MAE 289, Introduction to Functional Analysis with Applications
Assignment 2
due 2/19

(Unless otherwise specified, all problems are worth 10 points. Not all problems will be graded.)

• Section 1.4, Problem 4.
• Section 1.5, Problem 8.
• Section 1.6, Problems 4 and 6
• Section 2.1, Problem 14 (first part only, i.e. partition)
• Section 2.2, Problem 4
• Section 2.3, Problem 10 (You may assume that it is known that the set of finite sequences of rational numbers is countable.)

• E1. Show that $l_2 \subseteq l_\infty$.
• E2. Construct an infinite sequence of piece-wise linear, linearly independent functions in $C[0,1]$.
• E3. Is the set of continuous, piece-wise linear functions (consisting of a finite number of segments) a subspace of the normed (vector) space $C[0,1]$? Is it dense? Support your answers of course.

  Hint: You can use the fact that any continuous function on a compact set is uniformly continuous. A function, $f$, is uniformly continuous on a set, $A$, if given $\varepsilon > 0$, there exists $\delta > 0$ such that for any $x, y \in A$, $\|x - y\| < \delta$ implies $|f(x) - f(y)| < \varepsilon$.
• E4. Let the space $S_c[0,1]$ be the set of $x : [0,1] \to \mathbb{R}$ such that $x(t) + ct^2$ is convex. Prove that the space is closed under the operations of max-plus addition of functions (i.e. $[x \oplus y](t) \doteq \max\{x(t), y(t)\}$) and max-plus multiplication by a scalar $a \in \mathbb{R}$ (i.e. $[a \otimes x](t) \doteq a + x(t)$).
• E5. Prove that the following is an equivalent definition of compactness for subsets of a metric space. “$A$ is compact if given any open cover of $A$, $\{G_\lambda : \lambda \in \Lambda\}$ and $M < \infty$ such that $\delta(G_\lambda) \leq M$ for all $\lambda \in \Lambda$, there is a finite subcover.” Here, the notation $\delta(G_\lambda)$ indicates the diameter of the set, i.e. $\delta(B) \doteq \sup_{x,y \in B} d(x, y)$. 