System Design: The Absentee in System Theory

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Dedicated to:

Osita Nwokah,
Friend, Scholar

With a Little Help From My Friends

Adhikari    Kiichiro    Roman
Aldrich     Lu         Sato
Callafon    Mingori    Sultan
Grigoriadis Murakami   Williamson
Helton      Pinaud     Yamashita
Hubble Space Telescope (next servicing mission)

- New Solar Array
- Old Controller
Technology: What Paved the Way?

• 1st Half of 20th Century: PHYSICS

• 2nd Half of 20th Century: ENGINEERING
  (Component Technology)
  (Control is a component)

• Next?: SYSTEMS
  (Interdisciplinary Technology)
Outline

• Designing Systems
• Designing Models for Systems
• Inspirations From ART NATURE
• Designing Controlled Structures
Pin the Tail on the Performance Limiting Technology

Manufacturing

Modeling

Computing

Physics

Sensing

Signal Processing

Control

Systems Theory
Systems Design Today

- **Universities Teach Component Technology**
  - materials, dynamics, signal processing, control
- **Leading to: uncoordinated multidisciplinary tasks**
  - Manufacture components, then
  - Model components (physics), then
  - Connect, Measure, Actuate components, then
  - Control the interconnected components
- **Problem:** dealing with sufficient rather than necessary
- **Systems** approach needed

- **Michael Faraday:** “Begin with the **whole**, then construct the **parts**”
When is the Whole LESS Than the Sum of the Parts?

• The answer: usually

• Today, components are overdesigned to compensate for the lack of coordination in their design

• A Misconception: “The best system is made from the best components”

• Often, more gain in integrating two disciplines, than gained by new technology in either discipline

• Examples …………..
Given a System Requirement
Where Should We Invest?

- How accurately to Model component X?
- How accurately to Manufacture component X?
- Is Component X even Necessary?
- How should the components be Connected?

• Control is a Component technology
Finite Precision Computing

\[ \delta x = A(x + e) + Bu \]
\[ y = C(x + e) \]

\[ x = T \vee \text{: No Clue about Basis } T \text{ From Physics} \]
\[ y = G(z)u + G_e(z, T)e \]

- Useless to Model Better Than Error \( e(\text{unbounded over } T) \)

- How We.... MODEL One Component Affects The Dynamics of Another
Unified Signal Processing/Control

\[ \dot{x} = A(x + e) + Bu \]
\[ y = C(x + e) \]

\[ x = T \nu \]
\[ y = G(z)u + G_e(z, T)e \]

\[ \min_T \| G_e(T) e(\beta) \| \]

\[ \exists (T^{-1}(Exx^T)T^{-T})_{ii} = 1 \]

No Clue about Basis T From Physics

\[ Ee_i^2 = \frac{1}{12} 2^{-2} \beta \]

- **Component technology:** Design \((A,B,C)\), then \(T\)
- **Systems technology:** Design \((A,B,C,T)\) jointly

Control: **Coupled ARE**\((\beta)\)

[References: Mullis/Roberts 76, Williamson 86, Liu/Grigoriadis/Skelton 88, Gevers 92, Bamieh 94]
Hubble Space Telescope

- Limited by control
- Improvement @ no cost by unifying 2 disciplines

Using 24 bits: 100 times less (pointing variance)/(control energy)
Using 4 bits: 10,000 times less (pointing variance)/(control energy)

change existing coefficients (no cost solution)

[IEEE TAC, Vol 37, No.9, 1992, JGCD, vol 18, No.2, 1995]
Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]

\[
\min_{\mathbf{K}} \quad E \mathbf{u}^T \mathbf{u}
\]

\[
E y y^T \leq Y
\]

Guarantee Performance Y

Convex, given Y

\[
p_i \leq p_i \leq \bar{p}_i
\]

\[
\dot{x} = A(p)x + B(p)u + Dw
\]

\[
y = Cx
\]

\[
[A \quad B] = M + \sum p_i M_i
\]

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Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]

\[
\min_{p, K} \mathbb{E} u^T u
\]

\[
E y y^T \leq Y
\]

Update
Plant \( p \),
Control \( K \)

Not Convex

\[
p_i \leq p_{\bar{i}} \leq \bar{p}_i
\]

\[
\min_{K} \mathbb{E} u^T u
\]

\[
E y y^T \leq Y
\]

Guarantee
Performance \( Y \)

Convex, given \( Y \)

\[
\dot{x} = A(p)x + B(p)u + Dw,
\]

\[
y = Cx
\]

\[
[A \ B] = M + \sum p_i M_i
\]
Optimal Mix of Plant/Control Design

[Grigoriadis, Zhu, Skelton, 1992]

\[
\min_{p, K} \begin{bmatrix} E u^T u \\ E x x^T = X \end{bmatrix} \quad \text{Update Plant } p, \text{ Control } K
\]

\[
\min_K \begin{bmatrix} E u^T u \\ E y y^T \leq Y \end{bmatrix} \quad \text{Guarantee Performance } Y
\]

Convex, given \( X \)

\[
p_i \leq \bar{p}_i
\]

Convex, given \( Y \)

\[
[A \ B] = M + \sum p_i M_i
\]

\[
\begin{aligned}
\dot{x} &= A(p)x + B(p)u + Dw, \\
y &= Cx
\end{aligned}
\]
Optimal Mass/Damping/Stiffness/Control

[Lu, Skelton, Computer Aided Civil Infrastructure Engr, vol 13, 1998]
Optimal Mix of Physics/Information, Structures/Control

Most structure, least actuators

Control Energy, Subject to $\|y\|_2 \leq Y$

mass structure

Optimal Number Actuators

• Less energy, more robust
• More complex controller
• Avoiding integer program……
The Economic Design Problem

\[ W = \sigma Z \]

Finite Signal-to-Noise ratio

\[ E w_j w_k = W \delta_{jk}, \quad E_{zz}^T = Z \]

\[ \$ = \sum_i p_i \sigma_i^{-1} \]

Min \( \$ \), subject to \( E_{yy}^T \leq Y \) \( \sigma, K \)

Convergent algorithm, but no global

If \( \sigma_1^{-1} \gg \sigma_2^{-1} \), then delete sensor 2, or off-the-shelf

[Lu, Skelton, Automatica, to appear]
Outline

• Designing Systems
• Designing Models for Systems
• Inspirations From ART NATURE
• Designing Controlled Structures
Component Modeling

[Hu/Skelton, Computers and Structures, 1985]

From physics, choose $\Phi$ for small $e$

From systems criteria, choose $\Phi$ to depend on $u, w$

 recalls $x = T v$

Choose $T$ to depend on $w(b)$
Is System Modeling Just Physics, Physics?

Or can Mother Nature be fooled?

Plant P

Model M

Controller C(M)

\[ u \]

\[ + \]

\[ - \]

\[ e \]

\[ y \]

\[ \omega \]

\[ j \]

\[ M(\omega) \]

\[ P(\omega) \]

\[ N(\omega) \]

\[ \int (8y^2 + u^2) dt \]

\[ [\text{Yousuff, 85}] \]

\[ \Delta M + M \]

\[ \text{Desired focus} \]

\[ \text{Robust Control Theory} \]

\[ C(P) = C(N) = C(M) \]
Control Models : How Much Info is Really Necessary?

Minimize $\sum_{k=1}^{N} (y_k^T Q y_k + u_k^T R u_k)$

Subject to $x_{k+1} = Ax_k + Bu_k$, $y_k = Cx_k$

[Shi, Skelton, DATA-BASED CONTROL, ‘94], [Furuta, ‘93], [Ikeda, ‘99]

**Theorem**

**Optimal Controller Requires Only**

$CA^i B$, $i = 0,1,2, \ldots, N - 1$

- Only Errors in $CA^i B$ Affect Control Performance
- Any QMC from data yields the optimal control
- Why compute Markov Parameters, Use Data Directly
Data Equivalent Models

[Skelton, Zhu, Qmarkov COVER, 1991]

Does there exist any linear model to Match the input/output data?

IFF \( R - HH^T \geq 0 \)

\[
R_i = Ey_{k+i}y_k^T, \quad H_i = Ey_{k+i}u_k^T
\]

\[
R = \begin{bmatrix}
R_0 & R_1^T & R_2^T & R_3^T \\
R_1 & R_0 & R_1^T & R_2^T \\
R_2 & R_1 & R_0 & R_1^T \\
R_3 & R_2 & R_1 & R_0
\end{bmatrix}, \quad H = \begin{bmatrix}
H_0 & 0 & 0 & 0 \\
H_1 & H_0 & 0 & 0 \\
H_2 & H_1 & H_0 & 0 \\
H_3 & H_2 & H_1 & H_0
\end{bmatrix}
\]

Many Models Equivalent to the One From Physics
Absentee: A System Modeling Theory

- The size of the error is **Not Continuous** from Component (OL) to System (CL)
  - Unbounded OL error, but zero CL error
  - Smaller OL errors ⇒ smaller CL errors. Hence….

- **Good Component Models ⇒ Good System Models**
  - There might exist a Better model for System Design than the Actual model from physics.
  - **Bad News:** Investments in Component Modeling may Not Help System Modeling
  - **Good News:** Good System Models can be Simpler Than Component Models (Yes, There Exists an Optimal Size)

- Should control design occur **Before, After, or During** plant modeling?
The Best System Model

• May have little to do with the best Component Model

- Best Optics Model
- Best Fluids Model
- Best Structures Model
- Best Materials Model
Outline

• Designing Systems
• Designing Models for Systems
• Inspirations From ART
• Designing Controlled Structures
Integrating Material, Structure, Control

• The Theory of Continua is inadequate to account for the strength of materials.

• Challenge for Man-made Systems: Architecture
  – Information Architecture: selection of sensors, actuators, and feedback paths.
  – Material Architecture: selection of the material geometry

• Look to Biological and Natural systems, where
  – Mechanical, Chemical, and Electrical forces are involved in complex patterns of information flow, sensing, and feedback
Inspiration From Art

Kenneth Snelson
Needle Tower, 1968
Kroller Muller Museum
The Netherlands

After 30 Years of
• Forcing Continua
• Adding Actuators to
  Old Paradigms:
  Beams, Plates, Shells

Eureka !!!!
• No Joints
• No Load Reversals
• No Friction
• No Member Bending
• Easy to Change Equilibrium

“Tensegrity”
= Tension + Integrity
Mammalian Cell Cytoskeleton

Ingber, 98
Scientific American
Carbon Nanotubes, Fullerenes

Strength From Geometry

[Yakobson, Smalley, American Scientist, July 97]

Smalley, 1996 Nobel Prize
Buckyballs and Fullerenes

Geometry

mechanical properties,
electrical properties

[Yakobson, Smalley, American Scientist, July 97]
A Tensegrity Found in Nature,

Spider Fiber: Nature’s Strongest

Amino Acids:
Discontinuous compressive
Continuous tension

What’s in Common?
Fullerenes,
Spider fiber
Cell Cytoskeleton
Snelson’s Artform

Tensegrity topology
Outline

- Designing Systems
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**Definition: Tensegrity Systems**

2N Points form a Tensegrity Geometry if the set of Points are stabilizable with pretensioned axially loaded members connecting the points, with no more than two compressive members attached to a Point.

**Class D:** Discontinuous Compressive Members  
**Class C:** Continuous Compressive Members

- **C3T3, Class C**
- **C2T4, Class D**
- **3D: No Tensegrity**
A Tensegrity Geometry
(To Form A Tube)

Points Not in a Plane
A Tensegrity System
A Tensegrity System

3 Local Parameters + Equilibrium Constraint

Symmetric Geometry: 2 Free Parameters
Shell Class of Tensegrity Systems
Shell Class Tensegrity Structures

Over lap (°f stage height)

Overlap = 100%

Overlap = 0%

$\delta = 90^\circ$

$\alpha$ (degrees)

$\delta$ (degrees)

Vertical strings

Structural Systems and Control Laboratory
School of Engineering, UCSD
Tensegrity Geometry

\[( h, \alpha, \delta ) = \text{Stable equilibrium} \]

\[
F ( h, \alpha, \delta ) t = 0, \quad t > 0
\]

\[
| F^T F | = 0
\]

\[
h = \frac{1}{2 \tan \delta \cos \left( \alpha + \frac{\pi}{6} \right)}
\]

\[
\left\{ \begin{array}{l}
- \frac{L}{\sqrt{3}} + L \sin \delta \cos \left( \alpha + \frac{\pi}{6} \right) + \sqrt{\frac{L^2}{3} - 3 L^2 \sin^2 \delta \cos^2 \left( \alpha + \frac{\pi}{6} \right)}
\end{array} \right.
\]

- Pugh, 1976
- Pelligrino, Calladine, 1986
- Motro, 1986
- Furuya, 1992
- Coughlin, Stamenovic, 1997
- Skelton, 1993 - 1999
- Oppenheim, 1998
- Williamson, Skelton, 1999
Tensegrity Paradigm for Structural Control

- Changing the Shape With Less Control Energy
  - Construct a Tensegrity Geometry with a specified shape
  - Actuate the tendons (rest lengths) to avoid straining the structure, moving from one equilibrium to another
Shape Control with Tensegrity

Theorem [99]

Let $q_1$, $q_2$ be two tensegrity geometries, associated with the same nullspace of $F(q)$. Then,

- There exists a continuum of tensegrity geometries between $q_1$ and $q_2$.
- There exist tendon controls to change the geometry from $q_1$ to $q_2$ without changing potential energy.

\[
\ddot{q} + (K_r(q) + K_p(q))q = B(q)(u + v) + Dw
\]

\[
\dot{u} = K^{-1}(Rq - u) \frac{(Rq - u)^T KR \dot{q}}{(Rq - u)^T (Rq - u)} , \quad Rq > u
\]
Controlling (Torturing) Structures

- **Existing approach:** (DARPA SMART WING)
  - Design Control After Structure
  - Twist the structure against its equilibrium. This requires work (7 deg limit, 20 deg desired)

- **New Paradigm:** Unify at more fundamental level
  - Change shape by changing the equilibrium
Minimal Mass Tensegrity Structures

\[ m_i = b m_0 \]

\[ b = \frac{\pi E_0 (\tan^2 \delta)}{\sigma} \left( \sum_{j=2}^{i} \frac{1}{(\cos^2 \delta)^{j-1}} \right) + m_B \]

\( \sigma = \text{Tensile strength} \)

\[ m_B = \left( 2 \cos^5 \delta \right)^{-\frac{i}{2}} \]

\( \delta = 10 \)
Theorem

The compressive stiffness of the C4T1 Structure is equal to the tensile stiffness of the tendons of the shortest tendons

- Controlling the tension of the shortest tendon controls the compressive stiffness of the entire structure
- All compressive members carry the same load
- Infinite buckling strength for finite $i$
- Minimal mass occurs at smaller $i$ than infinite strength
Advantages of The Tensegrity Paradigm

- **All Members Axially Loaded**
  - Global bending without member bending

- **All Members Uni-Directionally Loaded (Pretension)**
  - No reversal of load direction (no friction, hysteresis)

- **Structural Efficiency**
  - Strength to mass very high
  - Inspired by Art and Biological forms

- **Easy to Integrate Structure/Control**
  - More accurate models (hence more precise control)
  - A structural member also serves as sensor, actuator
  - Actuator/Sensor architecture easily optimized
  - Change shape with little work (one equilibrium to another)
Two-Stage Tensegrity: Tendon Control

Controlled tendons

DC Motors
Deployable Tensegrity Heat Shield
Single-Stage Tensegrity: Pneumatic
Tensegrity Suspension for Disk Drives

Tension members

Compression members

Disk
Conclusions

To Universities and funding agencies:

- Give the **Soul of Control** a **Body: System Design**
- Give **Modeling** a Purpose: **Systems Modeling**
- **Function following Form**
  - Snelson’s Tensegrity Artform Inspires a New Paradigm to Integrate Mechanics, Structures, and Control
Conclusions

• After Component Technologies mature, the next quantum leap in technology must come from a scientific method to do Interdisciplinary System Design

• Why wait til component maturity to invest in a scientific method for Systems Design

• The biggest challenge: System Modeling

• System Design (and modeling) requires more than communicating what each discipline already knows

• We can exploit biological material architecture to

• Suggest a system design paradigm for designing to specific materials, thermal, electrical, mechanical properties:
What Your Data Never Told You

- Data is not Information
- Information is not Knowledge
- Knowledge is not Understanding
- Understanding is not Wisdom

[Howard Garner, Harvard]