Notes on the Solutions to Assignment 2

**Problem 2**

One propagates backward to obtain $P_t$ and $r_t$ for $t = 3, 2, 1$. From the terminal conditions, $P_4 = F$ and $r_4 = 0$. The backward propagation updates are:

$$
P_t = C + A'P_{t+1}A - A'P_{t+1}B\Sigma_{t+1}^{-1}B'P_{t+1}A, \quad \Sigma_{t+1} = D + B'P_{t+1}B,
$$

$$
r_t = r_{t+1} + \text{tr}[Q_v],
$$

$$
Q_v = \sqrt{\Lambda_{t+1}S_{t+1}^\prime \sigma Q\sigma S_{t+1}^{\prime}\Lambda_{t+1}},
$$

where $S_{t+1}, \Lambda_{t+1}$ are obtained from the standard decomposition of positive-definite $P_{t+1}$ as $P_{t+1} = S_{t+1}\Lambda_{t+1}S_{t+1}^\prime$.

The $\xi_2, \xi_3, \xi_4$ are normal random variables, where their distributions can be obtained from their dynamics, using the optimal controls obtained from the DP. That is, $\xi_2 = A\xi_1 + Bu_1 + w_1 = Ax + B\bar{u}_1(x) + w_1$ with $\bar{u}_1(x) = -\Sigma_2^{-1}B'P_2Ax$ yielding

$$
\xi_2 = [I - B\Sigma_2^{-1}B'P_2]Ax + w_1.
$$

Hence, the mean of $\xi_2$ is $[I - B\Sigma_2^{-1}B'P_2]Ax$ and the covariance is $Q$.

**Problem 3**

No. It is not too difficult to construct a counterexample.

**Problem 4**

Let $p, q \in \mathcal{P}$, and let $\lambda \in [0, 1]$. Let $r = \lambda p + (1 - \lambda)q$. It is sufficient to show that $r \in \mathcal{P}$. That is, it is sufficient to show that $r(x - \bar{x}) \leq f(x) - f(\bar{x})$ for all $x \in \mathbb{R}$. 
