MAE 290C, Spring 2017
HOMEWORK 1
Due Tue 04-29-2017 11:59PM (dropbox, email, google drive)
Provide source codes used to solve all questions

Problem 1. The application of the low-storage IMEX RKW3 method to Navier-Stokes is described in the Appendix of ref. (1). Follow the derivations made there to determine:

1. The leading-order factor in the third-order term of the viscous error as a function of the free parameter of the scheme.
2. The amplification factor for the viscous terms in the limit when the real part of the eigenvalues of L tend to infinity, also as a function of the free parameter of the scheme.
3. Implement the scheme to solve the equation
$$d_t u = -\left(i\frac{c\pi}{\Delta x} + \frac{\nu\pi^2}{\Delta x^2} \right) u,$$
for $\pi c \Delta t/\Delta x = 0.5$ and $Re_{\Delta x} = c\Delta x/\nu = 1$ with initial condition $u(0) = 1$. Solve for $t \leq 10$.

Problem 2. The Lotka-Volterra (a.k.a. predator – prey) model equations are:
$$d_t u = Au - Buv, \tag{1}$$
$$d_t v = Cuv - Dv, \tag{2}$$
where A, B, C and D are real positive parameters.

1. Determine the real and imaginary parts of the eigenvalues of the linearized system as a function of the linearization point (u_0, v_0), in the particular model obtained for $A = B = C = D = 1$.
2. Which numerical integration scheme would you use to solve this system of equations so that the numerical solution remains stable regardless of the initial conditions?
3. How would you determine Δt depending on the value of (u_0, v_0)?
4. Integrate the Lotka-Volterra system using the numerical integration scheme you selected in the previous question for $A = B = C = D = 1$ and the initial conditions $(u(0), v(0)) = (4, 1)$. Dynamically update Δt every 10 time steps in order to keep your numerical solution stable. Plot the solution and Δt in the range $0 < t < 50$.
5. Integrate the Lotka-Volterra system using the same numerical integration scheme but this time, use Δt equal to 1/10 the Δt from the previous question. Plot the solution and Δt in the range $0 < t < 50$. Do the solutions obtained with different Δt criteria differ? Why?

Problem 3. Write a code to solve the problem

$$d_t u = \frac{\nu\pi^2}{\Delta x^2} u$$

using an Implicit Euler, a Crank-Nicolson and a θ method with $\theta = 3/4$. Solve the problem for 10 time steps for $\frac{\nu\pi^2}{\Delta x^2}\Delta t = -0.1, -0.5, -1, -2, -4, -8$ and plot the solutions from the three numerical methods together with the exact analytical solution. Interpret the results, and explain if and why the nature of the CN solution changes for $\frac{\nu\pi^2}{\Delta x^2}\Delta t < -2$.

References

[1] Philippe R Spalart, Robert D Moser, and Michael M Rogers. Spectral methods for the navier-stokes equations with one infinite and two periodic directions. *Journal of Computational Physics*, 96(2):297 – 324, 1991.