1. (5) Writing everything out component-wise, show that for vector functions \( \mathbf{r}(t), \mathbf{v}(t) \in \mathbb{R}^3 \),

\[
\frac{d}{dt} [\mathbf{r} \times \mathbf{v}] = \dot{\mathbf{r}} \times \mathbf{v} + \mathbf{r} \times \dot{\mathbf{v}}
\]

where \( \times \) indicates the cross-product.

2. (10) Suppose that in some other universe, the relative acceleration between two bodies due to gravity took the form

\[
\ddot{\mathbf{r}} = \left[-k \sin(|\mathbf{r}|) + 2/|\mathbf{r}|^5\right] \mathbf{r}
\]

where \( k \) is some constant dependent on the masses. Would the relative motion of the bodies still necessarily lie in a plane? (Support your answer, of course.)

3. (10) A spacecraft is in orbit around Mars. Suppose that it was previously observed to have speed 5 km/sec while at a distance \( r = 5000 \) km from planet center. Suppose it is now at a distance \( r = 6000 \) km from planet center. What speed would you expect it to be traveling at now? Note that for Mars we have \( Gm_{Mars} = \mu_{Mars} = 42828.29 \) km\(^3\)/sec\(^2\).