MAE 289C, Introduction to Functional Analysis with Applications
Assignment 1
due 20 April

Not all problems may be graded.
Your supporting work must be included for full credit.
Potentially only a partial list of problems; more might be added.

1. (10 points) Try to obtain the Hölder inequality for $p = 1, q = \infty$ over domain $\mathcal{D} = (0, 1)$. How does the situation differ if you replace the essential supremum with supremum?
2. (10 points) Prove the Minkowski inequality in the case of $L_\infty(0, 1)$.
3. (10 points) Let $\mathcal{D} = \mathbb{R}^2$. Is $L_4(\mathcal{D}) \subseteq L_2(\mathcal{D})$? Is $L_2(\mathcal{D}) \subseteq L_4(\mathcal{D})$?
4. (5 points) Show that ℓ_p is separable for $p \in [1, \infty)$.
5. (5 points) Try to show that $L_\infty(0, 1)$ is not separable.
6. (10 points) Let $\mathcal{D} \doteq B_1(0) \subset \mathbb{R}^2$. Let $f : L_2(\mathcal{D}) \to L_4(\mathcal{D})$ be given by $f(x) = x^2$, that is, $f(x)[\omega] \doteq x^2(\omega)$ for all $\omega \in \mathcal{D}$. Does f actually map $L_2(\mathcal{D})$ into $L_4(\mathcal{D})$? If so, is f continuous on $L_2(\mathcal{D})$?
7. (15 points) Let $\mathcal{X} \doteq L_2(0, 1)$ and $\mathcal{Y} \doteq L_2(-1, 2)$. Fix some $y \in \mathcal{Y}$. For each $x \in \mathcal{X}$, let $f(x)[t] \doteq \int_0^1 y(t - r)x(r)\, dr$ for all $t \in (0, 1)$. Can you show that $f : \mathcal{X} \to \mathcal{X}$? If so, is f is continuous?