MAE 180 Orbital Mechanics
Take-Home Final
Due date/time: Friday, 13 Dec.; 6pm

You must show all work to get credit.

1. (6 points) Our spacecraft is in an elliptic Earth orbit with $a = 8000$ km and $e = 0.05$. It is ahead of where we would like it to be by about 4 minutes (time-wise). Construct the minimum total Δ_v pair of burns that will correct this within 4 days. What is the total Δ_v cost? (Use $\mu = 398600.$ km^3/sec^2 for this problem.)

2. (6 points) Suppose one vehicle is in a circular orbit around the Earth with semimajor axis $a = 8000$ km. Suppose we have another vehicle which at $t = 0$ is has relative position and velocity (in the body-centered system from class, origin at the vehicle position, the x-direciton being radially up from planet center, the y-direction being in the velocity direction, and the z-direction being out-of-plane, to complete the right-handed system) given by

$$\vec{r}^{\,bc} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0.1 \\ 0.05 \end{pmatrix} \quad \text{and} \quad \vec{v}^{\,bc} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

   in km and km/sec, respectively. What is the position of the second vehicle relative to the first at $T/4$ and at $T/2$, where T is the period of the first vehicle? (Again, use $\mu = 398600.$ km^3/sec^2.)

3. (12 points) For the same first vehicle, construct initial conditions for the second vehicle (in that same body-centered system) such that the second vehicle remains in the orbit plane of the first, and oscillates around the first in such a way that its maximum distance ahead of the first is 200 m, and its maximum distance behind is 100 m (behind by 100 m). (Of course, it may need to oscillate vertically as well as horizontally.) Using matlab, plot the x component of the trajectory of the second vehicle relative to the first in the body-centered system as a function of time, over at time period corresponding to one period of the first-vehicle orbit. Do the same for the y component. Lastly, plot the y component versus the x component over that time period.

4. (6 points) Suppose your organization is in the very early planning stages of a satellite launch. The current plan is to put the satellite

into an elliptical orbit with elements (neglecting the time of periapsis passage) after orbit insertion being

$$\begin{aligned}
a &= 8000.0 \text{ km}, \\
e &= 0.1, \\
i &= \pi/6 \text{ radians}, \\
\Omega &= \pi/2 \text{ radians}, \\
\omega &= \pi/2 \text{ radians}.
\end{aligned}$$

What would you expect the orbital elements to be exactly one day later? Suggest a small change in this planned orbit that would eliminate the issue with regard to the secular perturbations of the argument of perigee.

5. (10) You're planning to reduce your orbital energy upon arrival at Mars with the aid of aerobraking. You'll do this by very slightly intersecting the Martian atmosphere at each periapsis passage. Suppose that for your vehicle,

$$\frac{C_D S}{m_{s/c}} = 9.26 \times 10^{-9} \ \frac{\text{km}^2}{\text{kg}},$$

where C_D denotes the coefficient of drag, S denotes the effective area and $m_{s/c}$ denotes the vehicle mass. For Mars, take $\mu \simeq 42828.0$ km^3/sec^2, and employ the exponential atmospheric density model with coefficients given by reference density $\rho_0 = 4.7 \times 10^5$ kg/km^3 at reference radius $r_0 = 3429.0$ km, and with scale height $h = 10$ km.

Suppose we begin in an elliptical orbit with periapsis radius $r_p = 3460$ km and apoapsis radius $r_a = 10000$ km. For the sake of concreteness, suppose we begin and end integration of the braking when our vehicle descends and ascends past $r_e = 3600$ km. At what true anomaly values does this initial orbit hit r_e? At what times before and after the time of periapsis passage do these events occur?

Integrating only over the segment from where $r(t) \le r_e$, estimate the Δ_v resulting from the first aerobraking pass. Making the approximation that all that Δ_v occurred at periapsis, what would the resulting change in apoapsis radius be?

6. (10 points) Suppose that at 9AM PST, a satellite is located at $\vec{r} = (8000, 4000, 2000)^T$ km relative to the ECI coordinate system. Suppose you are standing at [geodetic] latitude 0.576 radians, longitude 4.241

radians and elevation 0 km relative to the Earth ellipsoid model, using equatorial radius $a_e = 6378.14$ km and polar radius $b_e = 6356.75$ km. Suppose that the ECI and ECEF systems were aligned at 1AM PST (earlier that morning). In the ENU system, where is the satellite relative to you? Is it above the horizon?