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An Introduction to the Mechanics of Tensegrity Structures

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Abstract

Tensegrity structures consist of strings (in tension) and bars (in compression). Strings are strong, light, and foldable, so tensegrity structures have the potential to be light but strong and deployable. Pulleys, NiTi wire, or other actuators to selectively tighten some strings on a tensegrity structure can be used to control its shape. This article describes some principles we have found to be true in a detailed study of mathematical models of several tensegrity structures. We describe properties of these structures which hold quite generally. We describe how pretensing all strings of a tensegrity makes its shape robust to various loading forces. Another property asserts that the shape of a tensegrity structure can be changed substantially with little change in the potential energy of the structure. Thus shape control should be inexpensive. This is in contrast to the control of classical structures which require substantial energy to change their shape.

1 Introduction

Tensegrity structures are built of bars and strings attached to the ends of the bars. The bars can resist compressive force and the strings cannot. Most bar-string configurations which one might conceive are not in equilibrium, and if actually constructed will collapse to a different shape. Only bar-string configurations in a stable equilibrium will be called **tensegrity structures**.

If well designed, the application of forces to a tensegrity structure will deform it into a slightly different shape in a way which supports the applied forces. Tensegrity structures are very special cases of trusses, where members are assigned special functions. Some members are always in tension and others are always in compression. We will adopt the words "strings" for the tensile members, and "bars" for compressive members. A tensegrity structure's bars cannot be attached to each other through joints that impart torques. The end of a bar can be attached to strings or ball jointed to other bars.

The artist Kenneth Snelson [1] built the first tensegrity structure and his artwork was the inspiration for the first author's interest in tensegrity. Buckminster Fuller coined the word tensegrity from two words, tension and integrity.

2 The Benefits of Tensegrity

There exists a large literature on the geometry, artform, and architectural appeal of tensegrity structures, but there exists little on the dynamics and mechanics of these structures [2–5]. Work by [6] shows an array of stable tensegrity units connected to yield a large stable system, which can be deployable. Tensegrity structures for civil engineering purposes have been built and described in [7, 8]. Several reasons are given below why tensegrity structures, which employ many tendons for prestressibility should receive new attention from mathematicians and engineers, even though the concepts are fifty years old.

Tension Stabilizes:
A compressive member looses stiffness as it is loaded, whereas a tensile member gains stiffness as it is loaded.

Tensegrity Structures are Efficient:
Bendsoe and Kikuchi [9], Jarre [10], and others have shown that the optimal distribution of mass for specific stiffness objectives tends to be neither a solid mass of material with a fixed external geometry, nor material laid out in orthogonal components. Material is needed only in the essential load paths, not the orthogonal paths of traditional man-made structures. *Tensegrity structures* use longitudinal members arranged in very unusual (and non–orthogonal) patterns to achieve strength with small mass.

Tensegrity Structures are Deployable:
Materials of high strength tend to have very limited displacement capability. Piezoelectric materials are capable of only small displacement and "smart" structures using such

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sensors and actuators have only small displacement capability. Since the compressive members of tensegrity structures are either disjoint, or connected with ball joints, large displacement, deployability and stowage in a compact volume will be immediate virtues of tensegrity structures [11]. This feature offers operational and portability advantages.

Tensegrity Structures are Easily Tunable:
The same deployment technique can also make small adjustments for fine tuning of the loaded structures, or adjustment of a damaged structure. Structures that are designed to allow tuning will be an important feature of next generation mechanical structures, including civil engineering structures.

Tensegrity Structures Can be More Reliably Modeled:
All members of a tensegrity structure are axially loaded. Perhaps the most promising scientific feature of tensegrity structures is that while the *global* structure bends with external static loads, none of the *individual* members of the tensegrity structure experience bending moments. (In this report, we design all compressive members to experience loads well below their Euler buckling loads.) More reliable models can be expected for axially loaded members compared to models for members in bending [12].

Tensegrity Structures Facilitate High Precision Control:
Structures that can be more precisely modeled can be more preciously controlled. Hence, tensegrity structures might open the door to quantum leaps in the precision of controlled structures. The architecture (geometry) dictates the mathematical properties and hence these mathematical results easily scale to both large and small scales, from the nanoscale to the megascale, from applications in microsurgery to antennas, to aircraft wings, to robotic manipulators.

Tensegrity is a Paradigm that Promotes the Integration of Structure and Control Disciplines:
A given tensile or compressive member of a tensegrity structure can serve multiple functions. It can simultaneously be a load-carrying member of the structure, a sensor (measuring tension or length), an actuator (such as nickel-titanium wire), thermal insulator, or electrical conductor. In other words, by proper choice of *materials and geometry*, a grand challenge awaits the tensegrity designer: How to control the electrical, thermal, and mechanical energy in a material or structure? Tensegrity structures provide a promising paradigm for integrating structure and control design.

Tensegrity Structures are Motivated from Biology:
In spider fiber, amino acids of two types have formed hard β–pleated sheets that can take compression, and thin strands that take tension [13, 14]. The β–pleated sheets are discontinuous and the tension members form a continuous network. Hence, the nanostructure of the spider fiber is a Tensegrity Structure. Nature's endorsement of tensegrity structures warrants our attention because per unit mass, the spider fiber is the strongest natural fiber.

If tensegrity is nature's preferred building architecture (as claimed by the Harvard biologist D. Ingber), modern analytical and computational capabilities of tensegrity could make the same incredible efficiency possessed by natural systems transferrable to man-made systems, from the nano- to the megascale. This is a grand design challenge, to develop scientific procedures to create smart tensegrity structures that can regulate the flow of thermal, mechanical, and electrical energy in a material system by proper choice of materials, geometry, and controls. This report contributes to this cause by exploring the mechanical properties of simple tensegrity structures.

3 Definitions

This is an introduction to the mechanics of a class of prestressed structural systems that are composed only of axially-loaded members. We need a couple of definitions to describe tensegrity scientifically.

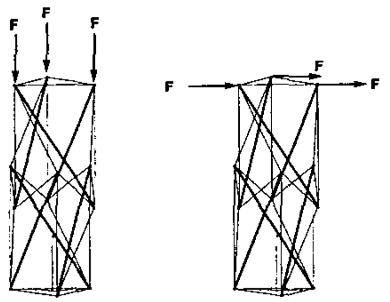
Definition 1 *A* Class k Tensegrity Structure *is a stable equilibrium of axially-loaded elements, with a maximum of k compressive members connected at the node(s).*

4 The Structure Analyzed in this Paper

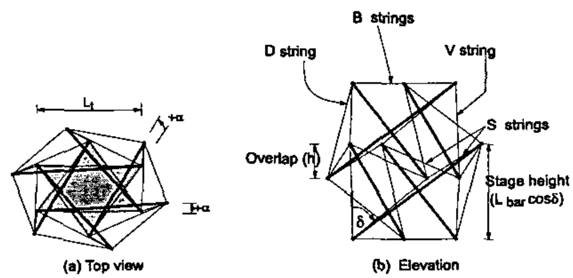
The basic example we analyzed is shown in Fig. 1, where thin lines are the strings and the thick lines are bars. The structure was analyzed under several types of loading. The mass and stiffness properties of such structures will be of interest under compressive loads, F, as shown. The 2–stage 3–bar was studied under two types of loading, axial and lateral. Axial loading is compressive while lateral loading results in bending.

5 Two–Stage Three–Bar Tensegrity Properties

The tensegrity unit studied here is the simplest three–dimensional tensegrity unit which comprises three bars held together in space by strings so as to form a tensegrity unit. A tensegrity unit comprising of three bars will be called a 3–bar tensegrity. A 3–bar tensegrity is constructed by using three bars in each stage which are twisted either in clockwise or in anti–clockwise direction. The top strings connecting the top of each bar support the next stage in which the bars are twisted in a direction opposite to the bars in the previous stage. In this way any number of stages can be constructed which will have an alternating clockwise and anti–clockwise rotation of the bars in each successive stage. This is the type of structure in Snelson's Needle Tower.



(a) 2–stage 3-bar - axial loads (left) and lateral loads (right)



(b) Top view and elevation of a 2–Stage 3–bar tensegrity

Figure 1: Tensegrity studied in this paper(Not to scale).

The four different types of strings are labeled S, V, D, and B in Fig. 1.

A typical 2–stage 3–bar type tensegrity is shown in Fig. 1 in which the bars of the bottom stage are twisted in the anti–clockwise direction. The coordinate system used is also shown in the same figure. The same configuration will be used for all subsequent studies on the statics of the tensegrity. The notations and symbols, along with the definition of angles α and δ, and overlap, h, between the stages, used in the following discussions are also shown in Fig. 1.

The tensegrity structure exhibits unique equilibrium characteristics. The sum of forces on a bar may be written $F(q)t = 0$, where F is a matrix, q describes the geometric configuration, and t is the vector of tensions in the strings. There exists a tension t that stabilizes the configuration q only if matrix F has a nullspace. This is called the "prestressability" condition. Obviously, if F has a nullspace, the tension vector t can be scaled by any positive scalar, increasing the potential energy of the system without changing its shape. Figure 2 shows the restrictions between the geometrical parameters α, δ, and h, which allow an equilibrium. Thus, any point on this equilibrium surface in Fig. 2 corresponds to a configuration that is prestressable.

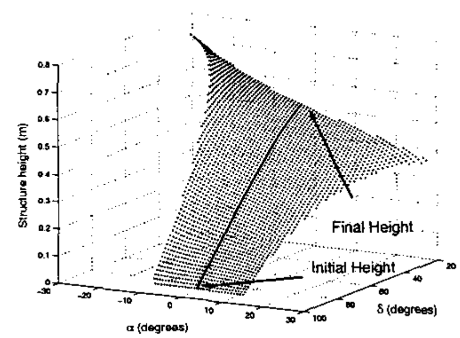


Figure 2: Equilibrium Surface with Deployment Path of 2–stage 3–bar Tensegrity Structure. ($b = .27$ m, $L_{bar} = .4$ m)

5.1 Load–deflection Curves and Axial Stiffness as a Function of the Geometrical Parameters

The load deflection characteristics of a two stage 2–stage 3–bar type tensegrity is next studied and the corresponding stiffness properties are investigated.

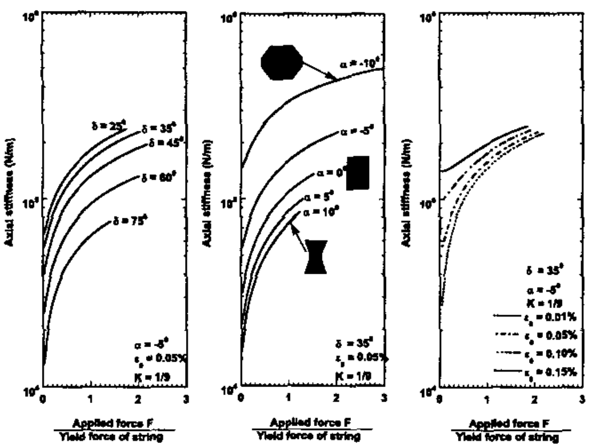
Figure 3 depicts the axial stiffness as a function of prestress, drawn for the case of a 2–stage 3–bar type tensegrity subjected to axial loading. The axial stiffness is defined as the external force acting on the structure divided by the axial deformation of the structure.

The characteristics of the axial stiffness of the tensegrity as a function of the geometrical parameters (i.e. α, δ) are next plotted in Fig. 3. The effect of the prestress on the axial stiffness is also shown in Fig. 3. In obtaining the Fig. 3, vertical loads were applied at the top nodes of the 2–stage tensegrity. The load was gradually increased until at least one of the strings exceeded its elastic limit. As the compressive stiffness and the tensile stiffness were observed to be nearly equal to each other in the present example, only the compressive stiffness as a function of the geometrical parameters, is plotted in Fig. 3. The change in the shape of the tensegrity structure from a "fat" profile to an "hour–glass" like profile with the change in α is also shown in Fig. 3(b).

The following conclusions can be drawn from Fig. 3:

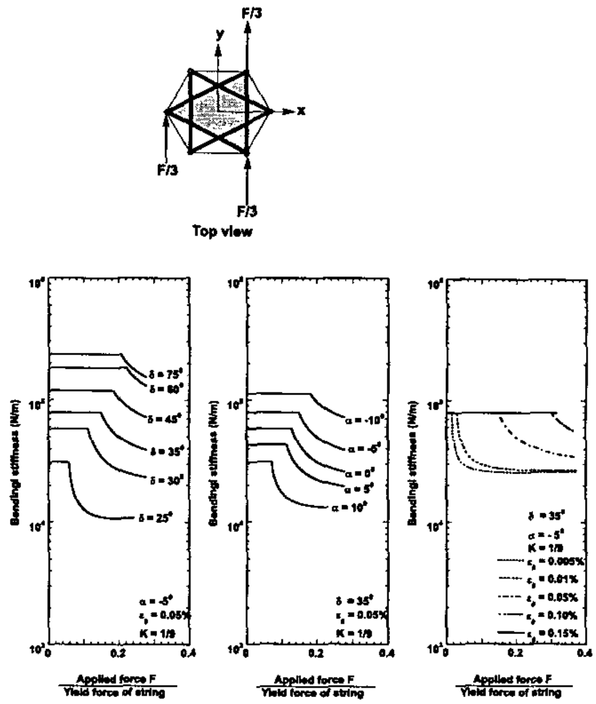
1. Fig. 3(a) suggests that the axial stiffness increases with the decrease in the angle of declination δ (measured from the vertical axis).
2. Fig. 3(b) suggests that the axial stiffness increases with increase in the negative angle α. Negative α means a "fat" or "beer–barrel" type structure whereas a positive α means an "hour–glass" type structure, as shown in Fig. 3(b). Thus a "fat" tensegrity performs better than an "hour–glass" type tensegrity subjected to compressive loading.
3. Fig. 3(c) suggests that the prestress has an important role in increasing the stiffness of the tensegrity in the

region of small external loading. However, as the external forces are increased, the effect of the prestress becomes practically negligible.



(a) (b) (c)

Figure 3: Axial stiffness of a 2–stage 3–bar tensegrity for (a) different δ with $\alpha = -5^0, \varepsilon_0 = 0.05\%, K = 1/9$, (b) different angle α with $\delta = 35^0, \varepsilon_0 = 0.05\%, K = 1/9$ and (c) different ε_0 with $\alpha = -5^0, \delta = 35^0, K = 1/9$. L_{bar} for all cases is 0.4m.



(a) (b) (c)

Figure 4: Bending stiffness of a 2–stage 3–bar tensegrity for (a) different δ with $\alpha = -5^0, \varepsilon_0 = 0.05\%, K = 1/9$, (b) different angle α with $\delta = 35^0, \varepsilon_0 = 0.05\%, K = 1/9$ and (c) different ε_0 with $\alpha = -5^0, \delta = 35^0, K = 1/9$. L_{bar} for all cases is 0.4m.

5.2 Load–deflection Curves and Bending Stiffness as a Function of the Geometrical Parameters

The bending characteristics of the 2–stage 3–bar tensegrity are presented in this section. The force is applied along the x–direction and then along the y–direction. The force is gradually applied until at least one of the strings exceeds its elastic limit.

The characteristics of the bending stiffness of the tensegrity as a function of the geometrical parameters (i.e. α, δ) are next plotted in Fig. 4. Figure 4 is plotted for lateral force applied in the y–direction, as shown in Fig. 4. The effect of the prestress on the bending stiffness is also shown.

The following conclusions about the bending characteristics of the 2–stage 3–bar tensegrity could be drawn from Fig. 4:

1. It is seen in Fig. 4, that the bending stiffness of the tensegrity with no slack strings is practically equal in both the x– and y–directions. However, the bending stiffness of the tensegrity with slack string is greater along y–direction than along the x–direction.

2. The bending stiffness of a tensegrity is constant and is maximum for any given values of α, δ and prestress when none of the strings are slack. However, as soon as at least one string goes slack (marked by sudden drop in the stiffness curves in Fig. 4), the stiffness becomes a nonlinear function of the external loading and decreases monotonically with the increase in the external loading. As seen in Fig. 4, the onset of strings becoming slack, and hence the range of constant bending stiffness, is a function of α, δ and prestress.

3. Fig. 4(a) suggest that the bending stiffness of a tensegrity with no slack strings increases with the increase in the angle of declination δ (measured from the vertical axis). The bending stiffness of a tensegrity with a slack string, in general, increases with increase in δ.

4. Fig. 4(b) suggest that the bending stiffness increases with the increase in the negative angle α. As negative α means a "fat" or "beer–barrel" type structure whereas a positive α means an "hour–glass" type structure, a "fat" tensegrity performs better than an "hour–glass" type tensegrity subjected to lateral loading.

5. Figs. 4(a,b) indicate that both α and δ play a very interesting and important role in not only affecting the magnitude of stiffness, but they also affect the onset of slackening of the strings (robustness to external disturbances). A large value of negative α and a large value of δ (in general) delay the onset of slackening of the strings, thereby increasing the range of constant bending stiffness. However, there exists a certain δ for which the onset of the slack strings is maximum.

6. Fig. 4(c) suggests that prestress does not affect the bending stiffness of a tensegrity with no slack strings. However, prestress has an important role in delaying the onset of slack strings and thus increasing the range of constant bending stiffness.

6 Controlling Properties

Tensegrity structures are natural candidates to be actively controlled structures since the control system can be embedded in the structure directly; for example tendons can act as actuators and/or sensors. Shape control of the tensegrity structure in Fig. 1 can be accomplished by moving along the equilibrium manifold shown in Fig. 2.

6.1 Deployment

An interesting application of this tensegrity is the controlled deployment of the structure from a near zero initial height to a greater height. Moving along the equilibrium manifold consists of moving along symmetrical prestressable configurations. We use an open loop control strategy based upon slowly moving from one stable equilibrium to another. Stability along the deployment path is assured only if this movement is slow enough. The necessary and sufficient conditions for a symmetric prestressable equilibrium to exist for $\alpha = 0$ are

$$h = \frac{L_{bar}\cos\delta}{2}, \quad 3L_{bar}\sin\delta > 2b, \quad 0 < \delta < \frac{\pi}{2}. \tag{1}$$

Therefore, under symmetrical reconfiguration, where $\alpha = 0$ and all rod declinations, δ, are equal, the total height of the structure is

$$\text{Total Height} = \frac{3L_{bar}\cos\delta}{2}. \tag{2}$$

The length of the S, V, D, and B strings are:

$$S = \sqrt{h^2 + \frac{b^2}{3} + L_{bar}^2\sin^2\delta - L_{bar}b\sin\delta} \tag{3}$$

$$V = \sqrt{L_{bar}^2 + b^2 - 2L_{bar}b\sin\delta} \tag{4}$$

$$D = \sqrt{L_{bar}^2 + \frac{b^2}{3} + h^2 - 2L_{bar}h\cos\delta - L_{bar}b\sin\delta} \tag{5}$$

$$B = b. \tag{6}$$

Therefore, we can prescribe a time varying function $\delta(t)$ to modify the objective: structure height. For example choosing $\delta(t) = 90° - 63°t$ satisfies the inequality constraints on δ in (1) for $0 < t < 1$, where $b = .27$m and $L_{bar} = .4$m. Substitution of (1) into the string length equations yields the open loop control laws for each tendon length

$$S(t) = \sqrt{L_{bar}^2 + \frac{b^2}{3} - \frac{3L_{bar}^2\cos^2\delta(t)}{4} - L_{bar}b\sin\delta(t)} \tag{7}$$

$$V(t) = \sqrt{L_{bar}^2 + b^2 - 2L_{bar}b\sin\delta(t)} \tag{8}$$

$$D(t) = S(t) \tag{9}$$

$$B(t) = b. \tag{10}$$

The structure deployment is shown in Fig. 5. Further detail can be found in [15].

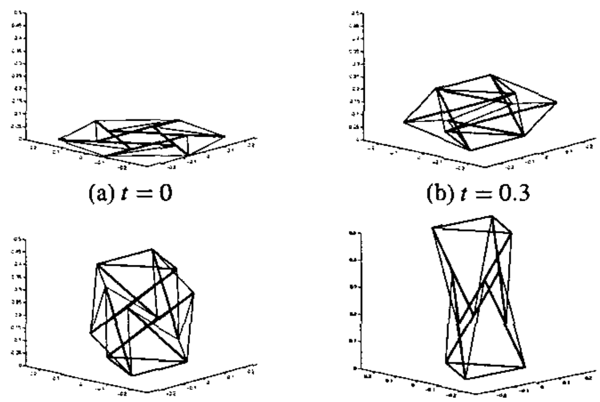

(a) $t = 0$ (b) $t = 0.3$ (c) $t = 0.6$ (d) $t = 1$

Figure 5: Deployment sequence followed on line shown in Fig. 2.

6.2 Stiffness Control

The stiffness analysis done in the previous sections, shows that stiffness is most effected by the geometrical parameters, α, δ. Since we have shown that reconfiguration of a tensegrity is possible, we now investigate the use of control to directly modify the mechanical properties of the tensegrity. The control of stiffness can be accomplished in exactly the same way as the deployment sequence. In fact, the deployment sequence presented is changing stiffness properties and geometrical properties at the same time. According to the equilibrium manifold shown in Fig. 2, we can simply choose a reconfiguration path that sets structure height to a constant and modifies only the angles α and δ (this in turn modifies stage overlap h). According to the stiffness plots shown in Figs. 3-4, we can move to different curves of higher or lower stiffness as we prefer, while changing only the shape of the structure and keeping the height constant. Keeping the height constant may be motivated by supporting some external load or used as a platform of some kind.

7 Conclusion

Tensegrity structures present a remarkable blend of geometry and mechanics. Out of various available combinations of geometrical parameters, there exists only a small subset that guarantees the existence of the tensegrity. The choice of these parameters dictate the mechanical properties of the structure. The choice of the geometrical parameters have a great influence on the stiffness. Pretension serves the important role of maintaining stiffness until a string goes slack. The geometrical parameters not only affect the magnitude of the stiffness either with or without slack strings, but they also affect the onset of slack strings. We now list the major findings of this paper that also apply to other tensegrities we have studied [16].

Pretension vs. Stiffness Principle

This principle states that increased pretension increases robustness to uncertain disturbances:

> **When a load is applied to a tensegrity structure the stiffness does not decrease or barely decreases as the loading force increases, unless a string goes slack.**
>
> **The greater the pretension, the greater the load required to make a string go slack.**

The bending stiffness of a tensegrity without slack strings is not affected appreciably by the amount of pretension.

Small Control Energy Principle

The second principle is that the shape of the structure can be changed with very small control energy along the parameter set dictated by Fig. 2. This is due to the fact that shape changes are achieved by changing the equilibrium of the structure. Thus we are moving from one equilibrium to another. In this case, control energy is not required to hold the new shape. This is in contrast to the control of classical structures, where shape changes required control energy to work against the old equilibrium.

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