MAE 289A: Mathematical Analysis for Applications
Assignment 1
Due Tuesday, 22 Oct. 2013

Problems to hand in (Not all problems may be graded.)

1. Protter and Morrey, Sec. 1.4, Problem 1.
2. Protter and Morrey, Sec. 6.1, Problem 5.
3. Protter and Morrey, Sec. 6.1, Problem 9.
4. Let $\mathcal{X} = \Re^2$ (our space is the x-y plane). Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ represent generic elements of $\mathcal{X} = \Re^2$. Draw $B_1((1,1))$ for the four metrics:
   (a) The Euclidean metric, i.e. $d_2(x,y) = \left[(x_1 - y_1)^2 + (x_2 - y_2)^2\right]^{\frac{1}{2}}$.
   (b) The metric, d_1, given by $d_1(x,y) = |x_1 - y_1| + |x_2 - y_2|$.
   (c) The metric, d_∞, given by $d_\infty(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$.
   (d) The metric, d_4, given by $d_4(x,y) = \left[(x_1 - y_1)^4 + (x_2 - y_2)^4\right]^{\frac{1}{4}}$. (A rough sketch is fine.)
5. Protter and Morrey, Sec. 6.2, Problem 9. (When proving that two sets are equal, it's often best to prove $\subseteq$ and $\supseteq$ separately.)
6. Protter and Morrey, Sec. 6.3, Problem 9. You might want to use the fact that given the rationals are dense in $I\!R$. (That is, given any $x \in I\!R$ and $\delta > 0$, there exists a rational q such that $q \in B_\delta(x) = (x - \delta, x + \delta)$.)
7. Protter and Morrey, Sec. 6.4, Problem 1.
8. Protter and Morrey, Sec. 6.4, Problem 2a.
9. Suppose $\{p_n\}_{n=1}^{\infty} \subset A$ does not have any convergent subsequences. Prove that for any $p_k \in \{p_n\}_{n=1}^{\infty}$, there exists $\delta > 0$ such that $B_\delta(p_k) \cap \{p_n\}_{n=1}^{\infty} = \{p_k\}$ (the set consisting only of the point p_k). (You might find a proof by contradiction to be easiest.)

10. Let $S \subset I\!R$, $S \neq \emptyset$. Prove that if $z = \inf S > -\infty$, then there exists $\{x_n\}_{n=1}^{\infty} \subseteq S$ such that $x_n \to z$. (This also holds in the case with $\inf S = -\infty$.)

You may find one (or more) of these problems quite challenging. Don't worry – that's to be expected in a course like this.