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AN LMI OPTIMIZATION APPROACH TO THE DESIGN OF STRUCTURED LINEAR CONTROLLERS USING A LINEARIZATION ALGORITHM

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ABSTRACT

This paper presents a new algorithm for the design of linear controllers with special constraints imposed on the control gain matrix. This so called SLC (Structured Linear Control) problem can be formulated with linear matrix inequalities (LMI's) with a nonconvex equality constraint. This class of prolems includes fixed order output feedback control, multi-objective controller design, decentralized controller design, joint plant and controller design, and other interesting control problems.

Our approach includes two main contributions. One is that many design specifications such as $\mathcal{H}_\infty$ performance, generalized $\mathcal{H}_2$ performance including $\mathcal{H}_2$ performance, ℓ_∞ performance, and upper covariance bounding controllers are described by a similar matrix inequality. A new matrix variable is introduced to give more freedom to design the controller. Indeed this new variable helps to find the optimal fixed-order output feedback controller.

The second contribution uses a linearization algorithm to search for a solution to the nonconvex SLC problems. This has the effect of adding a certain potential function to the nonconvex constraints to make them convex. Although the constraints are added to make functions convex, those modified matrix inequalities will not bring significant conservatism because they will ultimately go to zero, guaranteeing the feasibility of the original nonconvex problem. Numerical examples demonstrate the performance of the proposed algorithms and provide a comparison with some of the existing methods.

1 Introduction

Control problems are usually formulated as optimization problems. Unfortunately, most of them are not convex [1], and a few of them can be formulated as linear matrix inequalities (LMI's). In this case, powerful algorithms can be used to find the optimal solution [2, 3]. There are two main approaches to solve a linear control problem. One is the change of variable technique [4, 13] and the other is to use the elimination lemma [1]. Indeed, the book [1] was written to show that more than twenty different control problems can be treated as the same linear algebra problem. In the LMI framework, one can solve several linear control problems in the form $\min_{\mathcal{K}} f(\mathbf{T}(\zeta))$, where $f(\cdot)$ is some suitably defined objective function and $\mathbf{T}(\zeta)$ is the transfer function from a given input to a given output of interest. Notice that there is no constraint on the control variable $\mathcal{K}$ for this problem. For this problem, one can find a solution efficiently with the use of any LMI solver. However the problem becomes difficult when one adds some structural constraints on $\mathcal{K}$. A typical example is a decentralized control problem or a fixed-order output feedback control problem. In this case, the problem can be formulated as

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$\min_{\mathcal{K}\in\mathbf{\Phi}} f(\mathbf{T}(\zeta))$ [14]. Any linear control problem with structure imposed on the controller parameter $\mathcal{K}$ will be called the "Structured Linear Control (SLC)" The SLC problem involves a large class of problems such as decentralized control, fixed-order output feedback, linear model reduction or linear filtering, the simultaneous design of plant and controller , norm bounds on the control gain matrix, and multi-objective control problems.

There are many attempts to solve the SOF(Static Output Feedback) problem, which is known to be NP-hard [1, 5–7, 9–11, 16, 17]. Most algorithms try to obtain a stable controller rather than find an optimal controller. This is one of the important SLC problems. General multi-objective control problems are also important and remain open to this date. Indeed, this problem can be formulated as a SLC problem, since this problem is equivalent to finding multiple controllers for multiple plants where we restrict all controllers to be identical. For the full-order output feedback case, it has been proposed to specify the closed-loop objectives in terms of a common Lyapunov function which can be efficiently solved by convex programming methods [4]. It is well known that this solution is very conservative. An extended approach has been proposed to relax the constraint on the Lyapunov matrix [12]. This idea has several advantages over the "Lyapunov shaping paradigm" [4]: the Lyapunov matrix is not involved in any product with the system matrices. However, they impose another constraint on newly introduced variables $\mathcal{G}_1 = \cdots = \mathcal{G}_N = \mathcal{G}$ to ensure that the controllers considered in each inequality be the same. It seems that this constraint is inevitable for the "linearizing change of variable technique" used in [4, 12]. These two approaches can not be applicable to the "fixed-order multi-objective controller synthesis problem".

Recently, a convexifying algorithm has been proposed [14, 15] with interesting features. This algorithm solves convexified matrix inequalities iteratively. These convexified problems can be easily obtained by adding convexifying potential functions to the original nonconvex matrix inequalities at each iteration. Although the convexifying potential function, which is positive semi-definite, is added, those convexified matrix inequalities will not bring significant conservatism because they will go to zero by resetting the convexifying potential function to zero at each iteration. Due to the lack of convexity, only local convergence can be guaranteed. However, this algorithm is easily implemented and can be used to improve available suboptimal solutions. Moreover, this algorithm is so general that it can be applicable to almost all SLC problems.

The main objective of this paper is to present the optimal controller for SLC problems using a linearization method. Using linear algebra, we present new system performance analysis conditions. The new performance analysis conditions have several advantages over the original performance analysis conditions. First of all, many design specifications such as general $\mathcal{H}_2$ performance including $\mathcal{H}_2$ performance, $\mathcal{H}_\infty$ performance, ℓ_∞ performance, and the upper covariance bounding controllers can be written in a very similar matrix inequality. We introduce a new matrix variable $\mathcal{Z}$ for several system performance analysis conditions. As a result, we have more freedom to find the optimal controller. Indeed, this new variable helps to find the optimal fixed-order output feedback controller.

The paper is organized as follows. Section 2 defines a framework for SLC problems and describes the new system performance analysis conditions. We derive the important class of a matrix inequality which introduces a new matrix variable. Based on this new system performance specifications, a new linearization algorithm is proposed in section 3. Two numerical examples illustrate the performance of the proposed algorithms as compared with the existing results in section 4 and then draw the conclusion.

2 System Performance Analysis

For synthesis purposes, we consider the following discrete time linear system.

$$\mathbf{P}\begin{cases}\mathbf{x}_p(k+1) = \mathbf{A}_p\mathbf{x}_p(k) + \mathbf{B}_p\mathbf{u}(k) + \mathbf{D}_p\mathbf{w}(k)\\ \mathbf{z}(k) \quad\;\; = \mathbf{C}_z\mathbf{x}_p(k) + \mathbf{B}_z\mathbf{u}(k) + \mathbf{D}_z\mathbf{w}(k)\\ \mathbf{y}(k) \quad\;\; = \mathbf{C}_y\mathbf{x}_p(k) + \mathbf{D}_y\mathbf{w}(k)\end{cases} \tag{1}$$

where $\mathbf{x} \in \Re^{n_p}$ is the plant state, $\mathbf{z} \in \Re^{n_z}$ is the controlled output, and $\mathbf{y} \in \Re^{n_y}$ is the measured output. We assume that all matrices have suitable dimensions. Our goal is to compute an output-feedback controller

$$\mathbf{K}\begin{cases}\mathbf{x}_c(k+1) = \mathbf{A}_c\mathbf{x}_c(k) + \mathbf{B}_c\mathbf{y}(k)\\ \mathbf{u}(k) \quad\;\; = \mathbf{C}_c\mathbf{x}_c(k) + \mathbf{D}_c\mathbf{y}(k)\end{cases} \tag{2}$$

where $\mathbf{x}_c \in \Re^{n_c}$ is the controller state and $\mathbf{u} \in \Re^{n_u}$ is the control input, that meets various specifications on the closed-loop behavior. By assembling the plant $\mathbf{P}$ and the controller $\mathbf{K}$ defined as above, we have the compact closed-loop system

$$\begin{bmatrix}\mathbf{x}(k+1)\\ \mathbf{z}(k)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K})\end{bmatrix}\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{w}(k)\end{bmatrix} \tag{3}$$

where the controller parameter $\mathcal{K}$ and the closed loop states $\mathbf{x}$ are

$$\mathcal{K} \triangleq \begin{bmatrix}\mathbf{D}_c & \mathbf{C}_c\\ \mathbf{B}_c & \mathbf{A}_c\end{bmatrix} \; ; \; \mathbf{x} \triangleq \begin{bmatrix}\mathbf{x}_p\\ \mathbf{x}_c\end{bmatrix}$$

and the closed loop matrices

$$\mathbf{A}_{cl}(\mathcal{K}) \triangleq \mathcal{A}+\mathcal{B}\mathcal{K}\mathcal{C}\ ;\ \mathbf{B}_{cl}(\mathcal{K}) \triangleq \mathcal{D}_p+\mathcal{B}\mathcal{K}\mathcal{D}_y$$
$$\mathbf{C}_{cl}(\mathcal{K}) \triangleq \mathcal{C}_z+\mathcal{B}_z\mathcal{K}\mathcal{C}\ ;\ \mathbf{D}_{cl}(\mathcal{K}) \triangleq \mathcal{D}_z+\mathcal{B}_z\mathcal{K}\mathcal{D}_y$$

are all affine mappings on the variable $\mathcal{K}$ and all matrices given by

$$\begin{aligned}
\mathcal{A} &\triangleq \begin{bmatrix}\mathbf{A}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{0_{n_c}}\end{bmatrix}, \mathcal{B} \triangleq \begin{bmatrix}\mathbf{B}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{n_c}\end{bmatrix}, \mathcal{C} \triangleq \begin{bmatrix}\mathbf{C}_y & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{n_c}\end{bmatrix}\\
\mathcal{B}_z &\triangleq \begin{bmatrix}\mathbf{B}_z & \mathbf{0}\end{bmatrix}, \quad \mathcal{D}_p \triangleq \begin{bmatrix}\mathbf{D}_p\\ \mathbf{0}\end{bmatrix} \quad \mathcal{C}_z \triangleq \begin{bmatrix}\mathbf{C}_z & \mathbf{0}\end{bmatrix}\\
\mathcal{D}_y &\triangleq \begin{bmatrix}\mathbf{D}_y\\ \mathbf{0}\end{bmatrix} \quad \mathcal{D}_z \triangleq \mathbf{D}_z
\end{aligned} \tag{4}$$

are constant matrices that depend on the only plant properties.

The multi-objective control problem is defined as the problem of determining a controller that meets several closed-loop design specifications at the same time. We assume that these design specifications are formulated with respect to closed loop transfer functions of the form $\mathbf{T}_i(\zeta) \triangleq \mathbf{L}_i\mathbf{T}(\zeta)\mathbf{R}_i$ where the matrices $\mathbf{L}_i, \mathbf{R}_i$ select the appropriate input/output channels or channel combinations. From the dynamic matrices of system (1), a state-space realization of the closed loop system $\mathbf{T}_i(\zeta)$ is obtained by defining new matrices as following

$$(\mathbf{D}_p)_i \triangleq \mathbf{D}_p\mathbf{R}_i\ \ (\mathbf{D}_y)_i \triangleq \mathbf{D}_y\mathbf{R}_i\ \ (\mathbf{D}_z)_i \triangleq \mathbf{L}_i\mathbf{D}_z\mathbf{R}_i$$
$$(\mathbf{B}_z)_i \triangleq \mathbf{L}_i\mathbf{B}_z\ \ (\mathbf{C}_z)_i \triangleq \mathbf{L}_i\mathbf{C}_z$$

in the closed-loop matrices (3). In this form, closed-loop system performance and robustness may be ensured by constraining the general $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of the transfer functions associated to the pairs of signals $\mathbf{w}_i \triangleq \mathbf{R}_i\mathbf{w}$ and $\mathbf{z}_i \triangleq \mathbf{L}_i\mathbf{z}$.

2.1 General $\mathcal{H}$ Control Synthesis

The purpose of this section is to define quantitative measures of system performance and to provide (computable) characterizations of the performance measures. System gains for the discrete-time system (3) can be defined as follows [1].

Energy-to-Peak Gain : $\Upsilon_{ep} \triangleq \sup_{\|\mathbf{w}\|_{\ell_2}\le 1} \|\mathbf{z}\|_{\ell_\infty}$.

Energy-to-Energy Gain : $\Upsilon_{ee} \triangleq \sup_{\|\mathbf{w}\|_{\ell_2}\le 1} \|\mathbf{z}\|_{\ell_2}$.

Pulse-to-Energy Gain :
$\Upsilon_{ie} \triangleq \sup_{\mathbf{w}(k)=\mathbf{w}_0\delta(k)\ ,\ \|\mathbf{w}_0\|\le 1} \|\mathbf{z}\|_{\ell_2}$.

where $\delta(\cdot)$ is the Kronecker delta : $\delta(k)=0$ for all $k\neq 0$. and $\|\mathbf{A}\|$ is the spectral norm of a matrix $\mathbf{A}$. These system gains are characterized in terms of algebraic conditions. The following matrix inequalities will be discussed.

$$\left.\begin{array}{l}\mathcal{X} > \mathbf{A}_{cl}(\mathcal{K})\mathcal{X}\mathbf{A}_{cl}^T(\mathcal{K})+\mathbf{B}_{cl}(\mathcal{K})\mathbf{B}_{cl}^T(\mathcal{K})\\ \mathbf{\Upsilon} > \mathbf{C}_{cl}(\mathcal{K})\mathcal{X}\mathbf{C}_{cl}^T(\mathcal{K})+\mathbf{D}_{cl}(\mathcal{K})\mathbf{D}_{cl}^T(\mathcal{K})\end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l}\mathcal{Y} > \mathbf{A}_{cl}^T(\mathcal{K})\mathcal{Y}\mathbf{A}_{cl}(\mathcal{K})+\mathbf{B}_{cl}^T(\mathcal{K})\mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{\Upsilon} > \mathbf{C}_{cl}^T(\mathcal{K})\mathcal{Y}\mathbf{C}_{cl}(\mathcal{K})+\mathbf{D}_{cl}^T(\mathcal{K})\mathbf{D}_{cl}(\mathcal{K})\end{array}\right\} \tag{6}$$

$$\begin{bmatrix}\mathcal{X} & \mathcal{Z}\\ \mathcal{Z}^T & \mathbf{\Upsilon}\end{bmatrix} > \begin{bmatrix}\mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K})\end{bmatrix}\begin{bmatrix}\mathcal{X} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K})\end{bmatrix}^T \tag{7}$$

$$\begin{bmatrix}\mathcal{Y} & \mathcal{Z}\\ \mathcal{Z}^T & \mathbf{\Upsilon}\end{bmatrix} > \begin{bmatrix}\mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K})\end{bmatrix}^T\begin{bmatrix}\mathcal{Y} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K})\\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K})\end{bmatrix} \tag{8}$$

Note that (5) describes an upper bound to the observability Gramian $\mathcal{X}$ and (6) describes an upper bound to the controllability Gramian $\mathcal{Y}$. The following results are useful to compute system gains. $\mathbf{T}_{wz}(\zeta)$ denotes the transfer function from the input $\mathbf{w}$ to the output $\mathbf{z}$.

Lemma 1. *[1] Let a positive scalar γ be given and consider the discrete-time system (3). Suppose the system is asymptotically stable. Then the following statements are true.*

(i) $\Upsilon_{ep} < \gamma$ iff there exist matrices $\mathcal{X} > \mathbf{0}$ and $\mathbf{\Upsilon} > \mathbf{0}$ such that $\gamma\mathbf{I} > \mathbf{\Upsilon}$ and (5) holds.

(ii) $\Upsilon_{ie} < \gamma$ iff there exist matrices $\mathcal{X}$ and $\mathbf{\Upsilon}$ such that $\gamma\mathbf{I} > \mathbf{\Upsilon}$ and (6) holds

(iii) $\Upsilon_{\mathcal{H}_2} \triangleq \|\mathbf{T}_{wz}(\zeta)\|_{\mathcal{H}_2} = \left\|\mathbf{C}_{cl}(\mathcal{K})\left(\zeta\mathbf{I}-\mathbf{A}_{cl}(\mathcal{K})\right)^{-1}\mathbf{B}_{cl}(\mathcal{K})+\mathbf{D}_{cl}(\mathcal{K})\right\|_2 < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{Z}$, $\mathcal{X}$, and $\mathbf{\Upsilon}$ such that $trace[\mathbf{\Upsilon}] < \gamma^2$ and either (5) or (6) hold.

(iv) $\Upsilon_{ee} \triangleq \|\mathbf{T}_{wz}(\zeta)\|_{\mathcal{H}_\infty} = \left\|\mathbf{C}_{cl}(\mathcal{K})\left(\zeta\mathbf{I}-\mathbf{A}_{cl}(\mathcal{K})\right)^{-1}\mathbf{B}_{cl}(\mathcal{K})+\mathbf{D}_{cl}(\mathcal{K})\right\|_\infty < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{X}$, and $\mathbf{\Upsilon}$ such that $\gamma^2\mathbf{I} > \mathbf{\Upsilon}$ and either (7) holds with $\mathcal{Z}=\mathbf{0}$ or (8) holds with $\mathcal{Z}=\mathbf{0}$.

(v) $\Upsilon_{\mathcal{H}_{2\infty}} < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{X}$ and $\mathbf{\Upsilon}$ such that $trace[\mathbf{\Upsilon}] < \gamma^2$ and either (7) with $\mathcal{Z}=\mathbf{0}$ holds or (8) holds with $\mathcal{Z}=\mathbf{0}$.

One can add more system performance criteria such as positive realness, but we do not consider those here for brevity. The statement (iii) characterizes the $\mathcal{H}_2$ control problem and the statement (iv) characterizes the $\mathcal{H}_\infty$ control problem. Thus, we shall also use another notation $\Upsilon_{\mathcal{H}_\infty} \triangleq \Upsilon_{ee}$. The statement (i) is often called by general $\mathcal{H}_2$ control problem [4]. Notice that we introduce the new design specifications $\Upsilon_{\mathcal{H}_{2\infty}}$ which is given by the statement (v) in Lemma 1. This measure is closely related to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ measures and can be interpreted by a unconstrained mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem. In this sense, we label all problems defined in Lemma 1 by *general $\mathcal{H}$ problem* . Note that all inequalities given in Lemma 1 have similar forms which can be parametrized by the matrix inequality $(\mathbf{\Theta} + \mathbf{\Gamma}\mathcal{K}\mathbf{\Lambda})\,\mathbf{R}\,(\mathbf{\Theta} + \mathbf{\Gamma}\mathcal{K}\mathbf{\Lambda})^T < \mathbf{Q}$. The following lemma is important to derive alternative analysis conditions.

Lemma 2. *[1] Let a matrix $\mathbf{B}$ and symmetric positive definite matrices $\mathbf{A}, \mathbf{C}, \mathbf{Q}$ and $\mathbf{R}$ be given. Then the following statements are equivalent.*

(i) $\mathbf{Q} - \mathbf{A} > \mathbf{0}$ and $\mathbf{R} - \mathbf{C} > \mathbf{0}$.

(ii) There exists a matrix $\mathbf{Z}$ such that

$$\begin{bmatrix} \mathbf{Q} & \mathbf{Z} \\ \mathbf{Z}^T & \mathbf{R} \end{bmatrix} > \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{bmatrix}. \tag{9}$$

Suppose the above statements hold. Then all matrices $\mathbf{Z}$ satisfying (9) are given by

$$\mathbf{Z} = \mathbf{B} + (\mathbf{Q} - \mathbf{A})^{\frac{1}{2}}\,\mathbf{L}\,(\mathbf{R} - \mathbf{C})^{\frac{1}{2}}$$

where $\mathbf{L}$ is an arbitrary matrix such that $\|\mathbf{L}\| < 1$.

Using Lemma 2, we can rewrite Lemma 1 as following compact form.

Theorem 1. *Consider the system (3). Suppose the system is asymptotically stable and a positive scalar γ is given. Then the following statements are true.*

(i) $\Upsilon_{ep} < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{Z}, \mathcal{X}$ and $\mathbf{\Upsilon}$ such that $\gamma\mathbf{I} > \mathbf{\Upsilon}$ and (7) holds.

(ii) $\Upsilon_{ie} < \gamma$ iff there exist matrices $\mathcal{Z}, \mathcal{X}$ and $\mathbf{\Upsilon}$ such that $\gamma\mathbf{I} > \mathbf{\Upsilon}$ and (8) holds.

(iii) $\Upsilon_{\mathcal{H}_2} < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{Z}$, $\mathcal{X}$ and $\mathbf{\Upsilon}$ such that $trace[\mathbf{\Upsilon}] < \gamma^2$ and (7) hold.

(iv) $\Upsilon_{\mathcal{H}_\infty} < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{X}$ and $\mathbf{\Upsilon}$ such that $\gamma^2\mathbf{I} > \mathbf{\Upsilon}$ and (7) hold with $\mathcal{Z} = \mathbf{0}$.

(v) $\Upsilon_{\mathcal{H}_{\infty 2}} < \gamma$ iff there exist matrices $\mathcal{K}$, $\mathcal{X}$ and $\mathbf{\Upsilon}$ such that $trace[\mathbf{\Upsilon}] < \gamma^2$ and (7) with $\mathcal{Z} = \mathbf{0}$.

Proof : (7) can be rewritten by $\begin{bmatrix} \mathcal{X} & \mathcal{Z} \\ \mathcal{Z}^T & \mathbf{\Upsilon} \end{bmatrix} > \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$ where

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K}) \\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K}) \end{bmatrix} \begin{bmatrix} \mathcal{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K}) \\ \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K}) \end{bmatrix}^T$$

Then the results follows immediately from Lemma 2. ∎

One of the interesting features of Theorem 1 is its compact form, and the fact that so many performance specifications have similar forms. Since we used the observability Gramian form, (7) appears in (i),(iii),(iv), and (v) in Theorem 1. We can easily see that the $\mathcal{H}_2$ norm and the $\mathcal{H}_\infty$ norm is closely related. Note that $\Upsilon_{2\infty}$ has the same constraints as (iii) except for the constraint $\mathcal{Z} = \mathbf{0}$. So this measure is constrained $\mathcal{H}_2$ control problem and $\mathcal{H}_\infty$ norm is the maximum singular value of $\mathbf{\Upsilon}$. The most important contribution in Theorem 1 is the introduction of new variable $\mathcal{Z}$. This new variable may help to find the optimal solution, since we enlarge the domain of the problem. It is well known in a variety of mathematical problems that enlarging the domain in which the problem is posed can often simpify the mathematical treatment. Many nonlinear problems admit solutions by linear techniques by enlaring the domain of the problem.

3 Linearization Algorithm

In the previous section, we have shown how several control problems can be written as linear matrix inequalities that preserve the affine dependence on the free variables with the addition of a nonconvex equality constraints. Since analytic condition is nonvex matrix inequality, we need to develop a new algorithm.

Theorem 2. *Let a scalar convex functional $f(\mathbf{X})$, a matrix functional $\mathcal{J}(\mathbf{X})$ and $\mathcal{H}(\mathbf{X})$ be given and consider the following nonconvex optimization problem.*

$$\min_{\mathbf{X}\in\mathbf{\Psi}} f(\mathbf{X})\,,\ \mathbf{\Psi} \triangleq \{\mathbf{X}|\ \mathcal{J}(\mathbf{X}) + \mathcal{H}(\mathbf{X}) < \mathbf{0}\} \tag{10}$$

Suppose $\mathcal{J}(\mathbf{X})$ are convex, $\mathcal{H}(\mathbf{X})$ are not convex, and $f(\mathbf{X})$ is a first order differentiable convex function bounded from below on the set $\mathbf{\Psi}$. Then this problem can be solved (locally) by iterating a sequence of convex sub-problems if there exists a matrix functional $\mathcal{G}(\mathbf{X}, \mathbf{X}_k)$ such that

$$\mathbf{X}_{k+1} = \arg\min_{\mathbf{X}\in\mathbf{\Psi}_k} f(\mathbf{X}) \tag{11}$$

where

$$\begin{gathered}\mathbf{\Psi}_k \triangleq \{\mathbf{X} \mid \mathcal{J}(\mathbf{X}) + \mathrm{LIN}\,(\mathcal{H}(\mathbf{X}), \mathbf{X}_k) + \mathcal{G}(\mathbf{X}, \mathbf{X}_k) < \mathbf{0}, \\ \mathcal{H}(\mathbf{X}) \le \mathcal{G}(\mathbf{X}, \mathbf{X}_k) + \mathrm{LIN}\,(\mathcal{H}(\mathbf{X}), \mathbf{X}_k)\}\end{gathered}$$

where $\mathrm{LIN}\,(\star, \mathbf{X}_k)$ is the linearization operator of $\star$ at given $\mathbf{X}_k$.

Proof : First note that every point $\mathbf{X}_{k+1} \in \mathbf{\Psi}_k$ is also in $\mathbf{\Psi}$ since $\mathcal{J}(\mathbf{X})+\mathcal{H}(\mathbf{X}) \leq \mathcal{J}(\mathbf{X})+\mathrm{LIN}\left(\mathcal{H}(\mathbf{X}), \mathbf{X}_k\right)+\mathcal{G}(\mathbf{X}, \mathbf{X}_k) < \mathbf{0}$. As long as $\mathbf{X}_k \in \mathbf{\Psi}_k$, $f(\mathbf{X}_{k+1}) < f(\mathbf{X}_k)$ holds strictly until $\mathbf{X}_{k+1} = \mathbf{X}_k$. The fact that $f(\mathbf{X})$ is bounded from below ensures that this strictly decreasing sequence converges to a stationary point. ∎

The convex problem (11) is much simpler than (10) and there is no need to perform any kind of line search algorithm. This is a very simple, but powerful idea to solve a difficult nonconvex problem by relaxing nonlinear terms.

All matrix inequalities given in the previous sections are convex except for the term $-\mathcal{Y}^{-1}$. Note that this nonconvex term is always negative definite. One can ask that "How can we linearize this nonconvex term $\mathcal{Y}^{-1}$ at given $\mathcal{Y}_o > \mathbf{0}$?". Since our variables are matrices, we need to develop the taylor series expansion for matrix variables. Following lemma provides the linearization of $\mathcal{Y}^{-1}$.

Lemma 3. *The linearization of the matrix* $\mathbf{Y}^{-1} \in \Re^{n\times n}$ *about the value* $\mathbf{Y}_o^{-1}$ *is given by*

$$\mathrm{LIN}\left(\mathbf{Y}^{-1}, \mathbf{Y}_o\right) = \mathbf{Y}_o^{-1} - \mathbf{Y}_o^{-1}\left(\mathbf{Y}-\mathbf{Y}_o\right)\mathbf{Y}_o^{-1} \quad (12)$$

where $\mathrm{LIN}\left(\star, \mathbf{Y}_o\right)$ *is the linearization operator of the function* $\star$ *at given* $\mathbf{Y}_o$.

Since

$$\begin{aligned} & -\mathcal{Y}^{-1} - \mathrm{LIN}\left(-\mathcal{Y}^{-1}, \mathcal{Y}_o\right) \\ &= -\mathcal{Y}^{-1} + \mathcal{Y}_o^{-1} - \mathcal{Y}_o^{-1}\left(\mathcal{Y}-\mathcal{Y}_o\right)\mathcal{Y}_o^{-1} \\ &= -\left(\mathcal{Y}^{-1}-\mathcal{Y}_o^{-1}\right)\mathcal{Y}\left(\mathcal{Y}^{-1}-\mathcal{Y}_o^{-1}\right) \leq \mathbf{0}, \end{aligned}$$

we can set a matrix functional $\mathcal{G}(\mathbf{X}, \mathbf{X}_k) = \mathbf{0}$ and the equality is attained when $\mathcal{Y} = \mathcal{Y}_o$. Note that this provides the updating rules. Basically a linearization approach is to solve a sufficient condition and hence this approach is conservative. However, this conservatism will be minimized since we shall solve the problem iteratively. Due to the lack of convexity, only local optimality is guaranteed.

For the problem defined in the previous section, we have only one nonconvex term. In this case, this approach is the same as so called convexifying algorithm proposed in [14]. It is deserve to mention that the linearization algorithm is of convexifying algorithm. In ordet to use convexifying algorithm, we need to find a convexifying potential functional. There might exist many candidates for convexifying potential functional for a given nonconvex functional and some convexifying potentials may yield too conservative. Finding a nice convexifying functional is generally a difficult question. Linearization of nonconvex term $\mathcal{H}(\mathbf{X})$ may help to find such a convexifying potential function. Now we are ready to develop a new algorithm for .

Algorithm 1. Structured Linear Control

1. *Set* $\epsilon > 0$ *and* $k = 0$.
2. *Solve the following convex optimization problem.*

$$\mathcal{X}_{k+1} = \arg\min_{\mathcal{X},\mathcal{Z},\mathbf{\Upsilon},\mathcal{K}} \|\mathbf{\Upsilon}\|$$

$$\text{subject to} \left[\begin{array}{cc|cc} \mathcal{X} & \mathcal{Z} & \mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K}) \\ \mathcal{Z}^T & \mathbf{\Upsilon} & \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K}) \\ \hline \multicolumn{2}{c|}{(\star)^T} & \mathrm{LIN}\left(\mathcal{X}^{-1}, \mathcal{X}_k\right) & \mathbf{0} \\ & & \mathbf{0} & \mathbf{I} \end{array}\right] > \mathbf{0}$$

   where $\mathrm{LIN}\left(\mathcal{X}^{-1}, \mathcal{X}_k\right)$ *is given by (12).*
3. *If* $\|\mathcal{X}_{k+1} - \mathcal{X}_k\| < \epsilon$, *go to step 4. Otherwise, set* $k \leftarrow k+1$ *and go back to Step 2.*

We also summarize the algorithm [14] for the comparison purpose. Since the step 1 and 3 are the same as those in Algorithm 1, we describe the step 2 only.

Algorithm 2. *[14]*

2. Solve the following convex optimization problem.

$$\mathcal{Y}_{k+1} = \arg\min_{\mathbf{\Upsilon},\mathcal{K},\mathcal{Y}} \|\mathbf{\Upsilon}\|$$

$$\text{subject to} \left\{ \begin{array}{l} \begin{bmatrix} \mathrm{LIN}\left(\mathcal{Y}^{-1}, \mathcal{Y}_k\right) & \mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K}) \\ \mathbf{A}_{cl}^T(\mathcal{K}) & \mathcal{Y} & \mathbf{0} \\ \mathbf{B}_{cl}^T(\mathcal{K}) & \mathbf{0} & \mathbf{I} \end{bmatrix} > \mathbf{0} \\ \begin{bmatrix} \mathbf{\Upsilon} & \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K}) \\ \mathbf{C}_{cl}^T(\mathcal{K}) & \mathcal{Y} & \mathbf{0} \\ \mathbf{D}_{cl}^T(\mathcal{K}) & \mathbf{0} & \mathbf{I} \end{bmatrix} > \mathbf{0} \end{array} \right.$$

where $\mathrm{LIN}\left(\mathcal{Y}^{-1}, \mathcal{Y}_k\right)$ *is given by (12).*

Note that these two algorithm can also be used for the optimal fixed order control problem. Algorithm 1 and 2 may find an initial feasible solution. If they failed, we need to find an initial feasible solution in order to use those algorithms. Here, we also propose a new feasibility algorithm for the completeness of the proposed algorithm using the linearization approach. We describe the step 2 only for brevity.

Algorithm 3. Initialization

2. Solve the following convex optimization problem.

$$\mathcal{X}_{k+1} = \arg \min_{\mathbf{\Upsilon},\mathcal{K},\mathcal{X},\mathcal{Y},\mathcal{Z}} \|\mathbf{\Upsilon}\|$$

$$\textit{subject to} \begin{cases} \begin{bmatrix} \mathcal{X} & \mathbf{I} \\ \mathbf{I} & \mathcal{Y} \end{bmatrix} \geq \mathbf{0} \\ -\mathbf{\Upsilon} + \mathcal{Y} - \mathrm{LIN}\left(\mathcal{X}^{-1}, \mathcal{X}_k\right) < \mathbf{0} \\ \left[\begin{array}{cc|cc} \mathcal{X} & \mathcal{Z} & \mathbf{A}_{cl}(\mathcal{K}) & \mathbf{B}_{cl}(\mathcal{K}) \\ \mathcal{Z}^T & \mathbf{\Upsilon} & \mathbf{C}_{cl}(\mathcal{K}) & \mathbf{D}_{cl}(\mathcal{K}) \\ \hline & & \mathcal{Y} & \mathbf{0} \\ \multicolumn{2}{c|}{(\star)^T} & \mathbf{0} & \mathbf{I} \end{array}\right] > \mathbf{0} \end{cases}$$

Feasibility problem is not convex either, since there is the nonconvex term $-\mathcal{X}^{-1}$. But this nonconvex term is the same as the previous optimization problem. Hence we can linearize this term as we explained before.

Notice that the proposed algorithm is very similar to the one proposed in [5], which adopts cone-complementarity linearization algorithm. The new proposed algorithm minimized $trace[\mathcal{Y}+\mathcal{X}_k^{-1}\mathcal{X}\mathcal{X}_k^{-1}]$, while the cone complementarity linearization algorithm minimizes $trace[\mathcal{Y}\mathcal{X}_k + \mathcal{X}\mathcal{Y}_k + \mathcal{Y}_k\mathcal{X} + \mathcal{X}_k\mathcal{Y}]$. It is clear that the cone complementarity algorithm linearized at a matrix pair $(\mathcal{X}_k, \mathcal{Y}_k)$ and our algorithm linearized only at $\mathcal{X}_k$. Another feature is that we minimized $\|\mathcal{Y} - \mathcal{X}^{-1}\|$ and cone complementarity algorithm minimized $\|\mathcal{X}\mathcal{Y} + \mathcal{Y}\mathcal{X}\|$. Clearly we minimized the controllability Gramian $\mathcal{Y}$ and maximized the observability Gramian $\mathcal{X}$. This implies that our algorithm is suitable for solving the optimal $\mathcal{H}_2$ control problem, while the cone-complementarity algorithm is suitable for solving optimal $\mathcal{H}_\infty$ control problem. Because there always exists a positive scalar γ such that $\mathcal{X}\mathcal{Y} \leq \gamma\mathbf{I}$ [1]. Note that we can establish another feasibility algorithm since

$$\begin{aligned} \mathcal{X}\mathcal{Y} + \mathcal{Y}\mathcal{X} &= (\mathcal{X}+\mathcal{Y})(\mathcal{X}+\mathcal{Y}) - \mathcal{X}^2 - \mathcal{Y}^2 \\ &= (\mathcal{X}+\mathcal{Y})(\mathcal{X}+\mathcal{Y}) + \mathcal{X}_o^2 + \mathcal{Y}_o^2 \\ &\quad - \mathcal{X}\mathcal{X}_o - \mathcal{X}_o\mathcal{X} - \mathcal{Y}\mathcal{Y}_o - \mathcal{Y}_o\mathcal{Y} \\ &\quad - (\mathcal{X}-\mathcal{X}_o)(\mathcal{X}-\mathcal{X}_o) - (\mathcal{Y}-\mathcal{Y}_o)(\mathcal{Y}-\mathcal{Y}_o) \end{aligned}$$

Hence we can apply the linearization approach by ignoring the nonconvex term $-(\mathcal{X}-\mathcal{X}_o)(\mathcal{X}-\mathcal{X}_o) - (\mathcal{Y}-\mathcal{Y}_o)(\mathcal{Y}-\mathcal{Y}_o)$. This approach also linearizes at a matrix pair $(\mathcal{X}_o, \mathcal{Y}_o)$.

4 Illustrative Examples

4.1 Example 1 : Decentralized $\mathcal{H}_2$ Optimal Static Output Feeback Controller

Consider the following discrete-time plant

$$\mathbf{A}_p = \begin{bmatrix} 0.8189 & 0.0863 & 0.0900 & 0.0813 \\ 0.2524 & 1.0033 & 0.0313 & 0.2004 \\ -0.0545 & 0.0102 & 0.7901 & -0.2580 \\ -0.1918 & -0.1034 & 0.1602 & 0.8604 \end{bmatrix}$$

$$\mathbf{B}_p = \begin{bmatrix} 0.0045 & 0.0044 \\ 0.1001 & 0.0100 \\ 0.0003 & -0.0136 \\ -0.0051 & 0.0936 \end{bmatrix}, \mathbf{D}_p = \begin{bmatrix} 0.0953 & 0 & 0 \\ 0.0145 & 0 & 0 \\ 0.0862 & 0 & 0 \\ -0.0011 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{D}_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_z = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_z = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which is borrowed from [14]. Our objective is to minimize $\|\mathbf{T}_{wz}(\zeta)\|_{\mathcal{H}_2}$ using two decentralized static output feedback controllers. Hence the structure of a controller $\mathbf{D}_c$ is diagonal. This is a typical SLC problem. In order to use initialization algorithm 3, we set $\mathcal{X}_0 = \mathbf{I} + \mathbf{R}\mathbf{R}^T$ and $\mathcal{Y}_0 = \mathcal{X}_0^{-1}$, where $\mathbf{R}$ is a random matrix. After using the initialization Algorithm 3, we have run the algorithm 1 and 2. The precision ϵ has been set to 10^{-3}.

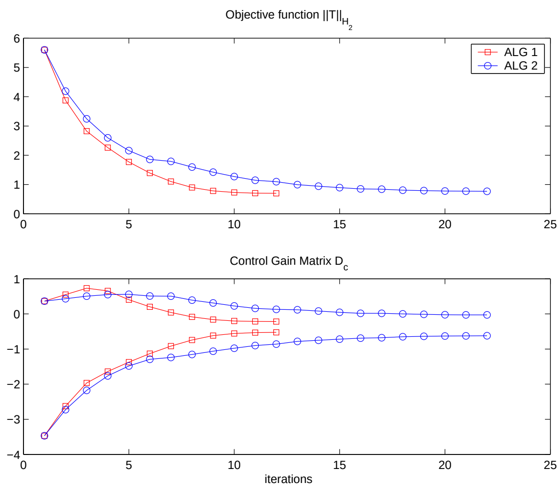

Figure 1. $\mathcal{H}_2$ performance of the optimal decentralized static output feedback controller

Figure 4.1 shows the performance of the decentralized static output feedback controller for both algorithms.

One can easily see that the behaviors of Algorithm 1 is better than Algorithm 2.

4.2 Example 2 : Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control

Now let's consider multi-objective controller. Design of feedback controllers that satisfy both $\mathcal{H}_2$ and $\mathcal{H}_\infty$ specifications is important because it offers robust stability and nominal performance and it is not always possible to have full access to the state vector. In this problem, we look for a unique static output feedback controller that minimizes an $\mathcal{H}_2$ performance cost while satisfying some $\mathcal{H}_\infty$ constraint. Consider the following simple discrete-time unstable plant

$$\begin{aligned}
\mathbf{x}_p(k+1) &= \begin{bmatrix} 2 & 0 \\ 1 & \frac{1}{2} \end{bmatrix} \mathbf{x}_p(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \mathbf{w}(k) \\
\mathbf{z}_1(k) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}_p(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \\
\mathbf{z}_2(k) &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}_p(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \\
y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}_p(k) + \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{w}(k)
\end{aligned}$$

which is also borrowed from [14]. By calculating the dynamic output feedback optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers, we obtain the following minimum achievable values for these norms $\min \|\mathbf{T}_{wz_1}(\zeta)\|_{\mathcal{H}_2} = 4.0957, \min \|\mathbf{T}_{wz_2}(\zeta)\|_{\mathcal{H}_\infty} = 6.3409$. Our objective is to design a static output feedback controller that minimizes $\|\mathbf{T}_{wz_1}(\zeta)\|_{\mathcal{H}_2}$ while keeping $\|\mathbf{T}_{wz_2}(\zeta)\|_{\mathcal{H}_\infty}$ below a certain level γ. We set $\gamma = 7$. We have run Algorithm 1 and 2 after the initialization. The specified precision ϵ is 10^{-4}. Figure 4.2 shows the performance of those algorithms. Again we can see that the behavior of Algorithm 1 is better than 2 for this problem. It is quite surprising that we achieved $\|\mathbf{T}_{wz_1}(\zeta)\|_{\mathcal{H}_2} = 4.1196, \|\mathbf{T}_{wz_2}(\zeta)\|_{\mathcal{H}_\infty} < 7$. This is just 0.5% worse than the $\mathcal{H}_2$ optimal dynamic output feedback controller.

5 Conclusion

We have addressed the SLC (Structured Linear Control) problem for linear discrete-time systems. New system performance conditions have been derived. This new results introduce the augmented matrix variable $\mathcal{Z}$. Hence the number of variables of the synthesis condition is maximized. It turns out that the behavior of new synthesis conditions is better than the original system synthesis.

In the SLC framework, these objectives are characterized as a set of LMI's with an additional nonconvex equality constraint. To overcome this nonconvex constraint, we adopt a linearization method. At each iter-

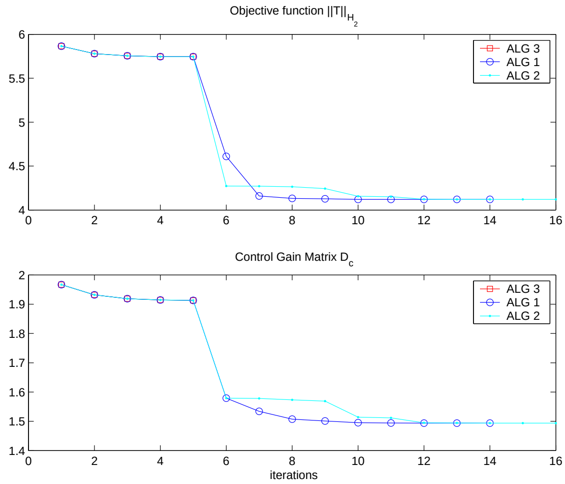

Figure 2. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance of the optimal static output feedback controller

ation, a certain potential function is added to the non-convex constraints to enforce them convex. Although we solved those sufficient conditions iteratively, this approach will not bring significant conservatism because they will converge to zero. Local optimality is guaranteed. Our approach can be applied to other linear synthesis problems as long as the dependence on the augmented plant on the synthesis parameters are affine. Two control design problems have been illustrated and compared with the existing method numerically.

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