The physics of vortex merger: Further insight

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Results of numerical simulations provide further insight on the physical mechanisms associated with symmetric vortex merger. The relative contributions of filament and exchange band fluid to the reduction in the vortex separation are determined and the latter is found to be dominant. A key underlying process is the interaction of strain rate and vorticity gradient near the center of rotation, through which a tilt in the vorticity contours is established. This leads to the entrainment of core fluid into the exchange band, which is transformed into a single vortex. *© 2006 American Institute of Physics*. [DOI: 10.1063/1.2201474]

The merging of two corotating vortices has both fundamental and practical significance. It is an elementary vortex interaction that plays a key role in the dynamics of shear layers and two- and three-dimensional turbulence. It may also occur in the wake of an aircraft, just downstream from the wing tip and an extended flap, thereby forming the primary trailing vortices.[1] As discussed below, there are differing views of the physical mechanism responsible for merger.

It is well known that a pair of (symmetric) vortices of equal circulation, Γ, and equal core size, a, separated by a distance, b, will rotate about each other due to the mutually induced velocity. If the aspect ratio a/b exceeds some critical value, $(a/b)_c$, vortex merger results. In general, the merging process (i.e., the process leading to $b=0$) in a viscous fluid consists of three phases.[2,3] During the first diffusive phase, the separation distance b remains constant while the cores grow by viscous diffusion. In the convective phase, when the vortices reach a critical size, b decreases significantly. Filamentation is observed during this phase. During the second diffusive phase, the two vorticity maxima are eventually reduced through viscous diffusion.

Two-dimensional symmetric vortex merger has been studied extensively in the past with much of the earlier work focused on the determination of $(a/b)_c$ (Refs. 1, 4, and 5). The physical mechanism associated with merger was considered by Melander *et al.*[6] They examine the flow in a corotating reference frame which reveals the differential motion and associated flow structure. This is illustrated here in Fig. 1(a), which shows the principal streamlines defining distinct regions in the flow: two *inner core* regions, the *exchange band*, and two *outer recirculation* regions. The inner core regions consist of streamlines encircling each individual vorticity maximum and correspond to the primary vortices. The exchange band consists of closed streamlines surrounding both inner core regions and corresponds to fluid circulating between the two vortices. The outer recirculation regions are the two separate circulation cells in which the fluid circulates in the opposite sense to that within the cores and exchange band (in the corotating frame). When vorticity enters these regions, the differential rotation leads to the formation of filaments. This is thought to modify the orientation of the vorticity contours with respect to the streamlines and lead to merger through an inviscid axisymmetrization process.[7] Although the importance of the exchange band was noted, Melander *et al.*[6] indicate that merger is driven by filament formation.

Recently, a number of studies have been carried out which reexamine the physical mechanisms of merger.[2,3,8–10] Meunier *et al.*[2,10] further consider the role of the filaments and explain the accompanying reduction in b in terms of conservation of angular momentum. A simple model is developed and, although the initial reduction in b is well predicted, it does not predict the dominant motion of the vortices suggesting that some other mechanism is present.[10] Cerretelli and Williamson[3] show that convective merger is due to the antisymmetric part of the vorticity field which is considered to be primarily associated with the filaments. Velasco Fuentes[9] finds that filamentation does not always lead to merger and in the case of steep vorticity profiles, merger begins before filamentation takes place. Huang[8] analyzes the flow in terms of Lagrangian flow structures and shows that the "sheet-like structure" emanating from the opposite vortex, which includes both filament and exchange band fluid, is responsible for the induced merging velocity. The formation of these structures are attributed to a misalignment of the major axes of the vortices and the connecting line between the vortex centroids which leads to a "mutual attraction" of fluid between the two vortices. How this misalignment is established was not explained.

From these previous studies, it is apparent that a number of differing views of the convective merger process exist. The objectives of the present study are to further investigate the physical mechanism and to develop a clearer understanding of this fundamental process. In particular, we distinguish and assess the relative contributions of the filament and exchange band regions to merging and elucidate the role of the rate of strain field.

Two-dimensional numerical simulations of a corotating vortex pair are performed for this study. The initial flow field consists of a superposition of two corotating Lamb-Oseen

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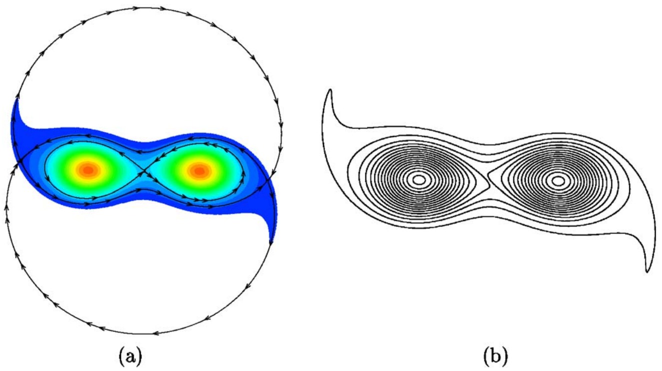
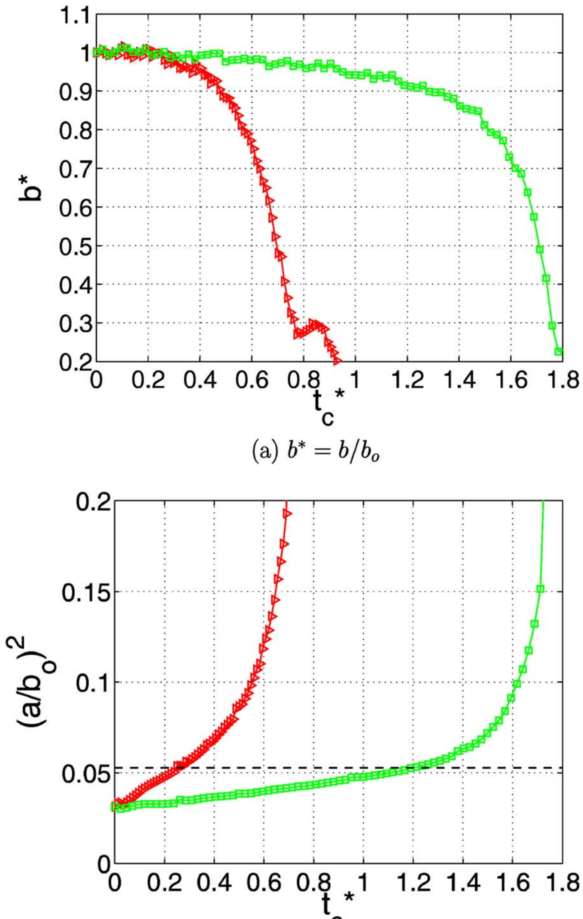


FIG. 1. Vortex pair for $\mathrm{Re}_\Gamma=5000$ at $t_c^*=1.34$. (a) Vorticity contours superimposed with streamlines in corotating frame. (b) Vorticity contour line plot.



(a) $b^*=b/b_o$

(b) $(a/b_o)^2$

FIG. 2. Time development of (a) separation distance, $b^*=b/b_o$. (b) Core size $(a/b_o)^2$ (dashed line corresponds to $(a/b)_c$). Symbols: ▷: $\mathrm{Re}_\Gamma=1000$. □: $\mathrm{Re}_\Gamma=5000$.

(Gaussian) vortices. The initial geometry of the vortex pair is specified by the aspect ratio, a_o/b_o, which in these simulations is 0.177. Here, a is based on the vorticity second moment, $a^2=\langle r^2\omega\rangle/\langle\omega\rangle$, where r is the radial distance from a vortex centroid, ω is vorticity, and $\langle\ \rangle$ indicates an area average. The range of Reynolds numbers considered is $1000 \leqslant \mathrm{Re}_\Gamma \leqslant 6500$ ($\mathrm{Re}_\Gamma=\Gamma_o/\nu$). A convective time scale of the flow is the rotational period, which is approximately that of a two-point vortex system, $T=2\pi^2 b_o^2/\Gamma_o$. In the results presented, the nondimensional time is $t_c^*=t/T$. Details of the numerical solution are given in Ref. 11.

Figure 2 shows the time development of the separation distance, $b^*=b/b_o$, and core radius, a^2/b_o^2. As in previous studies, the behavior of $b(t)$ and $a(t)$ designate the three phases of the merging process. In the first diffusive/adjustment phase, b remains relatively constant while a^2 grows linearly according to $a^2=4\nu t+a_o^2$. For $\mathrm{Re}_\Gamma=5000$, this phase occurs for $0<t_c^*\lesssim 1.1$. For $\mathrm{Re}_\Gamma=1000$, this phase is reduced due to a shorter viscous time scale. The vortices also adjust to the induced strain field which is shown in Fig. 3(a) ($t_c^*=0.41$), in which contours of ω are superimposed with the principal extensional strain rate (vectors). Note at this early time, the strain field resembles that of two separate vortices, i.e., a band of high strain surrounding the cores with an applied direction of 45° from the radial direction.[12] As diffusive growth of the cores continues, the strain bands overlap. The induced strain in this region strengthens and imposes a directionality on the flow which is asymmetric about the dividing plane of the vortices. The convective phase is associated with the predominant decrease in b and begins when $a/b_o=(a/b)_c\sim 0.23$ [dashed line in Fig. 2(b)] at which time a^2 deviates from its linear growth. For $\mathrm{Re}_\Gamma=5000$, this phase occurs for $1.1\lesssim t_c^*\lesssim 1.8$. During the convective phase, filamentation and significant deformation of the vortices are observed [Figs. 3(b) and 3(c)]. The convective phase terminates when b reaches approximately $0.20b_o-0.25b_o$ at which point the inward velocities at the centroids are nearly zero. As discussed in Ref. 3, the second diffusive phase (not shown) then proceeds with a slow reduction in b as the two ω maxima disappear by viscous diffusion.

As discussed, there is some uncertainty as to the specific source of the merging velocity, and thus $b(t)$. We address this directly by considering the contribution of each of the distinct flow regions, i.e., the inner cores, the exchange band, and the filaments [Fig. 1(a)]. The flow regions are identified in our analysis as follows. First, the inner recirculation (cores and exchange band) and outer recirculation (filaments) regions are distinguished by the *sign* of ω in the corotating frame, since the outer recirculation is associated with differential rotation. Next, the cores and exchange band are distinguished by considering the second invariant of the velocity gradient tensor (corotating frame), $II=(\omega^2/2-\mathsf{S}^2)/2$, where S is the strain rate tensor.[12] Thus, $II>0$ corresponds to rotation-dominated regions, which effectively characterizes the cores, and $II<0$ corresponds to strain-dominated regions, which characterizes the exchange band. Although our identification scheme is not based on a strict definition, a sensitivity analysis indicates that it appears to capture the main portion and behavior of these regions. Once the flow regions are identified, the inward velocity induced at the vortex centroids by each region is computed using the Biot-Savart law. These velocities are then integrated in time. Results are given in Fig. 4, which shows the contribution of each region to the *change* in $b^*(t)$. The contribution of the inner cores exhibits an oscillation during the diffusive/adjustment phase and a positive change in $b^*(t)$ during the

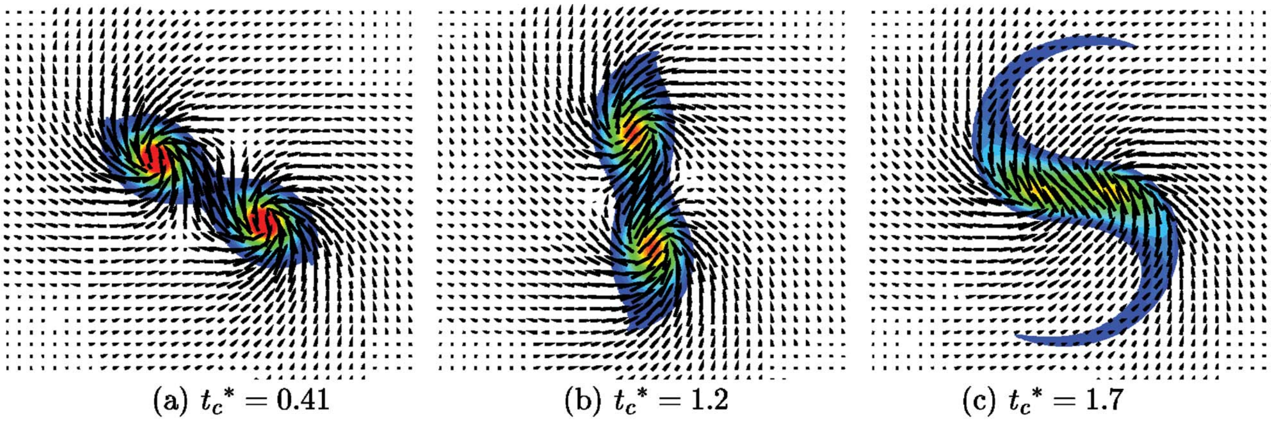


FIG. 3. Vorticity contours with superimposed principal extensional strain (vectors indicating magnitude of eigenvalue and direction of eigenvector) for $\mathrm{Re}_\Gamma=5000$.

main part of the convective phase; thus, it does not contribute to merger. The contribution of the filaments is significant in the early part of the convective phase; while the contribution of the exchange band dominates in the latter part and is responsible for the observed rapid reduction in b^*. The start of the exchange band effect on $b^*(t)$ correlates well with $a/b=(a/b)_c$ in Fig. 2(b), further indicating that it controls merging (at least for these Re_Γ). We note for $\mathrm{Re}_\Gamma=1000$ [Fig. 4(a)], the contribution of both exchange band and filaments occurs at nearly the same time; however, as Re_Γ increases, the time separation of the two processes increases. Estimates of $(a/b)_c$ based on filament behavior in high Re_Γ flows will therefore be inaccurate.

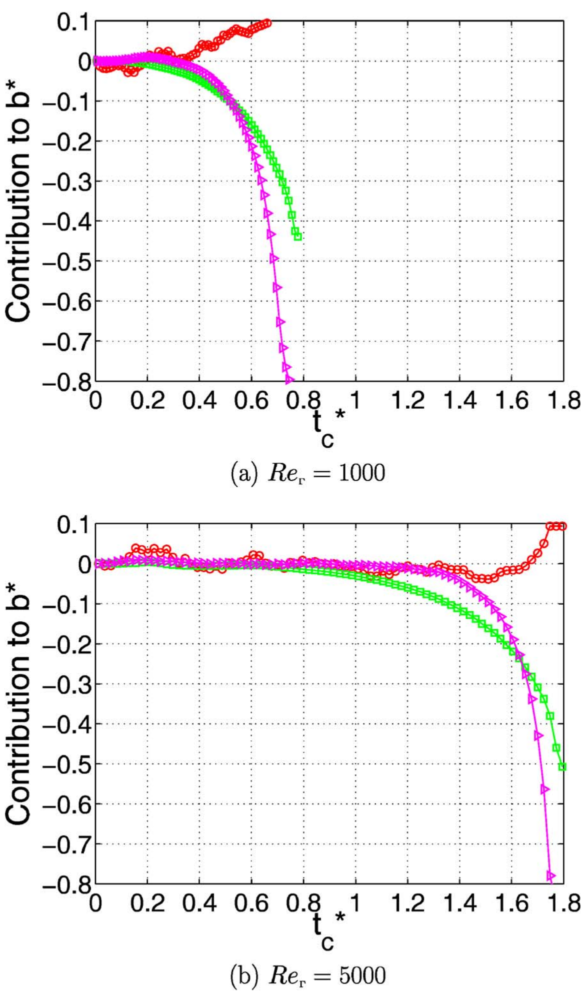


FIG. 4. Contribution of flow regions (○: inner cores, □: filaments, ▷: exchange band) to the change in separation distance, $b^*(t)$ in Fig. 2(a), for (a) $\mathrm{Re}_\Gamma=1000$, (b) $\mathrm{Re}_\Gamma=5000$.

The underlying physics are now discussed. The induced strain field of the two vortex system (Fig. 3) deforms the flow and produces antisymmetric ω, previously demonstrated to be the source of the merging velocity.[3] Our results indicate that both the filaments and exchange band contribute to the merging velocity, however, it is the exchange band that is the dominant contributor. The associated process is initiated by the establishment of a tilt in the ω contours with respect to the vortex connecting line near the center of rotation [Fig. 1(b)]. Vorticity now in the exchange band is then advected away from its source core and entrained. In time, the circulation of the exchange band increases at the expense of that of the inner cores, which become increasingly weak (decreasing *II*). At some point, the cores themselves are entrained. This corresponds to the rapid decrease in b^* associated with the exchange band (Fig. 4). Convective merger is complete when the circulation of (what was) the exchange band dominates (the region becomes rotation dominated) and a single vortex essentially exists.

The remaining issue is how the tilt in the ω contours is established. In two-dimensional flow, the interaction of ω and S is understood in terms of the vorticity gradient, $\nabla\omega$, which may undergo reorientation and amplification by S (note that $-\nabla\omega/|\nabla\omega|$ is the normal to an isovorticity line). The corresponding term in the equation for $|\nabla\omega|^2$ (see, e.g, Ref. 14) is $-\nabla\omega^T\mathsf{S}\nabla\omega$. Here, we consider the quantity $P_s=-(\nabla\omega^T\mathsf{S}\nabla\omega)/|\nabla\omega|^2$, whose sign indicates the relative orientation of $\nabla\omega$ with the principal strain axes and whose magnitude indicates the strain in the direction of $\nabla\omega$. A positive P_s corresponds to $\nabla\omega$ orienting towards the direction of the compressive strain, i.e., gradient amplification by compressive straining. Figure 5 shows the regions of positive/negative P_s, which exhibit the quadrupole structure associated with elliptic vortices,[14] and is asymmetric about the

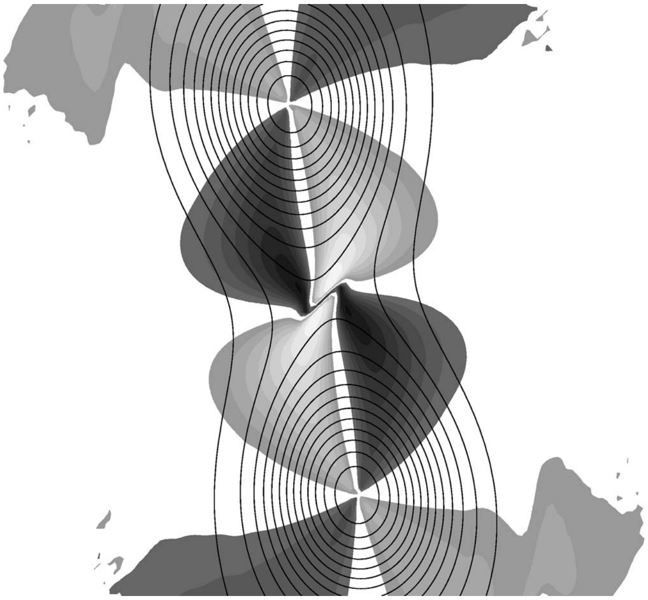


FIG. 5. Vorticity contours with gray shading corresponding to $\nabla\omega$ production term, $P_s = -(\nabla\omega^T \mathsf{S} \nabla\omega)/|\nabla\omega|^2$, for $\mathrm{Re}_\Gamma = 5000$ at $t_c^* = 1.2$ (light gray scale: $P_s > 0$, dark gray scale: $P_s < 0$).

dividing plane of the vortex pair. As discussed by Kimura and Herring,[14] in positive P_s regions, ω isocontours are squeezed together while at the same time, they are extended in the orthogonal direction. The opposite is true for negative P_s regions. We expect the effect to be more pronounced in strain-dominated regions, particularly where the strain bands overlap in the vicinity of the center of rotation (hyperbolic point). This is seen in Fig. 5 where, just above the origin, ω contours in $P_s > 0$ regions extend to the left, while ω contours in $P_s < 0$ regions contract to the left. This results in the tilting of the vortex to the left in this region. This is in agreement with Le Dizes and Verga,[13] who state that near the merging threshold, there is a corresponding misalignment of ω contours with respect to the streamlines at the hyperbolic points where $|\omega|$ is low (see their Fig. 14). We note that Kimura and Herring indicate that P_s plays a significant role in filament formation.[14]

In summary, our analysis reveals the key underlying processes responsible for convective merging: the coupled interaction of $\nabla\omega$ and S which leads to the entrainment of core ω by the exchange band. This is the missing mechanism in the model of Meunier *et al.*[10] Our analysis indicates that the "sheet-like structure" in Ref. 8 corresponds primarily to exchange band fluid and explains the critical process leading to its formation.

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