4–6]

$$\frac{d\epsilon_\theta}{dr} - \frac{\epsilon_r - \epsilon_\theta}{r} = 0. \tag{2}$$

This is the Saint–Venant compatibility condition in terms of strains for the problem under consideration.

The stress–strain relations in the case of plane stress (thin disk) are

$$\epsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta), \quad \epsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r), \tag{3}$$

with the inverse relations

$$\sigma_r = \frac{E}{1-\nu^2}(\epsilon_r + \nu\epsilon_\theta), \quad \sigma_\theta = \frac{E}{1-\nu^2}(\epsilon_\theta + \nu\epsilon_r). \tag{4}$$

The Young modulus of elasticity is E, and ν is the Poisson coefficient of lateral contraction. In the case of plane stain (long cylinder), E and ν are replaced by $E/(1-\nu^2)$ and $\nu/(1-\nu)$, respectively.

2.1. Displacement-based solution

The well-known solution of the considered problem is most commonly derived by using the displacement approach (*e.g*, [7,8]). The strain–displacement relations are substituted into constitutive relations Eq. (4), and these into the equilibrium equation Eq. (1). The resulting differential equation for the radial displacement is $u'' + u'/r - u/r^2 = 0$, having the solution

$$u = c_1 r + \frac{c_2}{r}. \tag{5}$$

The general boundary conditions at the inner and outer surface, expressed in terms of the displacement and its gradient, are

$$u(a_i) + \varphi_i a_i \left(\frac{du}{dr}\right)_{r=a_i} = \eta_i a_i, \tag{6}$$

where

$$\varphi_i = 0, \quad \eta_i = \frac{u_i}{a_i}, \qquad [\text{for prescribed } u(a_i) = u_i],$$
$$\varphi_i = \frac{1}{\nu}, \quad \eta_i = -\frac{1-\nu^2}{\nu}\frac{p_i}{E}, \qquad [\text{for prescribed } \sigma_r(a_i) = -p_i].$$

The corresponding integration constants c_1 and c_2 are readily found to be

$$c_1 = \frac{1}{c}\left[(1-\varphi_1)\eta_2 a_2^2 - (1-\varphi_2)\eta_1 a_1^2\right], \; c_2 = \frac{1}{c}[(1+\varphi_2)\eta_1 - (1+\varphi_1)\eta_2]a_1^2 a_2^2, \tag{7}$$

with the parameter $c = (1-\varphi_1)(1+\varphi_2)a_2^2 - (1+\varphi_1)(1-\varphi_2)a_1^2$.

2.2. Stress-based solution

The differential equation of equilibrium Eq. (1) contains two unknown stress components $(\sigma_r, \sigma_\theta)$, which makes the problem statically indeterminate. To provide the second needed differential equation, the Saint–Venant compatibility condition is recast by eliminating strain in terms of stress components. When Eq. (3) is substituted into Eq. (2), and with a help of the equilibrium condition Eq. (1), there follows:

$$\frac{d\sigma_\theta}{dr} - \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{8}$$

This is the Beltrami–Michell compatibility condition in terms of stresses for the considered problem. By comparing Eq. (2) and Eq. (8), it is seen that the Beltrami–Michell condition in terms of stresses has the same mathematical form as the Saint–Venant compatibility condition in terms of strains. By solving Eqs. (1) and (8), the radial and hoop stress components are

$$\sigma_r = C_1 + \frac{C_2}{r^2}, \quad \sigma_\theta = C_1 - \frac{C_2}{r^2}. \tag{9}$$

The constant C_2 can be given a physical interpretation in terms of the maximum inplane shear stress, $C_2 = \tau_{\max} a_1^2$, while C_1 represents the uniform average inplane normal stress.[1]

The general boundary conditions, expressed in terms of stresses, are

$$\sigma_r(a_i) + \alpha_i \sigma_\theta(a_i) = \beta_i, \tag{10}$$

where

$$\alpha_i = -\frac{1}{\nu}, \quad \beta_i = -\frac{E}{\nu}\frac{u_i}{a_i}, \qquad [\text{for prescribed } u(a_i) = u_i],$$
$$\alpha_i = 0, \quad \beta_i = -p_i, \qquad [\text{for prescribed } \sigma_r(a_i) = -p_i].$$

The corresponding integration constants C_1 and C_2 are readily found to be

$$C_1 = \frac{1}{C}\left[(1-\alpha_1)\beta_2 a_2^2 - (1-\alpha_2)\beta_1 a_1^2\right], \ C_2 = \frac{1}{C}[(1+\alpha_2)\beta_1 - (1+\alpha_1)\beta_2]a_1^2 a_2^2, \tag{11}$$

with $C = (1-\alpha_1)(1+\alpha_2)a_2^2 - (1+\alpha_1)(1-\alpha_2)a_1^2$.

Having determined the stress components, the radial displacement is calculated from $u = r\epsilon_\theta = r(\sigma_\theta - \nu\sigma_r)/E$, which gives

$$u = \frac{1}{E}\left[(1-\nu)C_1 r - (1+\nu)\frac{C_2}{r}\right]. \tag{12}$$

The connections between the integration constants c_1 and c_2 from the displacement-based approach, and C_1 and C_2 from the stress-based approach, follow by comparing Eqs. (5) and (12). The result is

$$c_1 = \frac{1-\nu}{E}C_1, \quad c_2 = -\frac{1+\nu}{E}C_2. \tag{13}$$

2.3. Strain-based solution

To derive the differential equation in strain domain, corresponding to equilibrium condition Eq. (1), we substitute the stress–strain relations Eq. (4) into Eq. (1), and use the Saint–Venant compatibility condition Eq. (2), to obtain

$$\frac{\mathrm{d}\epsilon_r}{\mathrm{d}r} + \frac{\epsilon_r - \epsilon_\theta}{r} = 0. \tag{14}$$

By comparing the differential equation of equilibrium Eq. (1) with Eq. (14), we conclude that the two equations, expressing the equilibrium in stress and strain domains, have the identical mathematical form. Since we have already shown that the compatibility condition in terms of strains Eq. (2) has the same form as the compatibility condition in terms of stresses Eq. (8), the solution of the differential equations Eqs. (2) and (14) is

$$\epsilon_r = k_1 + \frac{k_2}{r^2}, \quad \epsilon_\theta = k_1 - \frac{k_2}{r^2}, \tag{15}$$

in duality with Eq. (9).

The general boundary conditions, expressed in terms of strains, are

$$\varphi_i \epsilon_r(a_i) + \epsilon_\theta(a_i) = \eta_i, \tag{16}$$

where φ_i and η_i are defined by the expressions following Eq. (6). The corresponding integration constants k_1 and k_2 are readily found to be

$$k_1 = \frac{1}{k}\left[(1-\varphi_1)\eta_2 a_2^2 - (1-\varphi_2)\eta_1 a_1^2\right], \ k_2 = \frac{1}{k}[(1+\varphi_1)\eta_2 - (1+\varphi_2)\eta_1]a_1^2 a_2^2, \tag{17}$$

with $k = c$, given by the expression following Eq. (7). The corresponding radial displacement is

$$u = r\epsilon_\theta = k_1 r - \frac{k_2}{r}. \tag{18}$$

By comparing Eqs. (5), (12), and (18), the three sets of integration constants are related by

$$c_1 = k_1 = \frac{1-\nu}{E}C_1, \quad c_2 = -k_2 = -\frac{1+\nu}{E}C_2. \tag{19}$$

[1] By adding Eqs. (1) and (8), there follows $\mathrm{d}(\sigma_r + \sigma_\theta)/\mathrm{d}r = 0$. In a recent paper [9] this was used, in conjunction with Eq. (1), to derive a stress-based solution for the cylinder with a constrained inner and pressurized outer boundary.

3. Centrally symmetric Lamé problem

An analogous derivation proceeds in the case of a hollow sphere subjected to symmetric displacement or stress boundary conditions at the inner and outer surfaces. The Cauchy equation of equilibrium, in the absence of body forces, is

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}+2\frac{\sigma_r-\sigma_\theta}{r}=0, \tag{20}$$

where σ_r is the radial and $\sigma_\theta=\sigma_\phi$ is the hoop stress. The corresponding strains are related to the radial displacement $u=u(r)$ by $\epsilon_r=\mathrm{d}u/\mathrm{d}r$ and $\epsilon_\theta=\epsilon_\phi=u/r$. From these relations, the compatibility condition for strain components is $\epsilon_r=\mathrm{d}(r\epsilon_\theta)/\mathrm{d}r$, as in the case of axisymmetric problems, *i.e.*, Eq. (2). The stress–strain relations are

$$\epsilon_r=\frac{1}{E}(\sigma_r-2\nu\sigma_\theta),\quad \epsilon_\theta=\frac{1}{E}[(1-\nu)\sigma_\theta-\nu\sigma_r], \tag{21}$$

with the inverse relations

$$\sigma_r=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_r+2\nu\epsilon_\theta],\quad \sigma_\theta=\frac{E}{(1+\nu)(1-2\nu)}(\epsilon_\theta+\nu\epsilon_r). \tag{22}$$

3.1. Displacement-based solution

When the strain–displacement relations are substituted into constitutive relations Eq. (22), and these into the equilibrium equation Eq. (20), the resulting differential equation for the radial displacement is $u''+2u'/r-2u/r^2=0$, with the solution

$$u=c_1r+\frac{c_2}{r^2}. \tag{23}$$

The general boundary conditions at the inner and outer surface, expressed in terms of the displacement and its gradient, are

$$u(a_i)+\varphi_ia_i\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)_{r=a_i}=\eta_ia_i, \tag{24}$$

where

$$\varphi_i=0,\quad \eta_i=\frac{u_i}{a_i},\qquad [\text{for prescribed } u(a_i)=u_i],$$
$$\varphi_i=\frac{1-\nu}{2\nu},\quad \eta_i=-\frac{(1+\nu)(1-2\nu)}{2\nu}\,\frac{p_i}{E},\qquad [\text{for prescribed } \sigma_r(a_i)=-p_i].$$

The corresponding integration constants c_1 and c_2 are readily found to be

$$c_1=\frac{1}{c}[(1-2\varphi_1)\eta_2a_2^3-(1-2\varphi_2)\eta_1a_1^3],\ c_2=\frac{1}{c}[(1+\varphi_2)\eta_1-(1+\varphi_1)\eta_2]a_1^3a_2^3, \tag{25}$$

with $c=(1-2\varphi_1)(1+\varphi_2)a_2^3-(1-2\varphi_2)(1+\varphi_1)a_1^3$.

3.2. Stress-based solution

When Eq. (21) is substituted into Eq. (2), by incorporating the equilibrium condition Eq. (20), there follows:

$$\frac{\mathrm{d}\sigma_\theta}{\mathrm{d}r}-\frac{\sigma_r-\sigma_\theta}{r}=0, \tag{26}$$

which is the Beltrami–Michell compatibility condition for the considered problem and which has the identical mathematical form as the Saint–Venant compatibility condition Eq. (2) in terms of strains.

From the equations of equilibrium Eq. (20) and compatibility Eq. (26), there follows:

$$\sigma_r=C_1+2\frac{C_2}{r^3},\quad \sigma_\theta=C_1-\frac{C_2}{r^3}. \tag{27}$$

Thus, the average normal stress is uniform throughout the sphere and equal to C_1 (see also [10, p. 193] and [11, p. 251]). The general boundary conditions, expressed in terms of stresses, are

$$\sigma_r(a_i)+\alpha_i\sigma_\theta(a_i)=\beta_i, \tag{28}$$

where

$$\alpha_i = -\frac{1-\nu}{\nu}, \quad \beta_i = -\frac{E}{\nu}\frac{u_i}{a_i}, \qquad [\text{for prescribed } u(a_i) = u_i],$$
$$\alpha_i = 0, \quad \beta_i = -p_i, \qquad [\text{for prescribed } \sigma_r(a_i) = -p_i].$$

The corresponding integration constants C_1 and C_2 are readily found to be

$$C_1 = \frac{1}{C}[(2-\alpha_1)\beta_2 a_2^3 - (2-\alpha_2)\beta_1 a_1^3], \; C_2 = \frac{1}{C}[(1+\alpha_2)\beta_1 - (1+\alpha_1)\beta_2]a_1^3 a_2^3, \tag{29}$$

with $C = (2-\alpha_1)(1+\alpha_2)a_2^3 - (2-\alpha_2)(1+\alpha_1)a_1^3$.

Having determined the stress components, the radial displacement is calculated from $u = r\epsilon_\theta = r[(1-\nu)\sigma_\theta - \nu\sigma_r]/E$, which gives

$$u = \frac{1}{E}\left[(1-2\nu)C_1 r - (1+\nu)\frac{C_2}{r^2}\right]. \tag{30}$$

The connections between the integration constants (c_1, c_2), and (C_1, C_2), follow by comparing Eqs. (23) and (30), and are

$$c_1 = \frac{1-2\nu}{E}C_1, \quad c_2 = -\frac{1+\nu}{E}C_2. \tag{31}$$

3.3. Strain-based solution

By substituting the stress–strain relations Eq. (22) into Eq. (20), and by incorporating the Saint–Venant compatibility condition Eq. (2), there follows:

$$\frac{d\epsilon_r}{dr} + 2\frac{\epsilon_r - \epsilon_\theta}{r} = 0. \tag{32}$$

By comparing Eqs. (20) and (32), we conclude that these equations have the identical mathematical form. Since the compatibility condition in terms of strains Eq. (2) has the identical form as the compatibility condition in terms of stresses Eq. (26), the solution of the differential equations Eqs. (2) and (32) is

$$\epsilon_r = k_1 + 2\frac{k_2}{r^3}, \quad \epsilon_\theta = k_1 - \frac{k_2}{r^3}, \tag{33}$$

in duality with Eq. (27).

The general boundary conditions, expressed in terms of strains, are

$$\varphi_i \epsilon_r(a_i) + \epsilon_\theta(a_i) = \eta_i, \tag{34}$$

where φ_i and η_i are defined by the expressions following Eq. (24). The corresponding integration constants k_1 and k_2 are readily found to be

$$k_1 = \frac{1}{k}[(1-2\varphi_1)\eta_2 a_2^3 - (1-2\varphi_2)\eta_1 a_1^3], \; k_2 = -\frac{1}{k}[(1+\varphi_2)\eta_1 - (1+\varphi_1)\eta_2]a_1^3 a_2^3, \tag{35}$$

with $k = c$, given by the expression following Eq. (25). The radial displacement is

$$u = r\epsilon_\theta = k_1 r - \frac{k_2}{r^2}. \tag{36}$$

By comparing Eqs. (23), (30), and (36), the three sets of integration constants are related by

$$c_1 = k_1 = \frac{1-2\nu}{E}C_1, \quad c_2 = -k_2 = -\frac{1+\nu}{E}C_2. \tag{37}$$

4. Additional remarks on the compatibility conditions

There is a subtle point in the stress-based approach to solve the Lamé problem, if the equilibrium equation Eq. (1) is used in conjunction with the general compatibility condition from the two-dimensional elasticity, $\nabla^2(\sigma_r + \sigma_\theta) = 0$. When the stresses depend on r only, this condition is

$$\frac{d^2}{dr^2}(\sigma_r + \sigma_\theta) + \frac{1}{r}\frac{d}{dr}(\sigma_r + \sigma_\theta) = 0. \tag{38}$$

Two successive integrations give

$$\frac{d}{dr}(\sigma_r + \sigma_\theta) = \frac{C_0}{r}, \quad \sigma_r + \sigma_\theta = C_0 \ln r + C_1. \tag{39}$$

These expressions, with nonvanishing constants C_0 and C_1, apply, for example, to pure bending of curved circular bars ([3], p. 71), where the stresses are independent of the polar angle θ, but there is a nonvanishing θ-dependent circumferential displacement υ, in addition to the radial displacement u. For a fully axisymmetric Lamé problem, the constant $C_0 = 0$, although this cannot be recognized from Eq. (39) itself, except when there is no hole in the disk, or when there is a hole in an infinitely extended disk ($C_0 = 0$ in these cases to eliminate the unbounded stress contributions; in the latter case C_1 is also equal to zero to make stresses vanish at infinity). The procedure to specify C_0 would then be to apply the boundary conditions and express C_1 and C_2 in terms of C_0, and then derive two expressions for the radial displacement u, from each of the two strain expressions (ϵ_r and ϵ_θ). In order that these two expressions for u are the same, C_0 must be equal to zero.[2] In this way, however, the procedure ceases to be purely stress-based, as one presented in Section 2.2, based on the stronger form of the Beltrami–Michell compatibility condition Eq. (8), rather than Eq. (38).

The transition from the Saint–Venant compatibility condition of general two-dimensional elasticity to its form applicable to axisymmetric Lamé problem is easier. With no shear strain $\epsilon_{r\theta}$, and with the radial and hoop strains dependent on r only, the general Saint–Venant compatibility condition of plane elasticity (*e.g.*, [14, p. 669]) reduces to

$$r\frac{\mathrm{d}^2\epsilon_\theta}{\mathrm{d}r^2} + 2\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r} - \frac{\mathrm{d}\epsilon_r}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r} + \epsilon_\theta - \epsilon_r\right) = 0, \tag{40}$$

and the integration gives

$$r\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r} + \epsilon_\theta - \epsilon_r = k_0. \tag{41}$$

But the left-hand side is identically equal to zero, which can be verified by inspection, because $\epsilon_r = \mathrm{d}u/\mathrm{d}r$ and $\epsilon_\theta = u/r$. Therefore, $k_0 = 0$, reducing Eq. (41) to proper form of the Saint–Venant compatibility condition Eq. (2) for the problem under consideration.

Analogous remarks apply to the reduction of the general compatibility condition to centrally symmetric case. The Saint–Venant general compatibility conditions in the centrally symmetric case reduce to ([15, p. 21])

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r}\right) - r\frac{\mathrm{d}\epsilon_r}{\mathrm{d}r} = 0, \qquad r\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r} + \epsilon_\theta - \epsilon_r = 0. \tag{42}$$

However, the first of these can be rewritten as

$$r\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\epsilon_\theta}{\mathrm{d}r} + \epsilon_\theta - \epsilon_r\right) = 0, \tag{43}$$

which is identically satisfied whenever the second one is. Thus, as anticipated, there is just one Saint–Venant compatibility condition for the centrally symmetric case, given by the second of Eq. (42), *i.e.*, Eq. (2). Similarly, instead of

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}(\sigma_r + 2\sigma_\theta) + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}(\sigma_r + 2\sigma_\theta) = 0, \tag{44}$$

the proper form of the Beltrami–Michell compatibility condition, applicable to the Lamé problem of a hollow sphere, is given by Eq. (26).

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[2] See also the comments in [12, p. 443] and [13, p. 159].