MAE 290C, Spring 2017

HOMEWORK 3

Due June 10 11:59 PM (google drive, email, dropbox, etc)

Provide source codes used to solve all questions

PROBLEM 1

Consider the 2D Poisson's equation

$$\nabla^2 p = f$$

in a square domain of size N with homogeneous Dirichlet boundary conditions. The forcing function is given by

$$f(x, y) = \sum_{m=1}^{10} \sum_{n=1}^{10} \sin(2A\pi ix) \cos(2A\pi jy)$$

Discretize the problem with centered 2^{nd} order finite differences and solve it using a relaxation method for $N = 256$ grid points in each direction. Use alternate-direction implicit (Crank-Nicolson) integration in the relaxation time variable. Begin with the initial iteration condition $u(0) = f$. Given the shape of f, estimate the value of Δt that will provide balanced convergence as a function of A and run the ADI scheme for $0.01\Delta t_{bal} < \Delta t < 100\Delta t_{bal}$ for $A = 1$ and $A = 10$. Discuss the convergence of the solution as a function of Δt and A.

PROBLEM 2

Integrate numerically the linear wave equation

$$\partial_t u + c\partial_x u = 0,$$

in the domain $0 \leq x < 10$ with homogeneous initial conditions and $u(x = 0, t) = \sin(At)$ Solve using second-order centered finite difference schemes with $N = 200$ grid points, $\Delta t = 0.01$ and the following boundary conditions at the artificial exit:

1. Homogeneous boundary conditions, $u_N = 0$.
2. Linear extrapolating boundary conditions, $u_N = 2u_{N-1} - u_{N-2}$.
3. Quadratic extrapolating boundary conditions, $u_N = 3u_{N-1} - 3u_{N-2} + u_{N-3}$.
4. Homogeneous Neumann boundary conditions, $u_N = u_{N-1}$.
5. First-order upwinding convective boundary conditions,

$$d_t u_N = -c\frac{u_N - u_{N-1}}{\Delta x}.$$

6. Second-order upwinding convective boundary conditions,

$$d_t u_N = -c\frac{3u_N - 4u_{N-1} + u_{N-2}}{2\Delta x}.$$

Perform an analytical study of the reflection of waves generated by the each scheme at the artificial boundary and discuss the results obtained for $A = 0.1$ and $A = 3$. Can you rank each artificial boundary condition for each value of A the light of your analysis?