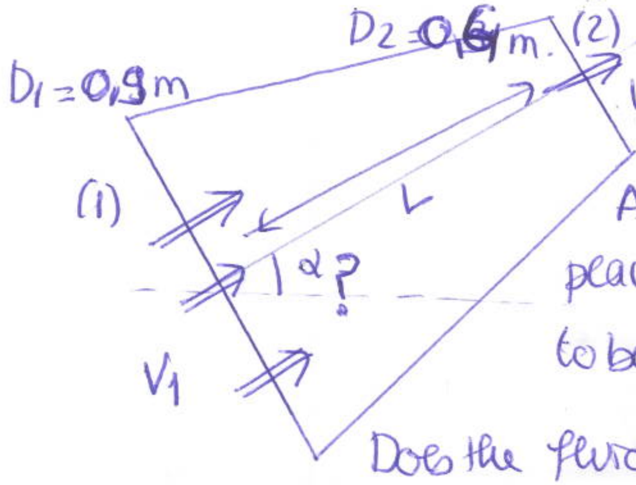
PROBLEM 1

A convergent nozzle of length $L = 10m$ is placed at an angle α (unknown) from the horizontal. Water flows at a flow rate $Q = 3 m^3/s$.



The pressure at the inlet (1) is 98kPa and at the outlet (2) is 12kPa.

At what angle α should the nozzle be placed if we want the friction head loss to be $h_f = 1,25m$?

Does the fluid flow upwards or downwards?

Energy equation $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$.

Continuity equation $Q = AV = A_1 V_1 = A_2 V_2$ $\left\{ \begin{array}{l} V_1 = \frac{4Q}{\pi D_1^2} \\ V_2 = \frac{4Q}{\pi D_2^2} \end{array} \right.$

$$(z_2 - z_1) = L \sin\alpha = \frac{(p_1 - p_2)}{\rho g} + \frac{(V_1^2 - V_2^2)}{2g} - h_f.$$

$$L \sin\alpha = \frac{(p_1 - p_2)}{\rho g} + \frac{8Q^2}{\pi^2 g}\left[\frac{1}{D_1^4} - \frac{1}{D_2^4}\right] - h_f.$$

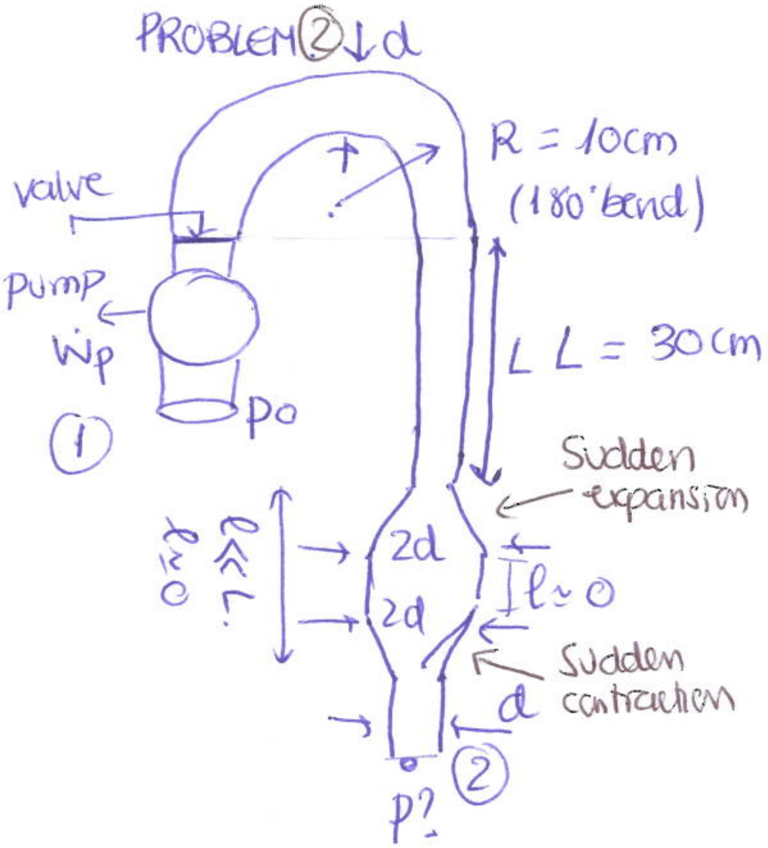
$D_1 = 0,9m$
$D_2 = 0,6m$

$$L \sin\alpha = \frac{(98-12)\cdot 10^3}{1000 \cdot 9,81} + \frac{8 \cdot 3^2}{\pi^2 \cdot 9,81}\left[\frac{1}{0,9^4} - \frac{1}{0,6^4}\right] - 1,25.$$

$$L \sin\alpha = 2,912 = 10 \cdot \sin\alpha \longrightarrow \boxed{\alpha = 16,93^\circ}$$

Since also the fluid is flowing upwards.

PROBLEM 2



- Laminar flow
- Blood $\rho = \rho_{water}$

$\mu = 4\mu_{water}$

- $p_0 = 120 \text{ mmHg}$
- $Q = 5 \text{ liters/min}$
- $d = 3\text{cm}$
- $\dot{W}_{pump} = 29{,}6 \text{ W}$

loss coefficients $\begin{cases} K_{valve} = 3 \\ K_{bend} = 0{,}28 \end{cases}$

Find the pressure P.

energy equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_{pump} + \Sigma h_m.$$

$$h_{pump} = \frac{\dot{W}_{pump}}{\rho g Q} \quad ; \quad h_f = \frac{V^2}{2g} \cdot f \frac{L}{d}$$

$$\Sigma h_m = h_{valve} + h_{bend} + h_{contr} + h_{exp} = \frac{V^2}{2g}\left[K_{valve} + K_{bend}\right] +$$

$$+ \frac{V^2}{2g} \cdot 0{,}42 \left(1 - \frac{d^2}{(2d)^2}\right) + \frac{V^2}{2g}\left(1 - \frac{d^2}{(2d)^2}\right)$$

* continuity $Q = V \cdot A$. $\quad V_2 = V = \dfrac{4Q}{\pi d^2}$

assume $V_1 \simeq 0$

$p_1 = p_0 \qquad \Delta Z = 0{,}3 m = z_1 - z_2.$

$p_2 = p?.$

$$\frac{p}{\rho g} = \frac{p_0}{\rho g} + \Delta z + \frac{\dot{W}_{pump}}{\rho g Q} - \frac{8Q^2}{g\pi^2 d^4}\left[f\frac{L}{d} + K_{valve} + K_{bend} + \right.$$

$$\left. + 0{,}42\left(1 - \frac{d^2}{4d^2}\right) + \left(1 - d^2/4d^2\right) \right]$$

$$p_0 = 120 mmHg = \frac{120}{760} \cdot 101325 \ Pa = 15998{,}68 \ Pa.$$

$$Q = \frac{5 \ liters}{min} = \frac{5 \cdot 10^{-3}}{60} \ m^3/s = 8{,}33 \cdot 10^{-5} m^3/s.$$

$\rho = 1000 \ Kg/m^3; \ \mu = 0{,}004 \ Kg/ms$

$$f = ? \quad Re_d = \frac{\rho V d}{\mu} = \frac{4Q\rho}{\pi d \mu} = 883{,}84 \rightarrow laminar \quad f = \frac{64}{Re_d} = 0{,}0724.$$

$$P = p_0 + \rho g \Delta z + \frac{\dot{W}_{pump}}{Q} - \frac{8Q^2\rho}{\pi^2 d^4} \cdot \left[f L/d + K_v + K_b + 1{,}42 \cdot 3/4 \right]$$

$p = 1{,}2087 \cdot 10^4 \ Pa \quad \Rightarrow \quad \boxed{p = 90 \ mm \ Hg.}$

MAE 101B, Spring 2007
Midterm I, 11:00 AM - 11:50 AM

Guidelines: Solve only TWO problems and indicate on your answer-book which problems are to be graded. Closed-book, closed-notes, no-calculator exam. Give all formulae used and **explain your steps** in each problem. When there are numbers in the problem, insert them into formulae (using SI units), but there is no need to calculate numerical answers. **Attach question paper** to the exam that you turn in.

1. There is flow in the x-direction in a channel between two flat plates as shown in the figure. Assume fully-developed, laminar, incompressible flow.

30pts **a.** Start from the x-momentum equation given below and show that dp/dx is a constant.

$$\frac{\partial u}{\partial t} + (\mathbf{v} \cdot \nabla)\, u = -\frac{1}{\rho}\frac{dp}{dx} + \nu \nabla^2 u$$

20pts **b.** The velocity is given to be,

$$u = -\frac{h^2}{2\mu}\frac{dp}{dx}\left(1 - \frac{y^2}{h^2}\right) .$$

The volumetric flow rate per unit span (width) is measured to be $\dot{Q}$. What is the new value of flow rate if h is doubled to $2h$, keeping dp/dx at the same value?

50 points

2. Consider a tank of water that is emptying as shown in the figure. There is fully-developed turbulent flow in the duct of diameter $10\,cm$ and length $1\,m$. Ignore minor losses. $\rho = 1000\,kg/m^3$, $\nu = 10^{-6} m^2/s$. Roughness is $\epsilon = 2\,mm$.

35pts **a.** The velocity is $V = 10\,m/s$. What is the corresponding water level, h?

15pts **b.** In part (a), the fully rough assumption is valid. Somewhat later in time, the flow is still turbulent but the fully rough assumption cannot be made. Explain the *physical* reason.

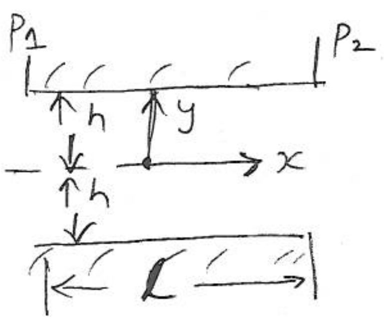
50 points

3. Water flows through a smooth-walled duct as shown in the figure. Assume turbulent flow and a sharp-edged entrance.

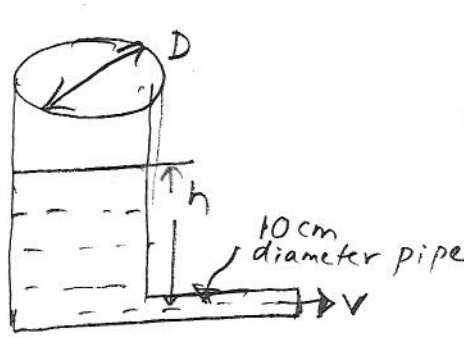
25pts **a.** We measure $\dot{Q} = 0.003 m^3/s$ *without* the diffuser. What would $\dot{Q}$ be with the diffuser added to the duct. Ignore major and minor losses.

25pts **b.** Repeat part (a) including major and minor losses. Give equations and solution procedure; no numerical answer is required.

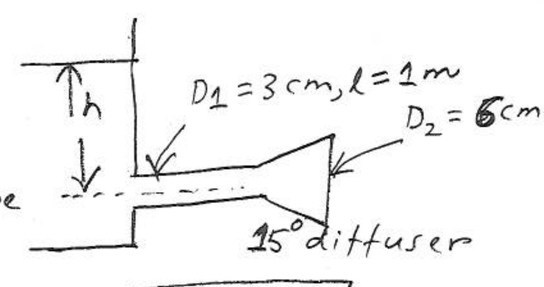
50 points



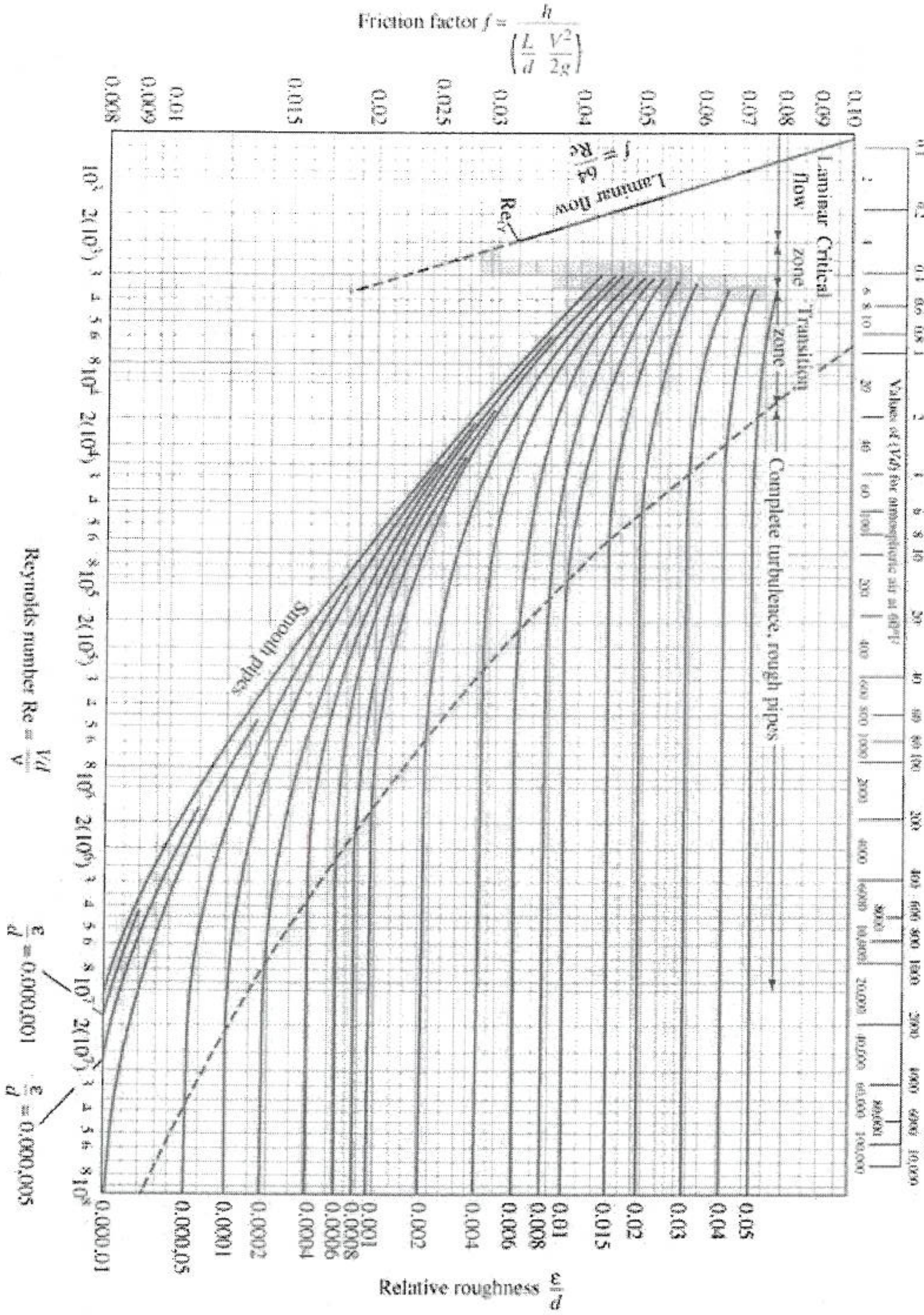
Problem 1



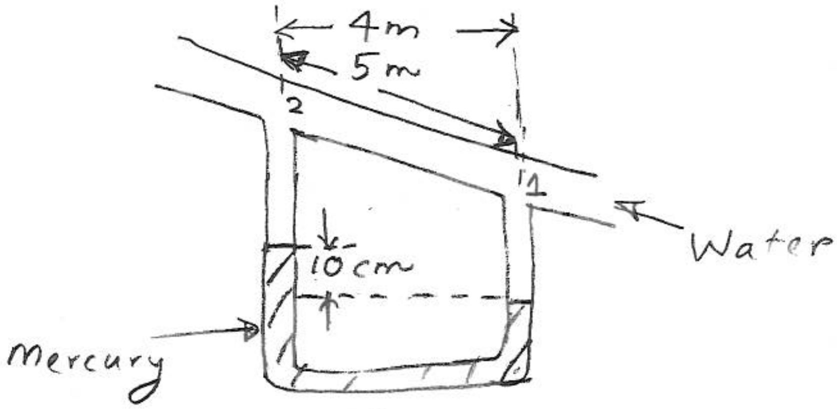
Problem 2



Problem 3



Friction factor $f = \dfrac{h}{\left(\dfrac{L}{d}\ \dfrac{V^2}{2g}\right)}$
Values of (Vd) for water at 60°F (velocity, ft/s × diameter, in)
Values of (Vd) for atmospheric air at 60°F
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
Laminar flow
$f = \dfrac{64}{Re}$
Re_c
Smooth pipes
Reynolds number $Re = \dfrac{Vd}{\nu}$
$\dfrac{\varepsilon}{d} = 0.000,001$
$\dfrac{\varepsilon}{d} = 0.000,005$
Relative roughness $\dfrac{\varepsilon}{d}$



Problem 4

MID I

a) 30
b) 20

1. $\frac{\partial u}{\partial t} + (\underline{v}\cdot\underline{\nabla})u = -\frac{1}{\rho}\frac{dp}{dx} + \nu\nabla^2 u$ ← given eqn.

Assume: steady $(\partial u/\partial t \to 0)$
$p = f(x)$ only
fully developed $(\partial u/\partial x = \partial v/\partial x = 0)$
laminar
incompressible
$v = 0$ (1-D flow)

full eqn: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$ 10 pts

Cancel using assumptions:

$$\underset{2}{0} + \underset{2}{0} + \underset{2}{0} + \underset{2}{0} = \frac{-1}{\rho}\frac{dp}{dx} + \nu\left(\underset{2}{0} + \frac{\partial^2 u}{\partial y^2} + \underset{2}{0}\right)$$ 12 pts for simplification

then:

$$\underbrace{\frac{dp}{dx}}_{\text{fcn of } x} = \rho\nu\underbrace{\frac{\partial^2 u}{\partial y^2}}_{\mu}\Big\}\text{ fcn of } y$$ 8 pts

a) The only way that a fcn (x) is a fcn (y) is if it is constant $\therefore \frac{dp}{dx} = \text{constant}$

b) $u = -\frac{h^2}{2\mu}\frac{\partial p}{\partial x}\left(1 - \frac{y^2}{h^2}\right)$ 7 pts

u if $V\alpha h^2$

$$\boxed{\dot{Q} = \int u\,dA = w2\int_0^h u\,dy} = 2w\int_0^h\left(\frac{-h^2}{2\mu}\frac{\partial p}{\partial x}\left(1 - \frac{y^2}{h^2}\right)\right)dy$$

$$\dot{Q} = \frac{-2wh^2}{2\mu}\frac{\partial p}{\partial x}\int_0^h\left(1 - \frac{y^2}{h^2}\right)dy = \frac{-wh^2}{\mu}\frac{\partial p}{\partial x}\left[y - \frac{y^3}{3h^2}\right]_0^h$$

$$\dot{Q} = -\frac{wh^2}{\mu}\frac{\partial p}{\partial x}\left[h - \frac{h}{3}\right] = \frac{-2}{3}\frac{w}{\mu}\frac{\partial p}{\partial x}h^3$$ 8 pts for $\dot{Q} \propto h^3$

So if $h \to 2h$, $\dot{Q} \to 2^3\dot{Q} = 8\dot{Q}$

5 pts for answer

5

2. Energy Eqn: — 5 pts for attempt at writing energy eqn

5 pts for correct form.

(a) ≥ 35 pts

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

Assume: $p_1 = p_2$ — 3 pts

$V_1^2 \sim 0$ ← 2 pts b/c $D >> d$ OR [not necessary b/c $V_1 = V_2\left(\frac{d}{D}\right)^2$]

Know: $h_f = f\,\frac{L}{D}\frac{V^2}{2g}$

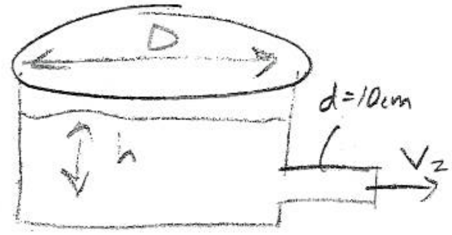
$\epsilon = 2mm = 0.002m$

$\rho = 1000\,kg/m^3$

$\nu = 10^{-6}\,m^2/s$

$d = 10\,cm = 0.1m$

$L = 1m$



a) If $V_2 = 10\,m/s$, what is h?
rewrite energy using assumptions:

10 pts $h = \frac{V_2^2}{2g} + f\,\frac{L}{d}\frac{V_2^2}{2g}$ (or) $h = \frac{V_2^2}{2g} + f\frac{L}{d}\frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

find friction factor f on Moody Chart using: (+2)

3 pts → $\frac{\epsilon}{d} = \frac{0.002m}{0.1m} = 0.02$ (+1)

and

3 pts → $Re = \frac{V_2 d}{\nu} = \frac{(10m/s)(0.1m)}{10^{-6}m^2/s} = 10^6$ (+1) +2

on chart: $f \sim 0.049$ ← +2

then: $h = \frac{100\,m^2/s^2}{2(9.81\,m/s^2)} + \frac{(0.049)(1m)(100\,m^2/s^2)}{(0.1m)(2)(9.81\,m/s^2)}$

b) = 15 pts.

9

2b) Why at a later time is fully rough assumption no longer valid?

As the water exits the tank the velocity will decrease because the gravitational head decreases as "h" decreases. This results in a decrease in Reynolds # and the Moody Chart shows that for same ε/d, if $Re \downarrow$ we go into partially rough regime.

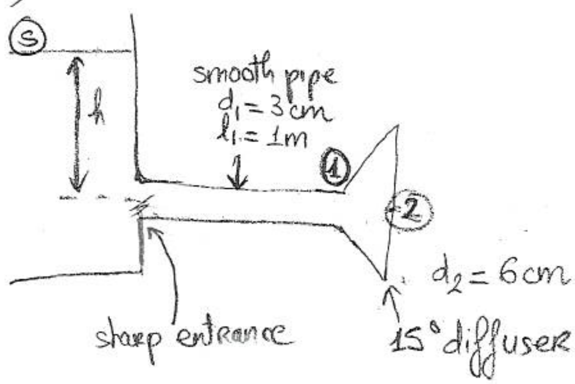
5 pts

5 pts

Physical reason is that when V (pipe velocity) $\downarrow$, Re # $\downarrow$ so that the viscous scale, $\nu/u^* \downarrow$. Eventually $\nu/u^* \downarrow$ to become comparable to ε so that the surface doesn't seem rough to the flow.

+5

3,



$\dot{Q}_{wo}$ = flow rate without diffuser $= .003 \frac{kg}{m^3}$

$\dot{Q}_{w}$ = flow rate with diffuser

(25) a) → Without diffuser; energy equation from (S) to (1) (5)

→ $P_s = P_1 = P_{atm}$; $V_s \approx 0$ m/s , Ignore losses

$$\Rightarrow h = \frac{V_1^2}{2g} \Rightarrow V_1 = \sqrt{2gh} \quad +5 \qquad (1)$$

→ with diffuser: energy equation from (S) to (2): same assumptions

$$\Rightarrow h = \frac{V_2^2}{2g} \Rightarrow V_2 = \sqrt{2gh} + 5 \Rightarrow V_1 = V_2 \qquad (2)$$

→ $Q_{wo} = V_1 A_1$ w/o diffuser ; $Q_w = V_2 A_2$ w/ diffuser $+5 \Rightarrow \frac{Q_w}{Q_{wo}} = \frac{A_2}{A_1}$

$$\Rightarrow Q_w = \left(\frac{d_2}{d_1}\right)^2 Q_{wo} = \left(\frac{6\text{cm}}{3\text{cm}}\right)^2 (.003\ m^3/s) = \boxed{.012\ m^3/s} \ +5$$

(25) b) → Without diffuser; energy equation from (S) to (1) (5)

→ $P_s = P_1 = P_{atm}$; $V_s \approx 0$ m/s ; Include friction and entrance losses

$$\Rightarrow h \underset{+5}{=} \left(1 + f_{wo}\frac{l_1}{d_1} + K_{entr}\right)\frac{V_1^2}{2g} = \left(1 + f_{wo}\frac{l_1}{d_1} + K_{entr}\right)\frac{16 Q_{wo}^2}{2\pi^2 g d_1^4} \qquad (3)$$

~~Solve for h~~ (+5)

→ With diffuser, energy equation from (S) to (2)

→ Include friction + entrance + diffuser losses

$$\Rightarrow h \underset{+5}{=} \frac{V_2^2}{2g} + \left(f_w \frac{l_1}{d_1} + K_{entR} + K_{diff}\right)\frac{V_1^2}{2g} \qquad (4)$$

→ $V_1 A_1 = V_2 A_2 \Rightarrow V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1$ (5), substitute Into (4)

$$\Rightarrow h = \left[\left(\frac{d_1}{d_2}\right)^4 + f_w\frac{l_1}{d_1} + K_{entR} + K_{diff}\right]\frac{V_1^2}{2g}$$

$$\Rightarrow V_1 = \sqrt{2gh}\left[\left(\frac{d_1}{d_2}\right)^4 + f_w\frac{l_1}{d_1} + K_{entR} + K_{diff}\right]^{-1/2}$$

$$\Rightarrow Q_w = \frac{\pi}{4} d_1^2 \sqrt{2gh}\left[\left(\frac{d_1}{d_2}\right)^4 + f_w\frac{l_1}{d_1} + K_{entR} + K_{diff}\right]^{-1/2} \qquad (5)$$

$\Rightarrow$ Substitute (3) Into (5),

$$\Rightarrow Q_W = Q_{WO} \left[\frac{1 + f_{WO}\frac{l_1}{d_1} + K_{entR}}{\left(\frac{d_1}{d_2}\right)^4 + f_W \frac{l_1}{d_1} + K_{entR} + K_{diff}} \right]^{1/2} \qquad (6)$$

5

$\rightarrow$ Q_{WO}, l_1, d_1 are given.

$\rightarrow$ K_{entR} and K_{diff} can be looked up from the book.

$\rightarrow$ for turbulent flow In smooth pipe, $f = fnc(Re)$ as defined In the book. f_{WO} can be computed directly since Q_{WO} Is known

$\rightarrow$ f_W can not be computed directly since $f_W = fnc(Q_W)$, so guess and check Q_W until (6) Is satisfied.

Hw 2

MAE 101B, Spring 2007

Homework 2

Due Thursday, April 19, in class

Guidelines: Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.
Please refrain from copying. Refer to the course outline for what constitutes copying

1. A turbine extracts 350 W of power in the configuration shown in the figure. The pipes are made of wrought iron. What is the flow rate Q in m^3/h?

2. Water flows out of a cylindrical tank of diameter D owing to gravitational head.

a) Assume turbulent flow with an average friction factor, f_0. Obtain the time for the water level to decrease from h to $h/2$.

b) Suppose the flow were laminar. Again, obtain the time for the water level to decrease from h to $h/2$.

3. Water flows through a sudden contraction between two pipes of diameter $D_1 = 50\,mm$ and $D_2 = 25\,mm$. The pressure drop between two points upstream and downstream of the contraction, respectively, is measured to be $4\,kPa$. What is the flow rate in m^3/s?

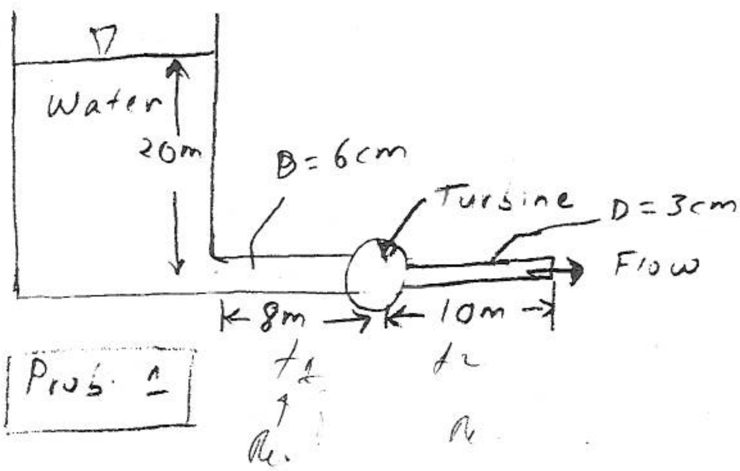
4. The dataset (download it from the 101B web site) gives the mean velocity, U, in turbulent channel flow. It is obtained by averaging the 3-D, unsteady data in a direct numerical simulation of channel flow by Hoyas and Jimenez (2006).

a) Analysis of the data gives $u^* = 0.02\,m/s$. Plot the data and identify the the following regions: viscous sublayer, logarithmic overlap region, and outer law profile. These regions were identified in the class and are also shown in Fig. 6.10 of White.

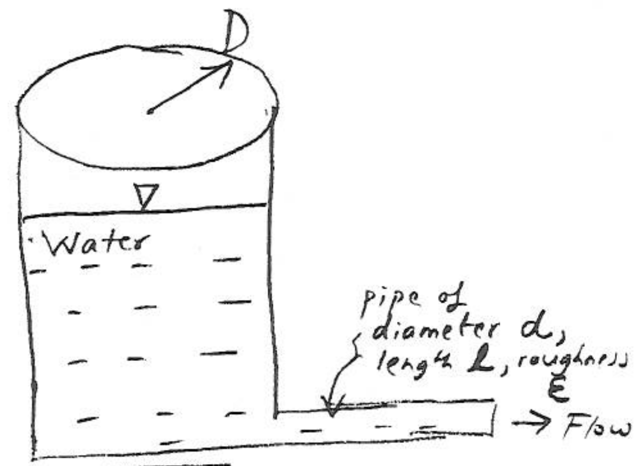
b) Suppose you were *not* given the value of u^*. Estimate u^* from the data.

Ungraded problems From text. 6.40, 6.148

u


Water
20m
D = 6cm
Turbine
D = 3cm
Flow
8m
10m

Prob. 1


D
Water
pipe of diameter d, length L, roughness ε
Flow

Prob. 2

MAE 101B, Spring 2007

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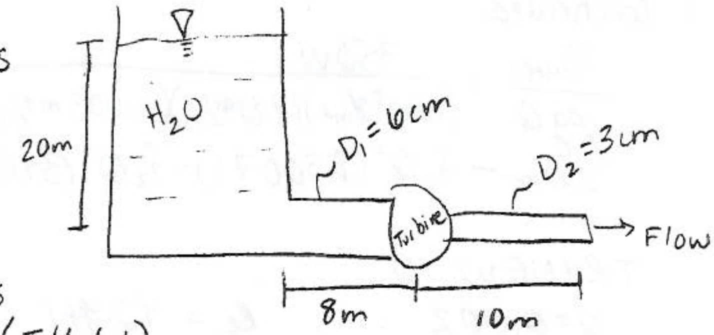
Ungraded problems From text. 6.40, 6.148

1. Given: Power = 350 W
wrought iron pipes



Find: Q (m^3/hr)

Assume: $V_1 \sim 0$; $p_1 = p_2$

Look up: $\rho_{H_2O} = 1000\ kg/m^3$

$\mu_{H_2O} = 0.001\ kg/m \cdot s$

$\varepsilon \approx 0.046\ mm$ (Table 6.1)

Equations: Energy => $\frac{p_1}{\rho g} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\bar{V}_2^2}{2g} + z_2 + h_{f2} + h_{f1} + h_{turbine}$ ①

head loss => $h_f = f \frac{L}{d} \frac{V^2}{2g}$ ②

$h_{turbine} = \frac{Power}{\rho g Q}$ ③

Solution:

Simplify ① : $z_1 - z_2 = \frac{V_2^2}{2g} + f_1 \frac{L_1}{d_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{d_2} \frac{V_2^2}{2g} + \frac{Power}{\rho g Q}$

* note: $Q = \frac{\pi}{4} d_1^2 V_1^2 = \frac{\pi}{4} d_2^2 V_2^2$

Now: $\frac{Power}{\rho g Q} = z_1 - z_2 - f_1 \frac{8 L_1 Q^2}{\pi^2 g d_1^5} - f_2 \frac{8 L_2 Q^2}{\pi^2 g d_2^5} - \frac{8 Q^2}{\pi^2 g d_2^4}$

To find the friction factors we need the relative roughness:

$\left(\frac{\varepsilon}{d}\right)_1 = \frac{0.046mm}{60mm} \approx 0.000767$ $\left(\frac{\varepsilon}{d}\right)_2 = \frac{0.046}{30} = 0.00153$

And the Reynolds number ... BUT WE NEED Q! Lets make a guess...

$Q = 0.003\ m^3/s$ then $Re_1 = \frac{4 \rho Q}{\pi \mu d_1} = 63662$ f_1 moody $= 0.0226$

CHART

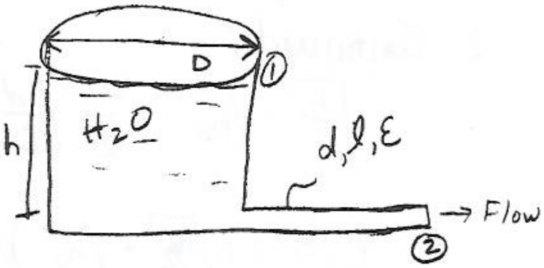
$Re_2 = \frac{4 \rho Q}{\pi \mu d_2} = 127{,}324$ f_2 moody ≈ 0.0238

or from eqn:

$\frac{1}{f^{1/2}} \approx -1.8 \log\left[\frac{6.9}{Re_d} + \left(\frac{(\varepsilon/d)}{3.7}\right)^{1.11}\right] \Rightarrow 0.0223 = f_1$

$0.0233 = f_2$

2. Given: see figure



Find: a) t for $h \to h/2$ (f_0)

b) t for $h \to h/2$ (laminar)

Assume: a) f_0 = avg friction factor

b) laminar

$V_{surface} \sim 0$

No minor losses

Equations: Energy $\Rightarrow$ $$\frac{p_1}{\rho g} + \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\bar{V}_2^2}{2g} + z_2 + h_f$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$Q = VA$$

Solution: $p_1 = p_2 = p_{atm}$, $V_1 \simeq 0$, $z_1 - z_2 = h$

a) then: $$h(t) = \frac{\bar{V}_2^2}{2g} + f_0 \frac{l}{d} \frac{\bar{V}_2^2}{2g}$$

where: $$\bar{V}_2 = Q/A_2 = \frac{Q}{\frac{\pi}{4} d^2} = \frac{4Q}{\pi d^2}$$

So: $$h(t) = \left(\frac{4Q}{\pi d^2}\right)^2 \frac{1}{2g} + f_0 \frac{l}{d\,2g} \left(\frac{4Q}{\pi d^2}\right)^2$$

AND : $$Q = -\frac{\pi D^2}{4} \frac{dh}{dt}$$

Then: $$h(t) = \left(\frac{\cancel{4}}{\cancel{\pi} d^2}\right)^2 \left(\frac{-\cancel{\pi} D^2}{\cancel{4}}\right)^2 \left(\frac{dh}{dt}\right)^2 \left(\frac{1}{2g} + f_0 \frac{l}{2gd}\right)$$

$$\frac{dh}{dt} = \left[h(t) \left(\frac{d}{D}\right)^4 \frac{1}{(1/2g + f_0 l/2gd)}\right]^{1/2}$$

$$\int_{h_0}^{h_0/2} \frac{1}{\sqrt{h}}\, dh = \int_0^{t_2} \left[\left(\frac{d}{D}\right)^4 \frac{2g}{(1 + f_0 l/d)}\right]^{1/2} dt$$

$$-2\left\{\sqrt{h}\right\}_{h_0}^{h_0/2} = (d/D)^2 \left(2g/(1 + f_0 l/d)\right)^{1/2} \left\{t\right\}_0^{t_2}$$

2. Continued again

then $A = -a$ and $B(x) = a^2 + 4bh(t)$ and $x = h$

So:

$$-\frac{1}{2}\left\{\frac{2h^{1/2}}{a^2+4bh} - \left(\frac{2(-a)}{(a^2+4bh)^2} \ln\left|-a + (a^2+4bh)^{1/2}\right|\right\}_{h_0}^{h_0/2} = t\Big|_0^{t_2}$$

$$t_2 = -\frac{1}{2}\left\{\frac{2h_0^{1/2}}{\sqrt{2}(a^2+2bh)} - \frac{2h_0^{1/2}}{(a^2+4bh_0)} + \frac{2a}{(a^2+2bh_0)^2}\ln\left|-a+(a^2+2bh_0)^{1/2}\right| - \frac{2a}{(a^2+4bh_0)^2}\ln\left|-a+(a^2+4bh_0)^{1/2}\right|\right.$$

where $a = \frac{32\mu l}{\rho D^2}$ and $b = 2g\left(\frac{d}{D}\right)^4$

4. <u>Given:</u> V, Y data from experiment

$u^* = 0.02\ m/s$, $\nu = 10^{-6}\ m/s^2$

a) Use 1st 1/2 of data for bottom of channel.
Create spreadsheet where:

$y = Y + 0.1$, $y^+ = \frac{y u^*}{\nu}$, $u^+ = \frac{u}{u^*}$

see attatched plot (also plotted overlap line and $u^+ = y^+$ line)

b) If we were not given u^* we could estimate it.

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} = \left(\frac{\mu\, du/dy}{\rho}\right)^{1/2} = \nu^{1/2}\left(\frac{\Delta u}{\Delta y}\right)^{1/2}$$

We y vs. u then zoom in very near y=0 (the wall) to estimate $\frac{\Delta u}{\Delta y}$ (see attatched)

$\Delta u \sim 0.05$, $\Delta y \sim 0.000125$ near wall

$$u^* = (10^{-6})^{1/2}\left(\frac{0.05}{0.000125}\right)^{1/2} = 0.02\ m/s$$

6.40 Given: $\tau_{turb} = \varepsilon \frac{du}{dy}$

$\varepsilon = \rho K^2 y^2 \left|\frac{du}{dy}\right|$

$K = 0.41$

Assume: $\tau_{turb} \approx \tau_w$ near wall

Find: integrate to get eqn 6.28

Equation: $\tau_{turb} \sim \tau_w = \rho u^{*2} = \varepsilon \frac{du}{dy} = \left[\rho K^2 y^2 \left|\frac{du}{dy}\right|\right] \frac{du}{dy}$

solve for $\frac{du}{dy} = \frac{u^*}{Ky}$

integrate: $\int du = \frac{u^*}{K} \int \frac{dy}{y}$

or: $u = \frac{u^*}{K} \ln(y) + \text{constant}$

*to convert to the exact form we need expt. data