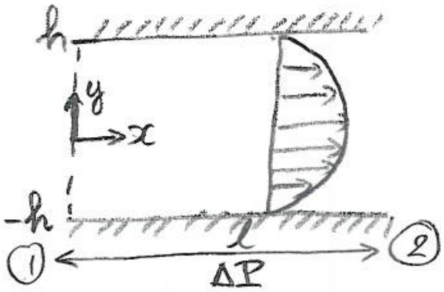
Homework I Solution

Problem 1:



a) $\rightarrow$ steady laminar: $\partial/\partial t = 0$

$\rightarrow$ 2-D: $\partial/\partial z = 0$

$\rightarrow$ fully-developed: $u = u(y)$ only $\Rightarrow \frac{\partial u}{\partial x} = 0$

$\rightarrow$ B.C.'s: $u(y = \pm h) = 0$

$\rightarrow$ continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0$

$\rightarrow$ x-momentum: $0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$ (1)

$\rightarrow$ y-momentum: $0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} \Rightarrow P = P(x)$

$\rightarrow$ from (1) $\Rightarrow \frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx} = \frac{1}{\mu}\frac{\Delta P}{l}$

$\rightarrow$ integrate, $\Rightarrow \frac{du}{dy} = \frac{1}{\mu}\frac{\Delta P}{l}y + A$

$\Rightarrow u = \frac{1}{2\mu}\frac{\Delta P}{l}y^2 + Ay + B$ where A, B are constant

$\rightarrow$ apply B.C.'s $\Rightarrow A = 0$; $B = \frac{-1}{2\mu}\frac{\Delta P}{l}h^2 \Rightarrow \boxed{u(y) = \frac{h^2}{2\mu}\frac{\Delta P}{l}\left(1 - \frac{y^2}{h^2}\right)}$

b) $\rightarrow V \equiv u_{avg} = \frac{1}{2h}\int_{-h}^{h} u\, dy = \frac{h^2}{3\mu}\frac{\Delta P}{l}$

$\rightarrow \tau_w = \mu\frac{du}{dy}\Big|_{y=h} = \frac{h\Delta P}{l}$

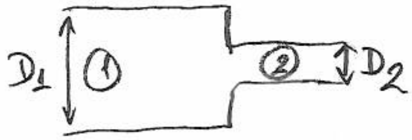
$\rightarrow$ momentum equation: $2h\Delta P - 2l\tau_w = 0 \Rightarrow \Delta P = \frac{\tau_w l}{h}$

$\rightarrow$ energy equation: $h_f = \frac{\Delta P}{\rho g} = \frac{\tau_w l}{\rho g h}$

$\rightarrow$ by definition, $h_f = f\frac{L}{D_h}\frac{V^2}{2g}$ where $D_h = 4h$ is the hydraulic diameter

$\Rightarrow f = \frac{8\tau_w}{\rho V^2} = \frac{8(h\Delta P/l)}{\rho V\left(\frac{h^2}{3\mu}\frac{\Delta P}{l}\right)} = \frac{24}{\frac{Vh}{\nu}} \Rightarrow \boxed{f = \frac{24}{Re_h}}$

Problem 2:



a) → from eqn 6.12 → $(h_f)_1 = \dfrac{128\mu L Q}{\pi \rho g D_1^4}$ (1)

→ $(h_f)_2 = \dfrac{128\mu L Q}{\pi \rho g D_2^4}$ (2)

→ energy equation in pipe 1 with no elevation and no velocity difference

$\Rightarrow (h_f)_1 = \left(\dfrac{\Delta P}{\rho g}\right)_1$ (3)

→ similar for pipe 2 $\Rightarrow (h_f)_2 = \left(\dfrac{\Delta P}{\rho g}\right)_2$ (4)

→ from (1) and (3) $\Rightarrow \left(\dfrac{\Delta P}{\rho g}\right)_1 = \dfrac{128\mu L Q}{\pi \rho g D_1^4} \Rightarrow \left(\dfrac{\Delta P}{L}\right)_1 = \dfrac{128\mu Q}{\pi D_1^4}$ (5)

→ from (2) and (4) $\Rightarrow \left(\dfrac{\Delta P}{\rho g}\right)_2 = \dfrac{128\mu L Q}{\pi \rho g D_2^4} \Rightarrow \left(\dfrac{\Delta P}{L}\right)_2 = \dfrac{128\mu Q}{\pi D_2^4}$ (6)

→ from (5) and (6) $\Rightarrow \left(\dfrac{D_2}{D_1}\right)^4 = \dfrac{(\Delta P/L)_1}{(\Delta P/L)_2} \Rightarrow \boxed{\dfrac{D_2}{D_1} = \dfrac{1}{3}}$

→ The ratio is independent of fluid property.

b) → for laminar flow, $Re \leq 2300$

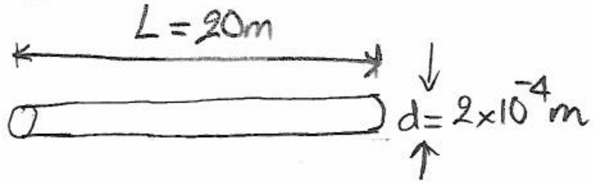
→ $Re = \dfrac{\rho V D}{\mu} = \dfrac{4\rho Q}{\pi \mu D} \Rightarrow D_{min} = \dfrac{4(1260\ kg/m^3)(2\ m^3/s)}{\pi(1.49\ kg/m\text{-}s)(2300)} = 0.94m$

$\Rightarrow D_{2,min} = 0.94m \Rightarrow \boxed{D_{1,min} = 2.82m}$

→ $\left(\dfrac{\Delta P}{L}\right)_1 = \dfrac{128\mu Q}{\pi D_1^4} = \dfrac{128(1.49\ kg/m\text{-}s)(2\ m^3/s)}{\pi(2.82\ m)^4} = \boxed{1.92\ N/m^3}$

→ $\left(\dfrac{\Delta P}{L}\right)_2 = \dfrac{128\mu Q}{\pi D_2^4} = \dfrac{128(1.49\ kg/m\text{-}s)(2\ m^3/s)}{\pi(0.94\ m)^4} = \boxed{156\ N/m^3}$

Problem 3:



a)

$\rightarrow$ Power $\equiv Q\Delta P = 0.1\ hp = 74.57\ J/s$

$\rightarrow$ Laminar flow $\Rightarrow Re \leq 2300$

$\rightarrow$ Energy eqn. $\Rightarrow h_f = \dfrac{\Delta P}{\rho g} = \dfrac{32\mu L V}{\rho g d^2} \Rightarrow \Delta P = \dfrac{32\mu L V}{d^2}$

$\rightarrow$ Power $= Q\Delta P = \left(\dfrac{\pi}{4} d^2 V\right)\left(\dfrac{32\mu L V}{d^2}\right) = 8\pi\mu L V^2 \Rightarrow V = \sqrt{\dfrac{\text{Power}}{8\pi\mu L}}$

$\rightarrow Re = \dfrac{\rho V d}{\mu} = \dfrac{\rho d}{\mu^{3/2}}\sqrt{\dfrac{\text{Power}}{8\pi L}}$

$\rightarrow$ If water is used, $Re = 2436 \rightarrow$ turbulent

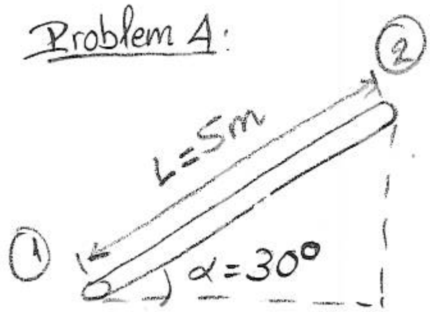
$\rightarrow$ If kerosene is used, $Re = 987 \rightarrow$ laminar

$\rightarrow$ kerosene is a better candidate for laminar flow.

b)

$\rightarrow \tau_w = \dfrac{8\mu V}{d} = \dfrac{8\mu^2 Re}{\rho d^2} = \dfrac{8(0.0016\ kg/m\text{-}s)^2(2300)}{(820\ kg/m^3)(2\times10^{-4}\ m)^2} = \boxed{1.44\ kPa}$

Problem 4:



a) → Energy equation

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$$

$$\Rightarrow \frac{P_1-P_2}{\rho g}=z_2-z_1+h_f$$

$$\Rightarrow h_f=\frac{P_1-P_2}{\rho g}+z_1-z_2$$

$$\Rightarrow h_f=\frac{(50\,kPa)-(30\,kPa)}{(1000\,kg/m^3)(9.81\,m/s^2)}-(5m)(\sin 30^\circ)$$

$$\Rightarrow h_f=-0.4613 \quad <0$$

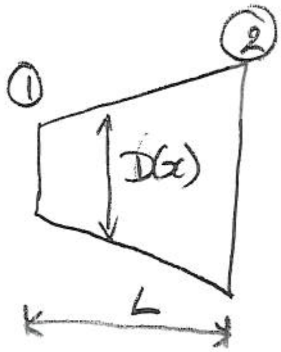
→ This Is Incorrect reading of pressure transducer because head loss can not be negative. Energy has to be conserved.

b) → $h_f=\frac{P_1-P_2}{\rho g}+z_1-z_2=\frac{(50\,kPa)-(20\,kPa)}{(1000\,kg/m^3)(9.81\,m/s^2)}-(5m)(\sin 30^\circ)$

$$\Rightarrow h_f=0.56\,m$$

$$\rightarrow f=h_f\frac{d}{L}\frac{2g}{V^2}=(0.56m)\frac{(0.05m)}{(5m)}\frac{2(9.81\,m/s^2)}{(10\,m/s)^2}=\boxed{0.0011}$$

Problem 5:



a) → the fluid exits at laminar transition point

$\Rightarrow (Re)_2 = 2300 \Rightarrow \frac{\rho D_2 V_2}{\mu} = 2300$

$\Rightarrow \frac{\rho D_2}{\mu}\left(\frac{4Q}{\pi D_2^2}\right) = 2300$

$\Rightarrow D_2 = \frac{4\rho Q}{2300\pi\mu} = \frac{4(1260\ kg/m^3)(1\ m^3/s)}{2300\pi(1.49\ kg/m\text{-}s)}$

$\Rightarrow D_2 = 0.47m$

$\rightarrow D(x=L) = D_2 \Rightarrow \frac{D_2}{L} = 0.6 \Rightarrow \boxed{L = 0.78m}$

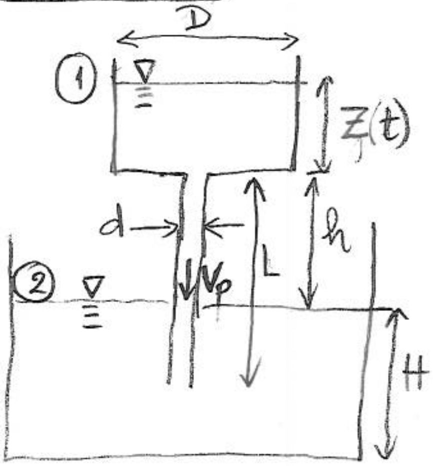
$\rightarrow D_1 = 0.5L = 0.39\,m$

b) Energy equation for constant pressure:

$\Rightarrow \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + h_f \Rightarrow h_f = \frac{V_1^2 - V_2^2}{2g} = \frac{16Q^2}{2\pi^2 g}\left(\frac{D_2^4 - D_1^4}{D_1^4 D_2^4}\right)$

$\Rightarrow h_f = \frac{16(1m^3/s)^2}{2\pi^2(9.81\ m/s^2)} \frac{(0.47m)^4 - (0.39m)^4}{(0.39m)^4(0.47m)^4} = \boxed{1.88m}$

Problem 6.27:



→ Energy eqn. at surface 1 and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

→ $P_1 = P_2 = P_{atm}$ and $V_1 = V_2 \approx 0$

$\Rightarrow h_f = z_1 - z_2 = Z(t) + h$

→ with $d \ll D$, friction loss is mostly from the pipe

$$\rightarrow h_f = \frac{32\mu L V_p}{\rho g d^2} = Z(t) + h \Rightarrow V_p = \frac{\rho g d^2}{32\mu L}\left[Z(t) + h\right]$$

→ mass conservation: $\frac{d}{dt}\left[\frac{\pi}{4}D^2 Z(t)\right] = -\frac{\pi}{4}d^2 V_p$

$$\Rightarrow \frac{\pi}{4}D^2\frac{dZ}{dt} = \frac{-\pi}{4}d^2\frac{\rho g d^2}{32\mu L}\left[Z(t) + h\right]$$

$$\Rightarrow \boxed{\frac{dZ}{dt} = \frac{-\rho g d^4}{32\mu L D^2}(Z + h)}$$

→ separate variables and integrate, $\int \frac{dZ}{Z + h} = -\int C dt$ where $C = \frac{\rho g d^4}{32\mu L D^2}$

$\Rightarrow Z = Ae^{-Ct} - h$ where A is a constant

→ at $t = 0$, $Z = Z_0 \Rightarrow A = Z_0 + h$

→ at $t = t_0$, $Z = 0 \Rightarrow 0 = (Z_0 + h)e^{-Ct_0} - h$

$$\Rightarrow t_0 = \frac{1}{C}\ln\left(\frac{Z_0}{h} + 1\right)$$

$$\Rightarrow \boxed{t_0 = \frac{32\mu L D^2}{\rho g d^4}\ln\left(\frac{Z_0}{h} + 1\right)}$$