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Interaction of two equal co-rotating viscous vortices in the presence of background shear

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Abstract

The interactions of two equal co-rotating vortices under the influence of both viscosity and uniform background shear are investigated using two-dimensional numerical simulations. A range of values of the shear strength parameter, $\zeta_0 = -S/\omega_0$, and initial aspect ratio, a_0/b_0, are considered for two values of circulation Reynolds number $\mathrm{Re}_\Gamma = |\Gamma_0|/\nu$. The primary effect of viscosity is to increase the core size $a(t)$ in time while the primary effect of shear is to vary the separation distance $b(t)$ in time. For sufficiently separated vortices, the motion of the vortices is well-described by a point vortex model with linear shear. The present simulations show that for a viscous symmetric vortex pair there are two distinct flow regimes, merger and separation, with the boundaries separating these regimes well-predicted by the point vortex model and largely independent of Re_Γ over the range tested. Results also indicate that the onset of merging occurs when $a(t)/b(t)$ attains the critical value $(a/b)_{cr}$ found for vortex pairs without shear.

(Some figures may appear in colour only in the online journal)

1. Introduction

Vortex pair interactions are fundamental processes in many complex flows of practical and scientific interest, such as aircraft wakes and two-dimensional turbulence. Understanding these interactions is therefore essential to understanding the more complicated flows. For example, it is known that two co-rotating vortices will merge to form a larger compound vortex if the aspect ratio (core size/separation distance), a/b, exceeds a critical value, $(a/b)_{cr}$.

Thus, vortex merging has been considered to be a key process in the inverse energy cascade of two-dimensional turbulence (Benzi *et al* 1987). However, a turbulent flow consists of a field of vortices and other structures, differing in size and strength, and the influence of these factors on the vortex merging process and turbulence dynamics is not well understood.

A great deal of research has been devoted to the study of the simplest vortex pair configuration: two equal co-rotating vortices, i.e., a symmetric vortex pair. Much of this research has focused on inviscid flow (e.g., Dritschel and Waugh 1992, Overman and Zabusky 1982, Saffman and Szeto 1980), while other studies have considered flows with viscosity (e.g., Brandt and Nomura 2007, Cerretelli and Williamson 2003, Melander *et al* 1988, Meunier *et al* 2002). If the vortices are sufficiently separated, their basic behavior is similar to that of two point vortices which rotate about each other. However, for finite-area vortices, the induced strain field will deform and tilt the vortices. The critical aspect ratio can be related to the relative strength of the induced strain rate to the vortex strength; a key factor for the physical mechanism of the onset of merger (Brandt and Nomura 2010). The presence of viscosity ensures that a given pair will achieve the critical aspect ratio given sufficient time to diffusively spread, producing merger in initially well-separated vortices. Increasing the effect of viscosity causes this process to occur more rapidly on a convective timescale, but the merging criterion remains relatively unchanged (Le Dizès and Verga 2002).

The majority of previous studies have considered a single vortex pair, with no external influences. In order to consider vortex merging in a turbulent flow, the influence of neighboring and remote vortices must be considered. A few studies have been carried out which approximate this influence as a simple background shear in which the pair interacts (Maze *et al* 2004, Perrot and Carton 2010, Trieling *et al* 2010). To date these studies have all considered inviscid vortex pairs.

The work of Trieling *et al* (2010) is of particular relevance as they studied the case of two equal finite-area Gaussian vortices in linear shear. Using contour dynamics simulations, they identified four possible interaction regimes: merger, periodic motion, separation without elongation, and separation with elongation; depending on the sign and strength of the background shear relative to the vorticity of the vortices. The basic motion of the vortices, and in particular, the delineation between separative and periodic regimes was found to be well-described by the point vortex model of Kimura and Hasimoto (1985). Conditions for the merging regime were determined by considering the shear-induced variations of the separation distance, b, and using known $(a/b)_{cr}$ values for no-shear flow (see section 2). Though some ambiguities remained, the value of $(a/b)_{cr}$ for the no-shear case was found to provide a reasonably effective criterion for vortices in shear as well.

The previous research has considered separately the effects of viscosity and linear shear on symmetric vortex interactions, but to date their combined effect has not been considered. The current research investigates vortex pair interactions with both effects present by investigating the possible regimes of interaction through a series of numerical simulations for a range of initial parameters.

In section 2, the modified point vortex analysis of Kimura and Hasimoto (1985) and Trieling *et al* (2010) is reviewed. In section 3, the setup and numerical method of the simulations are described. In section 4 results of the simulations are presented. These results are analyzed and the relevance of point vortex results to the viscous flow is demonstrated. Finally, section 5 summarizes the findings and conclusions.

2. Modified point vortex model

The motion of two point vortices in linear shear is described by the following equations (Kimura and Hasimoto 1985):

$$\frac{\mathrm{d}t_1}{\mathrm{d}t} = -\frac{\Gamma_2}{2\pi}\frac{y_1 - y_2}{b^2} + Sy_1 \tag{1}$$

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{\Gamma_2}{2\pi}\frac{x_1 - x_2}{b^2} \tag{2}$$

$$\frac{\mathrm{d}t_2}{\mathrm{d}t} = -\frac{\Gamma_1}{2\pi}\frac{y_2 - y_1}{b^2} + Sy_2 \tag{3}$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{\Gamma_1}{2\pi}\frac{x_2 - x_1}{b^2}, \tag{4}$$

where (x_i, y_i) are the coordinates of the ith vortex, Γ_i is the circulation of the ith vortex, $b^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ is the squared separation distance between the vortices, and $S = \mathrm{d}U/\mathrm{d}y$ is the uniform background shear. Considering symmetric vortices ($\Gamma_1 = \Gamma_2 = \Gamma$) and following the methods of Kimura and Hasimoto (1985), these equations can be integrated to find trajectories that are either closed or open, depending on the relative sign and strength of the shear and vortices. A nondimensional shear strength parameter, $\mu = S\, b_0^2/\Gamma$, can be considered (Trieling *et al* 2010) with a critical value determined from results of Kimura and Hasimoto (1985) to be,

$$\mu_{cr} = \left(\frac{S\, b_0^2}{\Gamma}\right)_{cr} = \frac{1}{\pi e}, \tag{5}$$

where e is the base of the natural logarithm and $b_0 = (x_2 - x_1)$ when $y_2 - y_1 = 0$.

Trajectories for initially horizontally-aligned vortices with various μ values are shown in figure 1. In the case of no shear ($\mu = 0$), the trajectory is a circle corresponding to periodic motion. When the shear is favorable ($\mu < 0$), the motion is always periodic since $\mu < 0 < \mu_{cr}$. The corresponding trajectories in figure 1 indicate that the vortex separation, initially b_0 when horizontally-aligned, reduces to a minimum as they revolve to become vertically aligned. When the shear is adverse ($\mu > 0$), the motion may be either separative or periodic. For weakly adverse shear ($0 < \mu < \mu_{cr}$), the motion is periodic but vortex separation instead increases to a maximum when they become vertically aligned. The case of $\mu = \mu_{cr}$ gives the critical separatrix for stationary flow; the vortices revolve to be vertically aligned and then remain in that position indefinitely. For strongly adverse shear ($\mu > \mu_{cr}$), the vortices instead follow open trajectories and their separation increases indefinitely.

The point vortex model has been found to effectively describe the motion of sufficiently separated finite-area vortices in the inviscid limit (Benzi *et al* 1987). Modifications to explicitly incorporate finite-area vortex parameters were made by Trieling *et al* (2010). By substituting $\Gamma = \pi a_0^2 \omega_0$ (where a_0 is the vortex radius and ω_0 is the peak vorticity) for circulation in the point vortex solution, the critical value of the ratio of shear to peak vorticity is found in terms of the initial aspect ratio a_0/b_0:

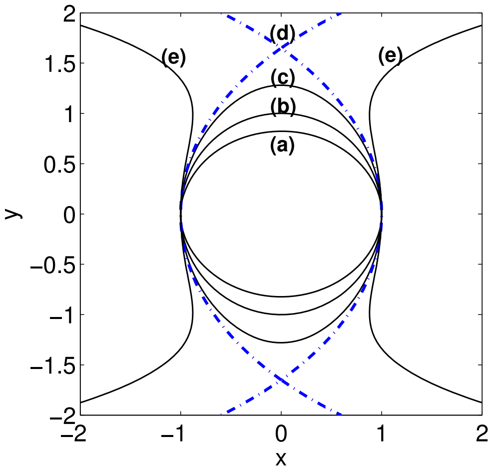

Figure 1. Point vortex trajectories of initially horizontally-aligned vortices ($y_1 = y_2 = 0$) for various initial shear strengths, computed by integrating equations (1)–(4) after substituting $\xi = x_2 - x_1$ and $\eta = y_2 - y_1$. The contours correspond to: (a) favorable shear ($\mu < 0$), (b) no shear ($\mu = 0$), (c) weakly adverse shear ($0 < \mu < \mu_{cr}$), (d) critical separatrix ($\mu = \mu_{cr}$), (e) strongly adverse shear ($\mu > \mu_{cr}$).

$$\left(\frac{S}{\omega_0}\right)_{cr} = \pi\mu_{cr}\left(\frac{a_0}{b_0}\right)^2 = \frac{1}{e}\left(\frac{a_0}{b_0}\right)^2. \tag{6}$$

Thus, the value of $\left(S/\omega_0\right)_{cr}$ depends solely on the initial aspect ratio (a_0/b_0). For a given a_0/b_0, separative motion will occur if S/ω_0 exceeds the corresponding value of $(S/\omega_0)_{cr}$. Results in Trieling *et al* (2010) from contour dynamics calculations demonstrate that (6) effectively distinguishes the separative and periodic regimes for finite-area vortices.

In flows without background shear, finite-area vortices will merge if their separation distance is less than the critical value, i.e., $b/a_0 < (b/a)_{cr}$. In flows with shear, and when the motion is not separative, this may be expected to hold true. As indicated by the point vortex model, the primary effect of shear is to vary $b(t)$ along the trajectory in the periodic regime. By considering $(b/a)_{cr}$ for the no-shear case, a simple merging criterion was formulated by Trieling *et al* (2010): merger will occur if the vortex separation distance is always less than the critical separation distance. This is a stricter condition than observed for the case of favorable shear, in which merger was found to occur if the minimum separation distance is less than the critical separation.

The point vortex model and finite-area results will be considered in our analysis of the case of viscous flow. The numerical simulations of these flows are described in the next section.

3. Setup and numerical simulations

Figure 2 shows the vortex pair initial condition: two like-signed Gaussian vortices of peak vorticity ω_0 and radius a_0 with an initial separation b_0 in a background shear flow of strength S. These are used to define several parameters which characterize the flow. The initial aspect

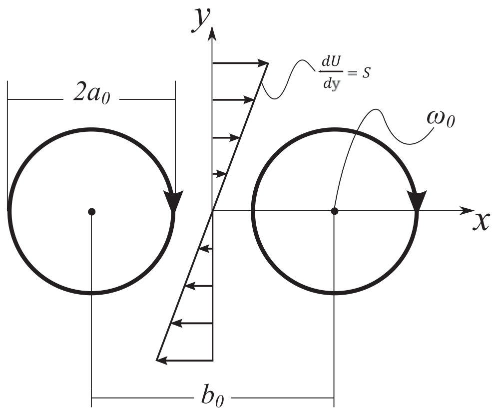

Figure 2. Flow initial condition, including Gaussian vortices of peak vorticity ω_0 and radius a_0 with initial separation b_0 in a uniform background shear of $S = \mathrm{d}U/\mathrm{d}y$. The case shown corresponds to $\zeta_0 = \omega_S/\omega_v = -S/\omega_0 > 0$, i.e. favorable shear.

ratio is a_0/b_0, where a is defined based on the second moment of vorticity (Meunier *et al* 2002). The circulation Reynolds number is $\mathrm{Re}_\Gamma = |\Gamma_0|/\nu$, where $\Gamma_0 = \pi a_0^2 \omega_0$. Here, the shear strength parameter, ζ_0, is defined as the ratio of the vorticity of the background shear to the characteristic vorticity of the vortices,

$$\zeta_0 = \frac{\omega_S}{\omega_v} = \frac{-S}{\omega_0}. \quad (7)$$

Thus, shear is considered to be favorable when $\zeta_0 > 0$ and adverse when $\zeta_0 < 0$.

Two-dimensional numerical simulations of the viscous vortex pair are performed using a combination of finite difference and pseudospectral approximations on a uniform staggered grid. The computational domain is periodic except in the shear direction where shear-periodic boundary conditions are employed. Details of the boundary conditions and numerical solution procedure are given in Gerz *et al* (1989).

Resolution tests found that using 1024^2 grid points, giving about 13 points across a vortex core ($2a_0$), was sufficient to capture the characteristic behavior of the vortex pair in the range of Reynolds numbers considered. In order to avoid unrealistic effects of neighboring vortices due to the periodic boundary condition, the size of the vortex pair is kept small relative to the domain size. An initial separation distance relative to the domain size $b_0/L = 1/24$ was found to be sufficient to minimize boundary effects. For further details of the numerical aspects, see Brandt and Nomura (2007).

4. Results and analysis

Figure 3 shows the time development of representative vortex pairs ($a_0/b_0 = 0.157$, $\mathrm{Re}_\Gamma = 5000$) for a range of $\zeta_0 = -S/\omega_0$ values. Results are given in terms of a convective timescale (based on period of revolution for two point vortices with no shear), $t^* = t/\left(2\pi b_0^2/|\Gamma_0|\right)$.

Early in time ($t^* \lesssim 1$) and when the vortices do not significantly interact, their motion can be described qualitatively by the point vortex model. The vortices rotate about each other in

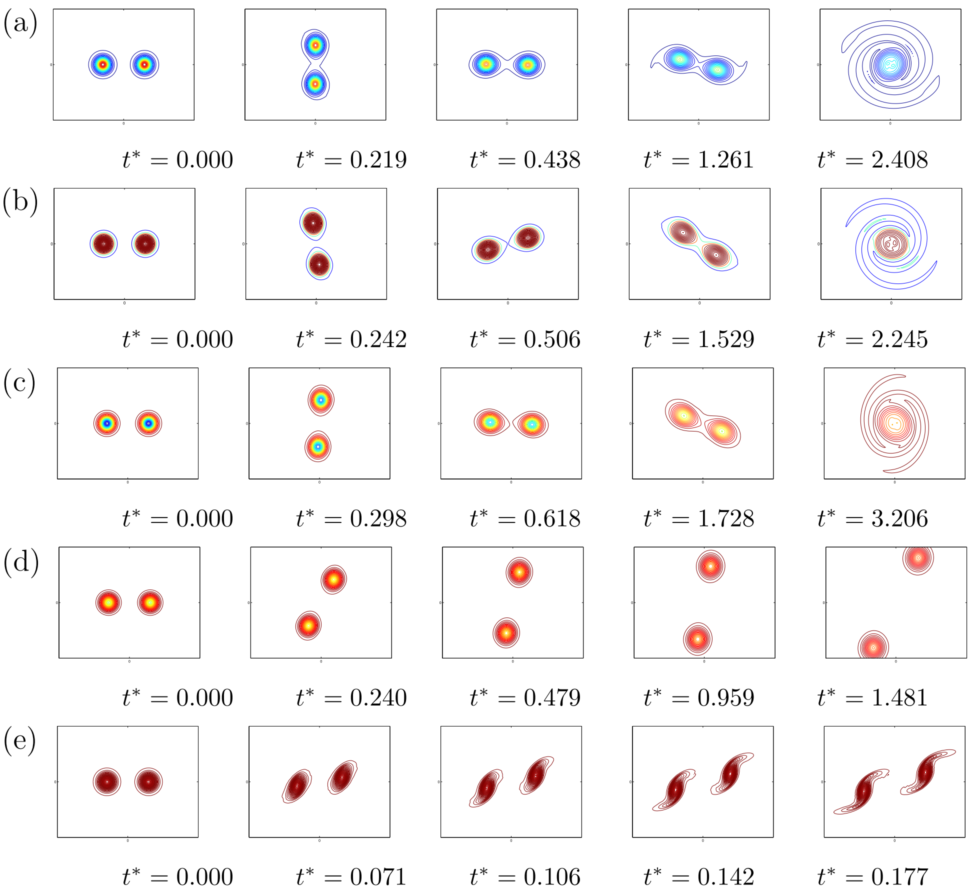

Figure 3. Vorticity contour plots showing time evolution of flows ($a_0/b_0 = 0.157$, $\mathrm{Re}_\Gamma = 5000$) for different $\zeta_0 = -S/\omega_0$: (a) $\zeta_0 = 0.0045$, (b) $\zeta_0 = 0$, (c) $\zeta_0 = -0.0045$, (d) $\zeta_0 = -0.0093$, (e) $\zeta_0 = -0.10$. For (a) and (b), the color red corresponds to the peak (negative) vorticity of the $\zeta_0 = 0.0045$ case and blue indicates lower-level (i.e. less negative) vorticity. For (c), (d) and (e), the color blue corresponds to the peak (positive) vorticity of the $\zeta_0 = -0.0045$ case and red indicates lower-level (positive) vorticity.

closed trajectories for favorable and weakly adverse shear (figures 3(a)–(c)) and separate for strong adverse shear (figures 3(d), (e)). Later in time, the vortices moving in closed trajectories eventually merge into a single vortex (figures 3(a)–(c)). For stronger adverse shear (figure 3(d), $\zeta_0 = -0.0093$), the vortices continue to move apart but are observed to retain their coherence (at least for the duration of these simulations), i.e., there is separation without elongation. For very strong adverse shear (figure 3(e), $\zeta_0 = -0.10$), the vortices are stretched out into filaments by the shear, i.e., there is separation with elongation. The observed interaction regimes are summarized in table 1.

Figure 4 shows the corresponding time development of the vortex separation, $b(t)$. Initially, the behavior of $b(t)$ is consistent with the point vortex trajectories (figure 1) and decreases/increases periodically for favorable/weakly adverse shear ($\zeta_0 = \pm 0.0045$). For strong adverse shear ($\zeta_0 = -0.0093$), the vortices exhibit separative trajectories and $b(t)$ increases monotonically and indefinitely. Later in time, the periodic motion breaks down and

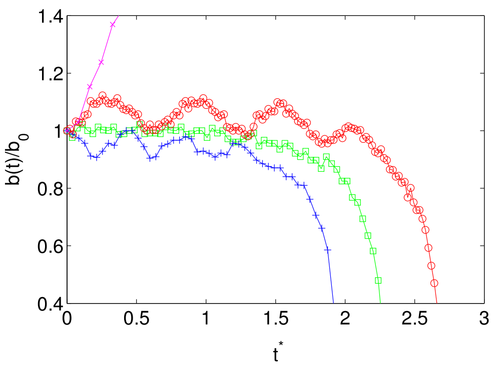

Figure 4. Normalized $b(t)$ for $\mathrm{Re}_\Gamma = 5000$, $a_0/b_0 = 0.157$. +: $\zeta_0 = 0.0045$, □: $\zeta_0 = 0$, ◯: $\zeta_0 = -0.0045$, ×: $\zeta_0 = -0.0093$.

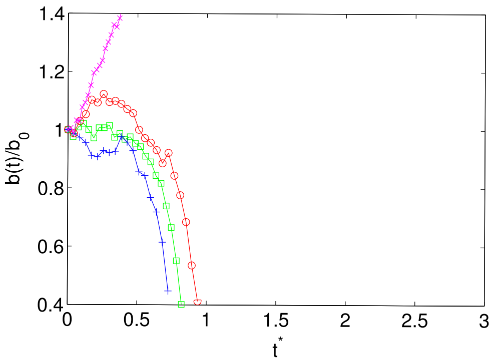

Figure 5. Normalized $b(t)$ for $\mathrm{Re}_\Gamma = 1000$, $a_0/b_0 = 0.157$. +: $\zeta_0 = 0.0045$, □: $\zeta_0 = 0$, ◯: $\zeta_0 = -0.0045$, ×: $\zeta_0 = -0.0093$.

Table 1. Outcome of interaction for vortex pairs of various initial $\zeta_0 = -S/\omega_0$ and $a_0/b_0 = 0.157$ for $\mathrm{Re}_\Gamma = 5000$ and $\mathrm{Re}_\Gamma = 1000$

ζ_0	Outcome, $\mathrm{Re}_\Gamma = 5000$	Outcome, $\mathrm{Re}_\Gamma = 1000$
0.0045	Merger	Merger
0	Merger	Merger
−0.0045	Merger	Merger
−0.0092	Merger	Merger
−0.0093	Separation without elongation	Separation without elongation
−0.10	Separation with elongation	Separation with elongation

we observe a rapid decrease in $b(t)$ corresponding to merger. The onset of merger is seen to occur earlier/later in favorable/weakly adverse shear with respect to the corresponding flow with no shear.

The effect of viscosity on flow development is investigated by performing simulations at $\mathrm{Re}_\Gamma = 1000$ ($a_0/b_0 = 0.157$). The results are included in table 1. The same interaction regimes occur: merging for favorable and weakly adverse shear, and separative for strongly adverse

Table 2. Critical $\zeta_0 = -S/\omega_0$ delineating separation and merger regimes for various a_0/b_0, from point vortex model predictions and empirical results.

a_0/b_0	Predicted $\zeta_{0,cr}$	Empirical $\zeta_{0,cr}$ $\mathrm{Re}_\Gamma = 5000$	Empirical $\zeta_{0,cr}$ $\mathrm{Re}_\Gamma = 1000$
0.105	−0.0041	−0.0040 ± 0.0001	−0.0040 ± 0.0001
0.157	−0.0091	−0.0092 ± 0.00004	−0.00926 ± 0.00009
0.235	−0.020	−0.020 ± 0.002	−0.020 ± 0.002

shear. The corresponding $b(t)$ behavior is shown in figure 5 where it is apparent that the lower Reynolds number accelerates establishment of the merging process.

The modified point vortex model in section 2 indicates that for finite-area inviscid vortices with a specified a_0/b_0, sufficiently strong adverse shear, i.e., $\zeta_0 < \zeta_{0,cr} < 0$, will result in separative motion. To test this criterion for viscous vortices, additional simulations are performed with different initial aspect ratios a_0/b_0. Equation (6) was used to predict the value of $\zeta_{0,cr}$ for a given a_0/b_0. For each a_0/b_0 considered, a series of simulations varying ζ_0 was performed until a pair was found to bracket the boundary between merging and separation regimes. An empirical estimate for $\zeta_{0,cr}$ was then obtained using the midpoint of the two bracketing ζ_0 values. The results are presented in table 2. The findings affirm the expectation: the value of $\zeta_{0,cr}$ between separation and merging varies with a_0/b_0, and in fact these values correspond quite well with the predictions based on the point vortex model.

The above results indicate that in viscous flow, for the range of Re_Γ considered and for the duration of the simulations, there are two distinct flow regimes: merger and separation (without or with elongation), and the boundary of the regimes is described well by the modified point vortex model in (6). This may be expected if we consider that, in the case of inviscid flow, the vortex pair evolves with constant $\mu = \mu_0$ (constant Γ) and therefore $\mu_0 > \mu_{cr}$ in (5) still distinguishes the separation regime for finite-area vortices. In the case of viscous flow, this criterion (and therefore, $\zeta_0 < \zeta_{0,cr} < 0$) may also remain valid for indicating separative motion, since Γ (and therefore μ) remains nearly constant until any significant interaction occurs (maximum computed deviation of Γ_0 before the onset of merger was 5.25%).

In inviscid flow, if $\mu < \mu_{cr}$, periodic motion will prevail if the aspect ratio a/b remains below the critical value for merger. In viscous flow, for $\mu < \mu_{cr}$ ($\zeta_0 > \zeta_{0,cr}$), then if $a/b < (a/b)_{cr}$ the vortices follow trajectories similar to the point vortex periodic regime for some duration of time. However, a primary effect of viscosity is that the cores will diffuse and grow in time. If the vortices are sufficiently separated, the viscous growth of the cores may be described by

$$a^2(t) = a_0^2 + 4\nu t. \tag{8}$$

Eventually this growth causes $a(t)/b(t) > (a/b)_{cr}$, resulting in merger. Stationary or continued periodic flow regimes are therefore not expected for finite Re_Γ. However, the early phase of development may be considered a quasi-periodic phase whose duration depends on Re_Γ (as indicated in figures 4 and 5). This is consistent with the initial quasi-steady phase of development in the no-shear flow, during which $b(t)$ remains nearly constant while $a(t)$ grows by diffusion (Brandt and Nomura 2007).

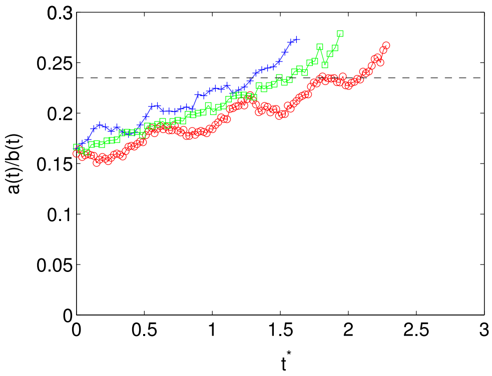

Figure 6. Aspect ratio $a(t)/b(t)$ as a function of convective time t^* for $\mathrm{Re}_\Gamma = 5000$. +: $\zeta_0 = 0.0045$, □: $\zeta_0 = 0$, ◯: $\zeta_0 = -0.0045$. Dashed line corresponds to $(a/b)_{cr} = 0.235$. Note that $a(t)$ is computed from simulation results using the azimuthal average of the radial location of maximum azimuthal velocity, then dividing by 1.12 to obtain an estimate for the second moment of vorticity (Brandt and Nomura 2007).

The onset of merger is identified as the time at which the vortices begin to significantly interact (transition from a diffusive-dominated to convective-dominated process), and this is effectively indicated by the deviation of $a^2(t)$ from its linear viscous growth (equation (8)) (see e.g. Brandt and Nomura 2007). This time is denoted here as $t^* = t^*_{cr}$ and values determined from the simulations (from computed $a^2(t)$) are shown in table 3. The results indicate that in favorable shear ($\zeta > 0$), $t^*_{cr} < t^*_{cr,\zeta=0}$, and in adverse shear ($\zeta < 0$), $t^*_{cr} > t^*_{cr,\zeta=0}$, indicating merger onset occurring earlier and later, respectively, than the time of merger onset observed in the no-shear case ($t^*_{cr,\zeta=0}$). At $\mathrm{Re}_\Gamma = 1000$, the vortices spread more rapidly relative to their advection and therefore the onset of merging occurs more quickly on the advective timescale than for $\mathrm{Re}_\Gamma = 5000$, otherwise the results are similar (table 3). We note that for these lower Reynolds number simulations, the vortices begin to interact very quickly on the t^* scale and so determination of the value of t^*_{cr} becomes difficult.

The time development of the vortex pair aspect ratio, $a(t)/b(t)$, is shown in figure 4 for $\mathrm{Re}_\Gamma = 5000$. For a pair of initially Gaussian vortices in viscous fluid with no shear, the onset of merger is found to occur at $(a/b)_{cr} = 0.235 \pm 0.006$ (Brandt and Nomura 2007), which is indicated by the dashed line in figure 6. Evaluating $a(t)/b(t)$ for $t^* = t^*_{cr}$ from table 3 gives $(a/b)_{cr}$ values of 0.230, 0.231, and 0.235 for $\zeta_0 = 0.0045$, 0, and -0.0045 respectively, for $\mathrm{Re}_\Gamma = 5000$. These results are in agreement with the reported $(a/b)_{cr}$ range.

Since favorable shear acts to periodically reduce $b(t)$, this will increase a/b values thereby promoting merger. Even in the inviscid case, Trieling *et al* (2010) found that favorable shear could induce merger even when $a_0/b_0 < (a/b)_{cr}$. In contrast, weakly adverse shear ($\zeta_{0,cr} < \zeta_0 < 0$) acts to periodically increase $b(t)$ which will tend to reduce a/b values and thereby impede merger. When viscosity is present, $a(t)$ grows in time, so when both shear and viscous effects are present $a(t)/b(t)$ increases but does not necessarily do so monotonically. The oscillatory behavior of $a(t)/b(t)$ may mean that the time of attaining $(a/b)_{cr}$ does not necessarily correspond to the start of merging, though no case was observed of $a(t)/b(t) = (a/b)_{cr}$ being attained and then the vortices failing to merge.

Table 3. Time to start of merging process t^*_{cr} for vortex pairs in the merging ζ_0 regime for $\mathrm{Re}_\Gamma = 5000$ and $\mathrm{Re}_\Gamma = 1000$ and $a_0/b_0 = 0.157$.

	$\mathrm{Re}_\Gamma = 5000$	$\mathrm{Re}_\Gamma = 1000$
ζ_0	t^*_{cr}	t^*_{cr}
0.0045	1.26	0.336
0	1.53	0.373
−0.0045	1.73	0.417

For the $\mathrm{Re}_\Gamma = 1000$ results, evaluating $a(t)/b(t)$ at the t^*_{cr} values from table 3 gives $(a/b)_{cr}$= 0.238, 0.233, and 0.229 for $\zeta_0 = 0.0045$, 0, and −0.0045 respectively. These values are also in agreement with the merging criterion found for vortex pairs without shear.

5. Conclusion

The interactions of two equal co-rotating vortices under the influence of both viscosity and uniform background shear have been investigated using numerical simulations. It is found that the observed interactions can be classified into two distinct regimes, merger and separation, depending on the relative significance of the background shear (as characterized by ζ_0) for a given vortex pair (as characterized by ζ_0 and a_0/b_0).

Early in the flow development when the vortices are sufficiently separated and Γ is constant, their motion is altered by the shear, as described by the point vortex model, while their cores grow by viscous diffusion. During this time, the primary effect of shear is to vary $b(t)$ in time, while the primary effect of viscosity is to increase $a(t)$ in time. If the shear is both adverse and sufficiently strong, i.e., $\zeta_0 < \zeta_{0,cr} < 0$, $b(t)$ will increase indefinitely: this is the separation regime. In this case, if the shear, and thus the corresponding strain rate, is very strong, the vortices will also begin to elongate. If the shear is only weakly adverse or favorable, i.e. $\zeta_0 > \zeta_{0,cr}$, the vortices will revolve along elliptical trajectories with $b(t)$ periodically increasing or decreasing, respectively, as $a(t)$ grows in time until $a(t)/b(t)$ reaches $(a/b)_{cr}$ and the vortices begin to merge into a single compound vortex: this is the merger regime. The value of $(a/b)_{cr}$ determined from the simulations is found to be within the range previously reported for the no-shear case, $(a/b)_{cr} = 0.235 \pm 0.006$, over the range of parameters tested. The boundary separating the merger and separation regimes, $\zeta_{0,cr}$, is accurately predicted by the point vortex model and varies with a_0/b_0. Both of these critical values were found to be largely independent of Re_Γ over the range considered. Therefore, for the purpose of determining whether a co-rotating viscous vortex pair in the presence of background shear will merge, $\zeta_0 > \zeta_{0,cr}\,(a_0/b_0)$ constitutes a sufficient criterion.

Although a study of the long time evolution of the vortex pair is beyond the scope of this paper, some remarks can be made based on the results presented here. As the vortices diffuse and spread, their peak vorticity will decrease. In the separation regime, since $|\zeta(t)| = |S/\omega(t)|$ increases in time, it is expected that vortices initially exhibiting separation without elongation would eventually exhibit elongation if simulations were to run long enough and boundary effects were to remain inconsequential. Likewise, in the merger regime, the compound vortex formed by merger would ultimately diffuse until it too would become weak enough to be deformed by the background strain and elongate.

Thus, in contrast with the case of inviscid flow where periodic motion, merger, and separation with and without elongation constitute distinct regimes determined by initial conditions (Trieling *et al* 2010), viscous vortices may evolve in time through these flow conditions. If sufficiently strong adverse shear is present, the vortices will separate and thereby limit any mutual interaction. Otherwise, the vortices will develop in a manner similar to the no-shear flow. Initially, a quasi-steady diffusive phase will occur in which the vortices revolve in an orbit; the effect of shear is to vary $b(t)$ along the orbit. This is followed by a convective/merging phase; where favorable/adverse shear promotes/hinders mutual interaction and merger. Finally, another diffusive phase is expected to occur in which the single vortex may ultimately be deformed and elongated by the background shear.

It is possible that the efficacy of the inviscid/point vortex predictions when viscous effects are present breaks down at very low Reynolds numbers, when the spreading is so rapid as to violate the 'sufficiently separated' requirement very shortly after the start of the simulation. In such cases merger may result even when their ζ_0 and a_0/b_0 might correspond to the separation regime as indicated by the point vortex criterion. Furthermore, it is known that interactions of unequal vortices in the absence of shear produce a richer variety of outcome regimes than do symmetric pairs (e.g. Brandt and Nomura 2010), so the regimes of interaction of such pairs when shear is present are undoubtedly more complex and nuanced than those presented here. These topics remain to be addressed in future work.

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