Path planning for the deployment of tensegrity structures

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1. ABSTRACT

Tensegrity structures consist of tendons (in tension) and bars (in compression). Tendons are strong, light, and foldable, so tensegrity structures have the potential to be light but strong and deployable. Pulleys, NiTi wire, or other actuators to selectively tighten some strings on a tensegrity structure can be used to control its shape. This article describes the problem of asymmetric reconfiguration of tensegrity structures and poses one method of finding the open loop control law for tendon lengths to accomplish the desired geometric reconfiguration. In addition, a practical hardware experiment displays the readiness and feasibility of the method to accomplish shape control of the structure.

Keywords: Tensegrity structures, deployment, reconfiguration, nonlinear programs.

2. INTRODUCTION

Tensegrity structures are built of bars and tendons attached to the ends of the bars.[1] The bars can resist compressive force and the strings cannot. Most bar-string configurations which one might conceive are not in equilibrium, and if actually constructed will collapse to a different shape. Only bar-string configurations in a stable equilibrium will be called tensegrity structures.[2–5]

If well designed, the application of forces to a tensegrity structure will deform it into a slightly different shape in a way which supports the applied forces. Tensegrity structures are very special cases of trusses, where members are assigned special functions. Some members are always in tension and others are always in compression. We will adopt the words "tendons" for the tensile members, and "bars" for compressive members.[6] A tensegrity structure's bars cannot be attached to each other through joints that impart torques. The end of a bar can be attached to strings or ball jointed to other bars.

Tensegrity structures are natural candidates to be actively controlled structures since the control system can be embedded in the structure directly; for example tendons can act as actuators and/or sensors.[7–9] Shape control of the tensegrity structure in Fig. 1 can be accomplished by moving along the equilibrium manifold shown in Fig. 2. The tensegrity unit studied here is the simplest three–dimensional tensegrity unit which comprises three bars held together in space by strings so as to form a tensegrity unit. A tensegrity unit comprising of three bars will be called a 3–bar tensegrity. A 3–bar tensegrity is constructed by using three bars in each stage which are twisted either in clockwise or in anti–clockwise direction. The top strings connecting the top of each bar support the next stage in which the bars are twisted in a direction opposite to the bars in the previous stage. In this way any number of stages can be constructed which will have an alternating clockwise and anti–clockwise rotation of the bars in each successive stage. This is the type of structure in Snelson's Needle Tower. The following review of symmetrical reconfiguration is convenient to allow the reader to visualize and gain insight into the asymmetrical reconfiguration of tensegrity structures.

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2.1. Controlling Symmetrical Tensegrity Structures

A typical 2–stage 3–bar type tensegrity is shown in Fig. 1 in which the bars of the bottom stage are twisted in the anti–clockwise direction. The coordinate system used is also shown in the same figure. This tensegrity structure can be described by the three parameters chosen because we desire purely symmetrical configurations. Due to the small number of parameters, we can conveniently visualize the allowable configurations of the tensegrity structure as an equilibrium surface plotted in the three dimensions of the parameters. The notations and symbols, along with the definition of angles α and δ, and overlap, h, between the stages, used in the following discussions are also shown in Fig. 1.

2.2. Symmetrical Deployment

An interesting application of this tensegrity is the controlled deployment of the structure from a near zero initial height to a greater height. Moving along the equilibrium manifold consists of moving along symmetrical prestressable configurations[10]. We use an open loop control strategy based upon slowly moving from one stable equilibrium to another. Stability along the deployment path is assured only if this movement is slow enough.

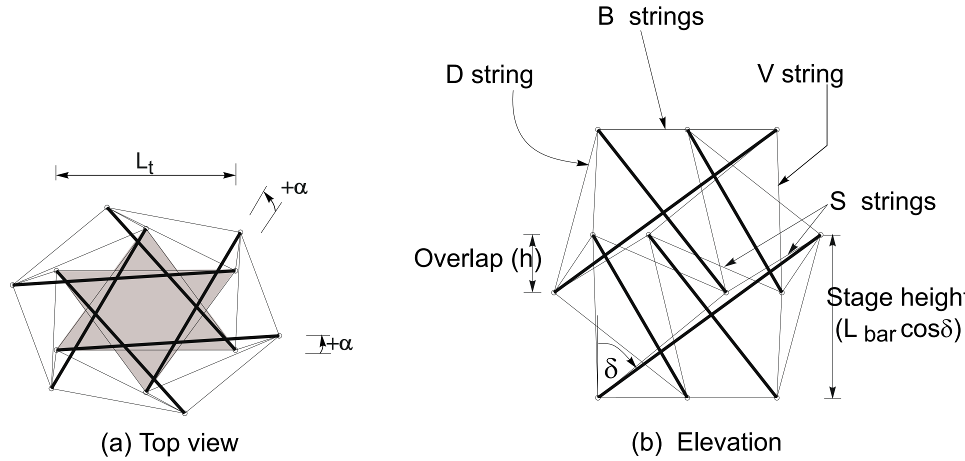

Figure 1. Tensegrity studied in this paper(Not to scale).

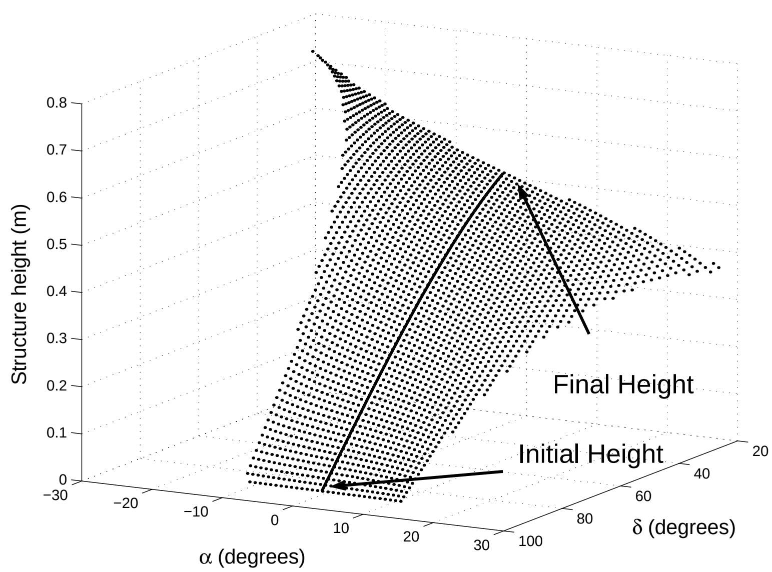

Figure 2. Equilibrium Surface with Deployment Path of 2–stage 3–bar Tensegrity Structure. ($b = .27$ m, $L_{bar} = .4$ m)

According to Sultan,[10] the necessary and sufficient conditions for a symmetric prestressable equilibrium to exist for $\alpha = 0$ are

$$h = \frac{L_{bar}\cos\delta}{2}, \quad 3L_{bar}\sin\delta > 2b, \quad 0 < \delta < \frac{\pi}{2}. \tag{1}$$

Therefore, under symmetrical reconfiguration, where $\alpha = 0$ and all rod declinations, δ, are equal, the total height of the structure is

$$\text{Total Height} = \frac{3L_{bar}\cos\delta}{2}. \tag{2}$$

The length of the S, V, D, and B strings are:

$$S = \sqrt{h^2 + \frac{b^2}{3} + L_{bar}^2\sin^2\delta - L_{bar}b\sin\delta}, \tag{3}$$

$$V = \sqrt{L_{bar}^2 + b^2 - 2L_{bar}b\sin\delta}, \tag{4}$$

$$D = \sqrt{L_{bar}^2 + \frac{b^2}{3} + h^2 - 2L_{bar}h\cos\delta - L_{bar}b\sin\delta}, \tag{5}$$

$$B = b. \tag{6}$$

Therefore, we can prescribe a time varying function $\delta(t)$ to modify the objective: structure height. For example choosing $\delta(t) = 90° - 63°t$ satisfies the inequality constraints on δ in (1) for $0 < t < 1$, where $b = .27$ m and $L_{bar} = .4$ m. Substitution of (1) into the string length equations yields the open loop control laws for each tendon length

$$S(t) = \sqrt{L_{bar}^2 + \frac{b^2}{3} - \frac{3L_{bar}^2\cos^2\delta(t)}{4} - L_{bar}b\sin\delta(t)} \tag{7}$$

$$V(t) = \sqrt{L_{bar}^2 + b^2 - 2L_{bar}b\sin\delta(t)} \tag{8}$$

$$D(t) = S(t) \tag{9}$$

$$B(t) = b. \tag{10}$$

3. ASYMMETRICAL RECONFIGURATION

Unfortunately, one shortcomming of the symmetrical parametrization for the two stage structure described is that the top and bottom triangles (top and bottom plates) are always parallel to each other and cannot deviate from that condition. In addition the center of mass of both plates is always aligned with the axis of symmetry. This is due to the fact that all bars are assigned the same parameter, therefore making the structure completely symmetric and representable by merely three parameters. If one wishes to modify the attitude or deviation of the top plate's center of mass away from the axis of symmetry, one has to abandon the symmetrical parametrization and investigate the possiblities in the asymmetric space. The asymmetrical parametrization involves using the maximum number of parameters to describe the structure, as opposed to the minimal set previously. Therefore, *six* independent parameters will be assigned to each bar member, increasing the parameter space to *thirty-six.* An investigation into the allowable space for each parameter would result in an equilibrium manifold represented by *thirty-six* parameters which is not practical. (We assume later that bar length is constant for all bars, reducing to five parameters per bar or thirty parameters total.) Due to the increase in independent parameters, the equilibrium manifold has become immensely large and a search to characterize this space would most likely be fruitless and yield no special insight. It is obvious that the symmetric parametrization is a special case of the asymmetric one.

Since a tensegrity structure does not exist outside of its equilibrium space, which is a smaller set than the inverse kinematic set, we need only compute the equilibrium space to implement the experiment. Also, the analytical inverse kinematics of this structure is not known at this time, therefore we adopt a numerical approach to solve this problem. Motivation for the numerical method to solve the path planning problem follows from the symmetrical parametrization shown in the introduction. Imagine two distinct points, each lying on the high dimensional equilibrium manifold and assume they can be connected by a multitude of smooth curves which also lie on the manifold. These curves essentially describe the reconfiguration of the geometric parameters and if it at least one curve can be selected, we have succeeded in accomplishing the objective to reconfigure the structure from one point to the other. It is highly likely that more than one curve exists due to the high dimensionality of the manifold. Therefore, we shall now propose a numerical procedure that can be used to choose a trajectory. This formulation allows for convenient specification of constraints that are important for the the initial equilibrium of the structure, the reconfiguration of the structure, and for the practical implementation in the laboratory. Of course, these constraints are chosen by the engineer to satisfy some objective or design constraints he may have in mind. At this time, no guarantee has been found that a solution exists for a given set of constraints and it is up to the intuition of the designer to make reasonable demands on the trajectory of the structure. The tensegrity equilibrium parameterization called tensegrity constitutive equations as described in Ref. 11 will be employed. The reader is advised to consult this reference for more details.

3.1. Tensegrity constitutive equations

Solution of a general tensegrity design problem involves finding the prestresable geometry of the structure and associated prestress element forces of the structure whose number of nodes, strings and total number of elements available are n_n, n_s, n_{el}. Once a maximum set of allowed element connections of a tensegrity structure and its associated oriented graph have been adopted, corresponding connectivity information are written in a form of member-node incidence matrix, $M \in \mathbf{R}^{3n_{el} \times 3n_n}$. Matrix M is a sparse block matrix whose i, j block is I_3 or $-I_3$ if the element i ends at or emanates from the j^{th} node, otherwise it is 0_3. After expressing element force t_i of a prestressed element i of a tensegrity structure as a product of an element vector g_i, and a scaling factor λ_i, called a force coefficient, and writing vector $g \in \mathbf{R}^{3n_{el}}$, formed by stacking up all the element vectors g_i, as a linear mapping of a nodal position vector, $p \in \mathbf{R}^{3n_n}$,

$$g = \begin{bmatrix} g_s \\ g_b \end{bmatrix} = Mp, \quad M = \begin{bmatrix} -S^T \\ B^T \end{bmatrix}, \quad S \in \mathbf{R}^{3n_n \times 3n_s},$$

the balance of the element forces at each of the nodes of the prestressed tensegrity structure can be written as,

$$C\Lambda Mp = 0, \quad \|\lambda\| > 0, \quad \lambda_i \geq 0, \quad i \in I_s, \quad C = \begin{bmatrix} S & B \end{bmatrix}. \tag{11}$$

where I_s is a set of indices of string elements. The vector $\lambda \in \mathbf{R}^{n_{el}}$ is formed by stacking up force coefficients λ_i. Linear operator $(\hat{\cdot})$ is defined as:

$$\hat{\cdot} : \mathbf{R}^n \to \mathbf{R}^{3n \times 3n}, \, \Lambda = blockdiag\{\lambda_i I_3\}, \, i = 1 \ldots n \tag{12}$$

Equation (11) represent a complete parametrization of non-necessarily stable equilibrium tensegrity structures. The solution of this problem can be obtained by casting this constrained zero finding problem as a nonlinear optimization problem.

$$\min_{p,\lambda} \|C\Lambda Mp\| \tag{13}$$

$$\|\lambda\| > 0, \quad \lambda_i \geq 0, \quad i \in I_s \tag{14}$$

We are now ready to show examples for specific reconfiguration problems where additional shape constraints are imposed.

4. EXAMPLES

In this section we demonstrate feasibility analysis of three different asymmetric reconfigurations and the possibility of accomplishing this using different number of actuators. It will be demonstrated that for the particular trajectory that is analyzed can be accomplished using *eighteen* actuators and *six* actuators.

Throughout the formal formulations of the problems it will be assumed that connectivity information of the two stage structure is given and fixed, in other words that connectivity matrices C and M are known and remain unchanged. In all examples for the given two stage tensegrity structure with constant bar lengths L_0 we want to compute control string lengths that correspond to the series of consecutive desired configurations that are close to each other.

4.1. Circular trajectory platform with 18 actuators

Example 1 In this example desired motion of the structure is characterized by a circular deviation of the top plate center of mass around the axis of symmetry. We want the structure to perform the following motion:

1. Move from the symmetric equilibrium configuration, where the top plate center of mass is at a given location, v^0,
2. reconfigure so that the center of the mass of the top plate moves along the x direction and reaches a targeted circular trajectory of given radius, r_{des}, from axis of symmetry,
3. return to the initial configuration after making the full revolution.

Formal specification of this requirement is defined and written as the motion constraint. Additional constraints must be satisfied. They are defined in group of constraints called geometry constraints:

1. Base nodes, p_{base}, fixed to an equilateral triangle of side b_0 in the inertial frame,
2. top tendons fixed to constant length b_0, (forming an equilateral triangular platform).

The division of constraints as "motion" and "geometry" constraints, although both are essentially constraints on structure geometry, is done for convenience that will simplify explanation later in the text.

Solution Lengths of the control strings corresponding to the i^{th} consecutive desired configuration are obtained by solving the sequence of optimization problems over $i = 1, \dots, n_x, \dots, n_c, \dots, n_f$, where n_x, n_c, n_f, are the number of points to the circular trajectory from $v_{cm}^{(0)}$, the number of points around the circular trajectory, and the number of points back to the initial configuration, respectively. Finally, the i^{th} problem to be solved is written as:

$$\lambda^i, p^i = arg \min \|C\Lambda M p\|_2^2 \tag{15}$$

subject to

$$\|\lambda\| > 0, \quad \lambda_l \geq 0, \quad l \in I_s, \tag{16}$$

$$\begin{cases} p_{\text{base}} = p_{\text{specified coordinates}} \\ l_{\text{top tendons}}(p) = b_0 \\ l_{bars}(p) = L_0 \end{cases} \tag{17}$$

$$\begin{cases} v_{cm}^i(p) = \begin{cases} v^0 & \text{if } i = 0, \\ v_{cm}^{i-1} + r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } 0 < i < n_x, \\ r_{\text{des}} \begin{bmatrix} \cos\theta^j & \sin\theta^j & 0 \end{bmatrix}^T + v_0 & \text{if } n_x \leq i < n_c, \\ v_{cm}^{i-1} - r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } i > n_c. \end{cases} \\ \theta^0 = 0, \quad \theta^j = \theta^{j-1} + 2\pi/(n_c - n_x), \quad \text{if } n_x \leq i < n_c \text{ and } j = 1, \dots, (n_c - n_x), \end{cases} \tag{18}$$

The initial configuration, p_0, is predetermined to be a symmetrical equilibria and is found using procedures shown in Ref. 11. At the end of each iteration the solution p^i is saved to compute the tendon lengths to reconstruct the tendon length trajectory. In addition, p^i is used as an initial guess to the solution $p^{i+1}$of the iteration that follows. This ensures every solution is the closest solution to its two neighboring solutions. The saved configuration data is used to easily compute the tendon length trajectory. This trajectory becomes the open loop control law that can be implemented using DC motors actuating the length of each tendon.

4.2. Circular trajectory with 6 actuators

Example 2 In this example we show that the same motion of the structure defined in the previous example can be accomplished by using six pairs of equivalent tendons available for length control. While formulation of the motion constrain remains the same, additional geometry constraints must be imposed. The additional geometry constraints are explained:

1. 3 pairs of Saddle tendons, S, equal, (see Fig. 1),
2. 3 pairs of Diagonal tendons, D, equal, (see Fig. 1),
3. 6 Vertical tendons, V, constant length. (see Fig. 1).

Solution First, reformulate geometry constraints to account for additional requirements imposed. The initial configuration, p_0, is predetermined to be a symmetrical equilibria and is found using procedures shown in Ref. 11.

$$\lambda^i, p^i = arg\min \|C\Lambda M p\|_2^2 \tag{19}$$

subject to

$$\|\lambda\| > 0, \quad \lambda_l \geq 0, \quad l \in I_s, \tag{20}$$

$$\begin{cases} p_{\text{base}} = p_{\text{specified coordinates}} \\ l_{\text{top tendons}}(p) = b_0 \\ l_{bars}(p) = L_0 \\ S_k(p) = S_{k+1}(p) \quad \text{for } k = 1,3,5 \\ D_k(p) = D_{k+1}(p) \quad \text{for } k = 1,3,5 \\ V_k(p) = V(p) \quad \text{for } k = 1,2,\ldots,6 \end{cases} \tag{21}$$

$$\begin{cases} v^i_{cm}(p) = \begin{cases} v^0 & \text{if } i = 0, \\ v^{i-1}_{cm} + r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } 0 < i < n_x, \\ r_{\text{des}} \begin{bmatrix} \cos\theta^j & \sin\theta^j & 0 \end{bmatrix}^T + v_0 & \text{if } n_x \leq i < n_c, \\ v^{i-1}_{cm} - r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } i > n_c. \end{cases} \\ \theta^0 = 0, \quad \theta^j = \theta^{j-1} + 2\pi/(n_c - n_x), \quad \text{if } n_x \leq i < n_c \text{ and } j = 1,\ldots,(n_c - n_x), \end{cases} \tag{22}$$

Similarly to the previous example, the tendon length of the control tendons is computed from the p^i obtained at the end of each iteration. This string length trajectory is then applied for the open loop control law implemented using DC motors actuating the length of each controlled tendon. Figure 3 shows the tendon changes required to reconfigure the structure.

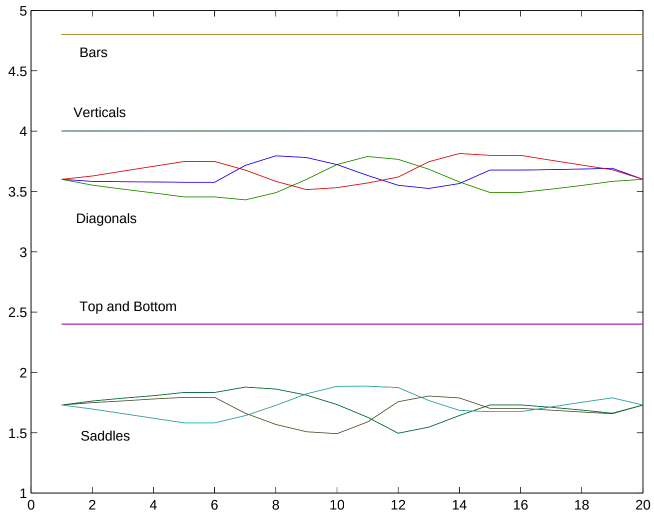

Figure 3. Tendon trajectory for 6 actuator circular trajectory tensegrity platform.

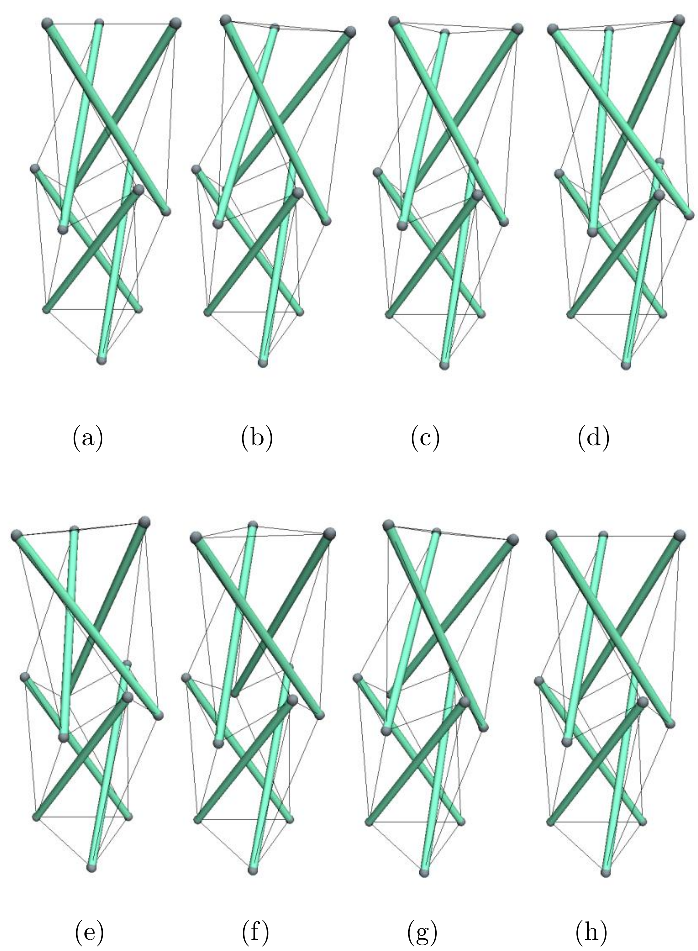

Figure 4. Sequence reconfiguration of tensegrity structure under asymmetrical reconfiguration in example 2.

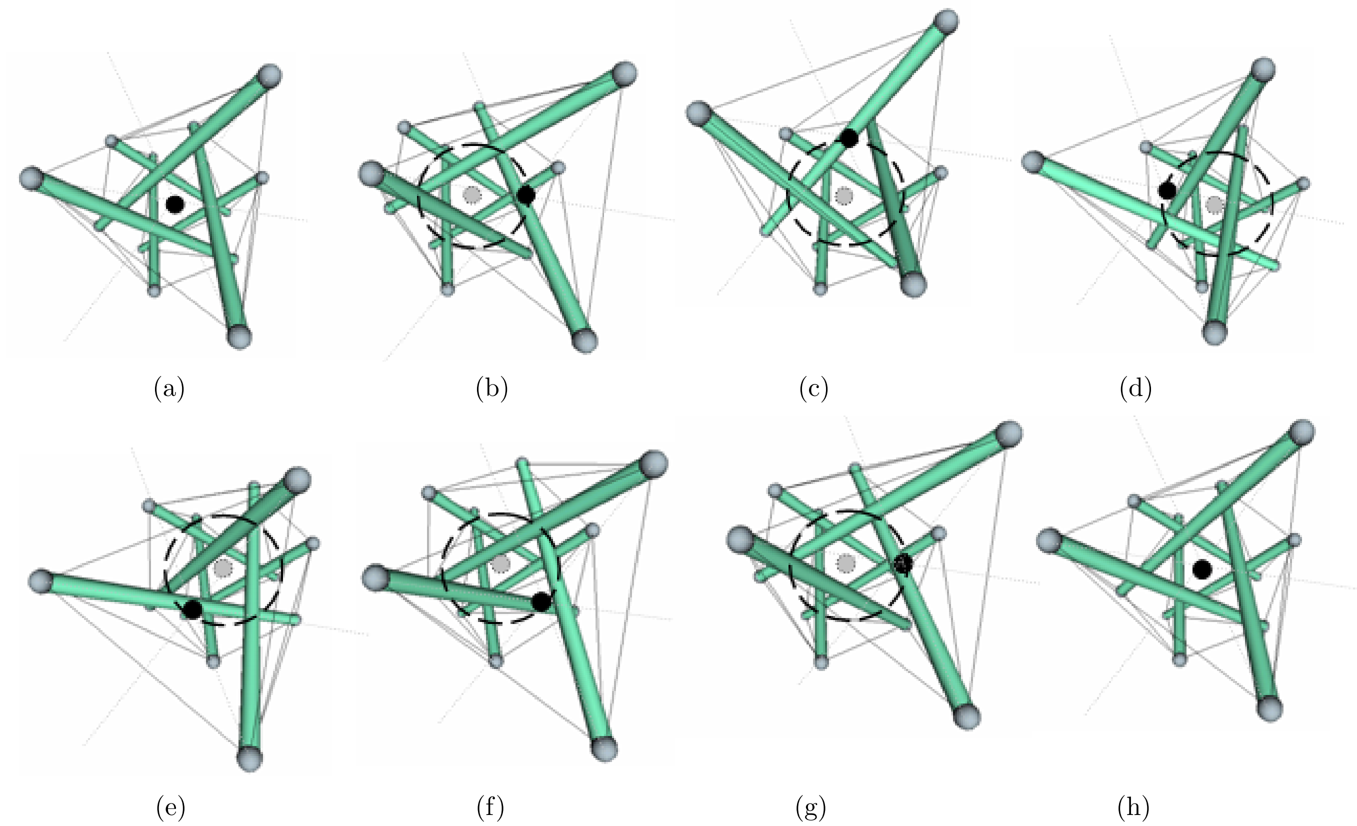

Figure 5. Top View of Figure 4. Black dots indicate top plate center of mass and white dots indicate bottom plate center of mass. Black circle drawn to illustrate trajectory.

4.3. Inclined elliptical trajectory with 6 actuators

Example 3 In this example we keep the same number of actuators and their location as in the structure of the previous example, namely the structure has 6 actuators controlling 6 pairs of equivalent tendon types. We only change specified desired motion of the structure. This time desired motion of the center of the mass of the plate is given in the following structure motion specification:

1. Top triangle center of mass, v_{cm}, deflected from axis of symmetry, v^0, to a desired distance or radius, r_{desired}, in the $\hat{x}$ direction.
2. Continue to deflect v_{cm} around v^0 along a prescribed **inclined** ellipse of radius, r_{des}, and height, s_{des}, for 1 revolution and then back to v^0.

Solution We only rewrite the motion constraints that have slightly changed from the previous example. The sequence of problems to be solved becomes:

$$\lambda^i, p^i = arg\min \|C\Lambda M p\|_2^2 \tag{23}$$

subject to

$$\|\lambda\| > 0, \quad \lambda_l \geq 0, \quad l \in I_s, \tag{24}$$

$$\begin{cases} p_{\text{base}} = p_{\text{specified coordinates}} \\ l_{\text{top tendons}}(p) = b_0 \\ l_{bars}(p) = L_0 \\ S_k(p) = S_{k+1}(p) \quad \text{for } k = 1,3,5 \\ D_k(p) = D_{k+1}(p) \quad \text{for } k = 1,3,5 \\ V_k(p) = V(p) \quad \text{for } k = 1,2,\ldots,6 \end{cases} \tag{25}$$

$$\begin{cases} v_{cm}^i = \begin{cases} v^0 & \text{if } i = 0 \\ v_{cm}^{i-1}(p) + r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } 0 < i < n_x \\ \begin{bmatrix} r_{\text{des}} \cos\theta_j & r_{\text{des}} \sin\theta_j & s_{\text{des}} \sin\theta_j \end{bmatrix}^T + v_0 & \text{if } n_x \leq i < n_c \\ v_{cm}^{i-1} - r_{\text{des}} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T / n_x & \text{if } i > n_c \end{cases} \\ \theta_0 = 0, \quad \theta_j = \theta_{j-1} + 2\pi/(n_c - n_x), \quad \text{if } n_x \leq i < n_c \text{ and } j = 1, \ldots, (n_c - n_x), \end{cases} \tag{26}$$

where and where the solution p_i overwrites p_{i-1} at each iteration and is saved to reconstruct the tendon length trajectory. This tendon length trajectory becomes the open loop control law that can be implemented using DC motors actuating the length of each tendon. Figure 6 shows the necessary tendon change to accomplish the desired reconfiguration.

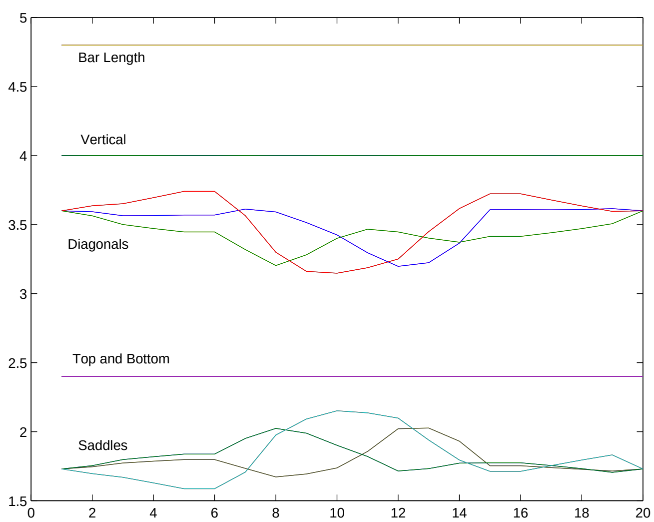

Figure 6. Tendon trajectory for 6 actuator inclined elliptical trajectory tensegrity platform.

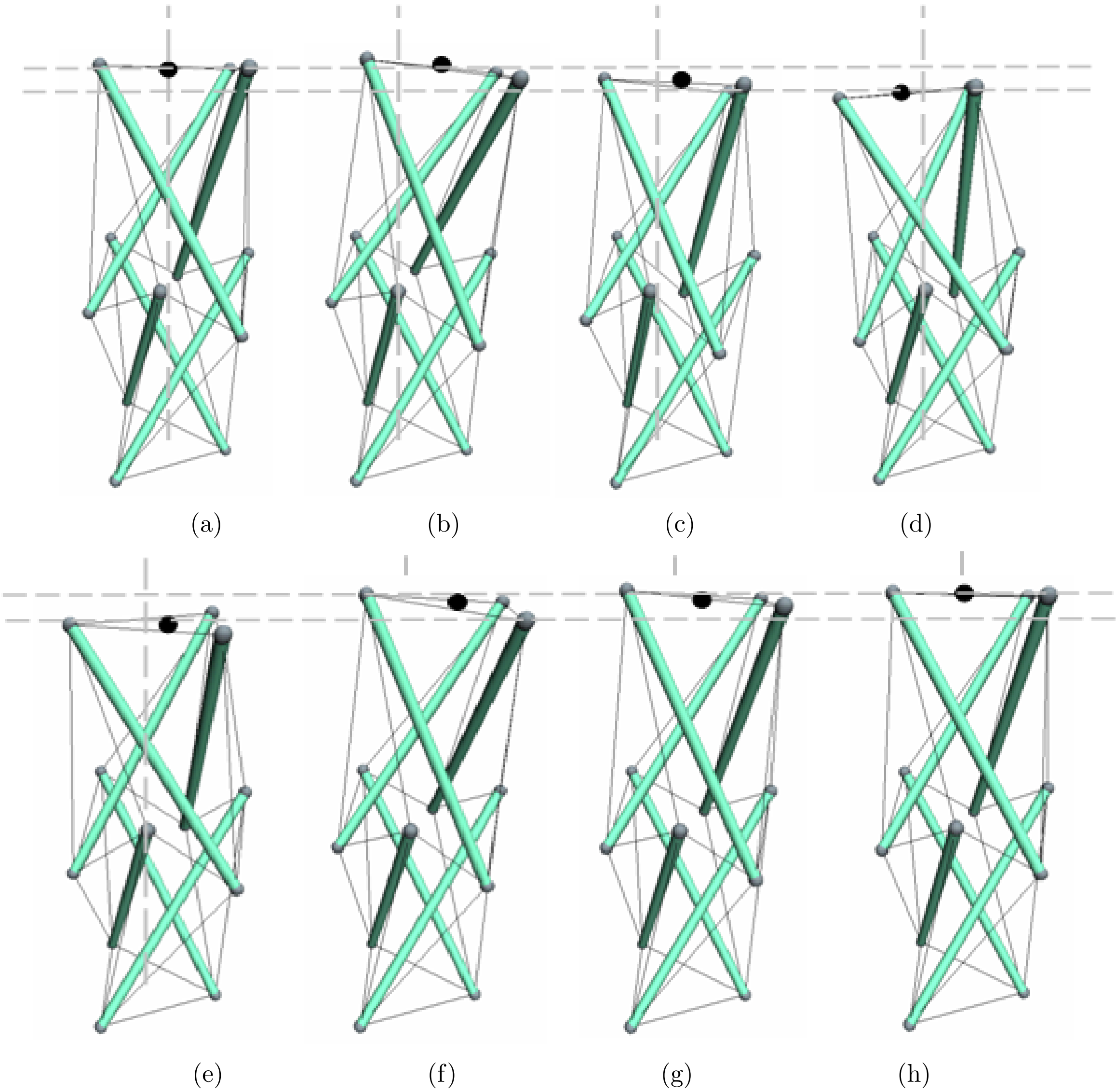

Figure 7. Sequence reconfiguration of tensegrity structure following an inclined elliptical trajectory.Top view is same as in Figure 5.

5. EXPERIMENTAL HARDWARE

The following experimental hardware resides in the Structural Systems and Control Laboratory at the University of California, San Diego's Dynamic and Controls Group in the Mechanical and Aerospace Engineering Department. The experiment demonstrates the practical feasibility of reconfiguration of a tensegrity structures utilizing D.C. motors as tendon actuators. The *six* motors control *one pair* of identical tendons in the structure. That is, one motor controls two saddle tendons, one motor controls two diagonal tendons, etc. Unfortunately, we are only able to present still photographs of the structure in this paper. For video of various reconfigurations of this structure, please email one of the authors.

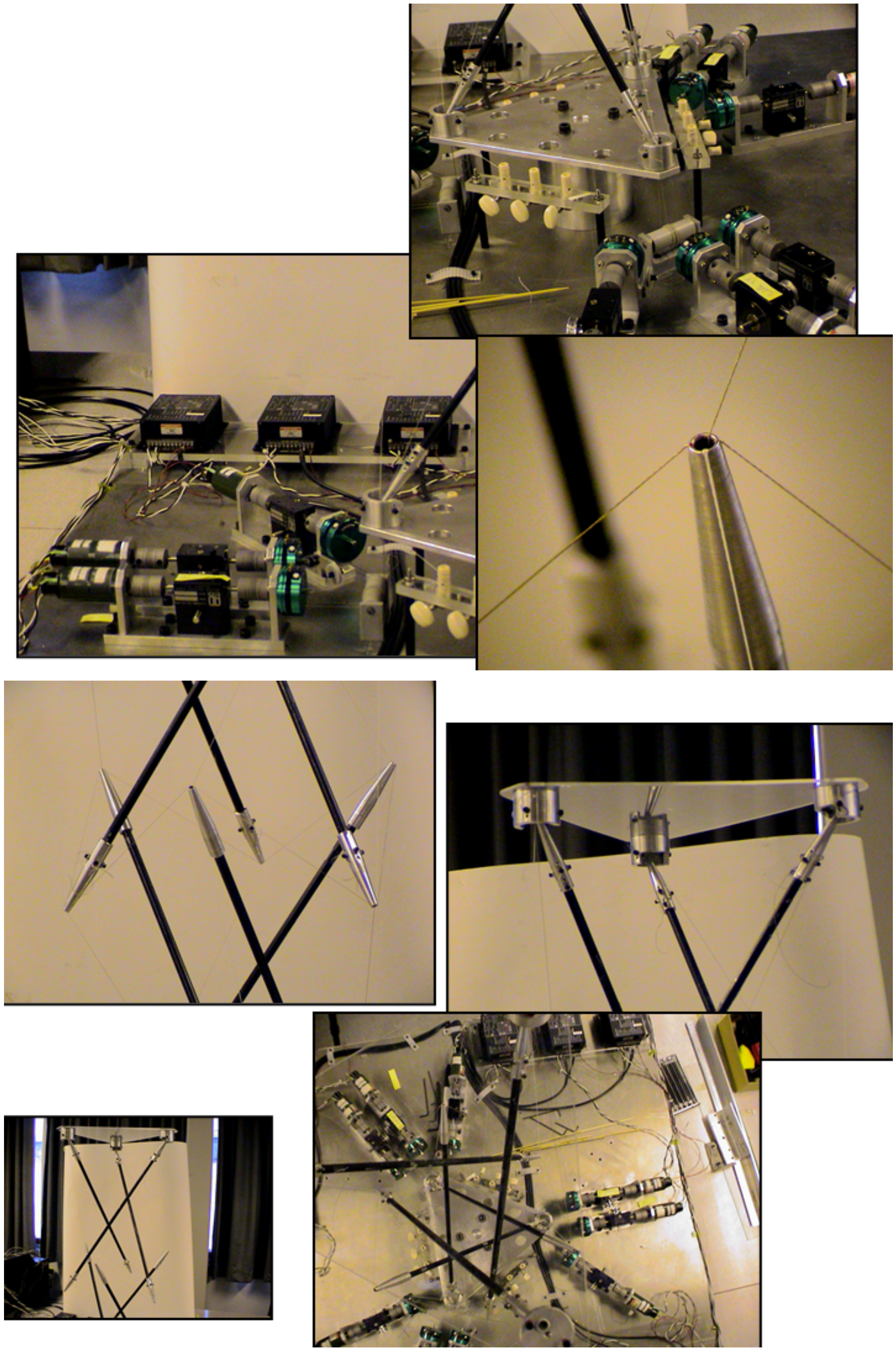

Figure 8. Experimental hardware for reconfiguration.

6. CONCLUSION

The tendon trajectories for reconfiguration of a tensegrity structure are computed using iterative nonlinear optimization programs which allow for easy specification of constraints on the geometry of the structure. The trajectory of the center of mass of the top plate is designated as the end-effector and its desired path can be specified as constraints on the geometry. The method posed in this paper illustrates the direct computation of the admissible static equilibrium configurations that are at the same time solutions to the inverse kinematic problem for the desired end-effector path. The motion of the structure is assumed to be quasi-static so the dynamics of the reconfiguration can be ignored.[12]

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