MAE 2

Homework #1 - Solutions

1)

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3.5 \end{pmatrix} = 2 * (-4) + 3 * (3.5) = 2.5$$

$$\begin{pmatrix} 2 \\ -4.5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -\pi \\ 3 \\ 7 \end{pmatrix} = 2 * (-\pi) + (-4.5) * 3 + 3 * 7 = 7.5 - 2\pi$$

. . . .

2) The magnitude (or length) of the vector $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$ is denoted by $|\vec{p}|$ and formulated as:

$$|\vec{p}| = \sqrt{p_1^2 + p_2^2 + p_3^2} \tag{1}$$

and the dot product of two vectors $\vec{r}$ and $\vec{p}$ is:

$$\vec{r} \cdot \vec{p} = |\vec{r}||\vec{p}|Cos\theta \quad \Longrightarrow \quad \theta = acos\left(\frac{\vec{r} \cdot \vec{p}}{|\vec{r}||\vec{p}|}\right) \tag{2}$$

And,

$$\vec{r} \cdot \vec{p} = 1.5 * 2 + 2 * (-2) + (-4) * 1 = -5$$

$$|\vec{r}| = \sqrt{1.5^2 + 2^2 + (-4)^2} = 4.7170 \quad ; |\vec{p}| = \sqrt{2^2 + (-2)^2 + 1^2} = 3$$

Plugging in these values in (2), one gets $\theta = 1.9319 \quad rad.$

. . . .

3) a) Let $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$ be the vector such that $\vec{r} \cdot \vec{p} = 8$. This implies:

$$\vec{r} \cdot \vec{p} = 2p_1 + 2p_2 + p_3 = 8 \tag{3}$$

Now, any vector $\vec{p}$ that satisfies (3) is a solution to this problem. Such a vector is $\vec{p} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$.

b) Given $|\vec{p}| = 10$, in addition to (3) we also have:

$$\sqrt{p_1^2 + p_2^2 + p_3^2} = 10 \quad \Longrightarrow \quad p_1^2 + p_2^2 + p_3^2 = 100 \tag{4}$$

To find a solution, let $p_3 = 0$. Then we have 2 equations in 2 unknowns:

$$\begin{aligned} p_1^2 + p_2^2 &= 100 \\ p_1 + p_2 &= 4 \end{aligned}$$

Solving these equations by substitution we get: $p_1 = 8.7823$ and $p_2 = -4.7823$. So one solution is:

$$\vec{p} = \begin{pmatrix} 8.7823 \\ -4.7823 \\ 0 \end{pmatrix}$$

Now, considering (3) and (4) we have 2 equations in 3 unknowns. (3) represents a plane and (4) represents a sphere with a radius of 10 centered at the origin. The intersection of the sphere with the plane is a circle and is the solution of this problem. Thus all the points on the circle satisfies (3) and (4) and we have an infinite number of solutions.

....

4) $Altitude = |\vec{r}| - radius_{earth}$

$$Altitude = \sqrt{8000^2 + 8000^2 + 2000^2} - 6378 = 5111.1 \quad km \tag{5}$$

....