MAE180A Spacecraft Guidance, Navigation, and Mission Design
Assignment 1
Due 9pm, Wednesday, 16 Oct.

Note: You must show all your work in order to get credit.

Problems to hand in. (Not all problems may be graded.)

Complete list of problems.

1. (5) Writing everything out component-wise, use the product rule for scalar functions to show that the following holds, where $\vec{r}(t), \vec{v}(t) \in \mathbb{R}^3$ denote vector functions of time.

$$\frac{d}{dt}\left[\vec{r} \times \vec{v}\right] = \dot{\vec{r}} \times \vec{v} + \vec{r} \times \dot{\vec{v}}.$$

2. (10) Suppose that in some other universe, the relative acceleration between two bodies due to gravity takes the form

$$\ddot{\vec{r}} = \left[-c_1(\exp(-|\vec{r}|) + 1)/|\vec{r}|^3\right]\vec{r}$$

where c_1 is some constant dependent on the masses. Would the relative motion of the bodies still necessarily lie in a plane? What about the case of

$$\ddot{\vec{r}} = \left[-c_1(\exp(-|\vec{r}|) + 1)/|\vec{r}|^3\right]\vec{r} - c_2\dot{\vec{r}},$$

where c_1, c_2 are constants? (Support your answers.)

3. (10) A spacecraft is in orbit around Mars. Suppose that it was previously observed to be moving with speed 5.6 km/sec while at a distance $r = 6000$ km from planet center. Suppose it is now at a distance $r = 8000$ km from planet center. No maneuvers have been performed. What speed would you expect it to be traveling at now? Note that for Mars we have $Gm_{Mars} = \mu_{Mars} \simeq 42828.4 \text{ km}^3/\text{sec}^2$.

Study Problems (not to hand in)

1. Suppose a spacecraft is in some unknown orbit around a planet, where the planet has $\mu_p = Gm_p = 1.2 \times 10^5 \, \frac{\text{km}^3}{\text{sec}^2}$, and m_p denotes the planet

mass. Suppose that at time t, the vehicle (planet-relative) position and velocity are

$$\vec{r}(t) = \begin{pmatrix} 5000 \\ 3000 \\ 1000 \end{pmatrix} \quad \text{and} \quad \vec{v}(t) = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}.$$

Using only what you know about specific energy (and ignoring any concern that the trajectory might be such that the vehicle would hit the planet), what is the maximum distance from the planet center that the spacecraft might achieve? (We will eventually find that that distance might not actually be reached with that specific initial position and velocity.) What if instead, at that same position and time, the velocity was

$$\vec{v}(t) = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}?$$

2. For the first case of Problem 1, what is the magnitude of the specific angular momentum? What is a unit vector in the direction of the specific angular momentum? What is the value of the "parameter"?

3. Suppose we have a vehicle in orbit about the Earth ($\mu \simeq 398600.4 \frac{\text{km}^3}{\text{sec}^2}$) with "parameter" begin $p \simeq 8640.0$ km and semimajor axis being $a \simeq 9000.0$ km. What is the eccentricity? What are the perigee and apogee distances (from planet center)?

4. For the same trajectory as in Problem 3, what are the period and mean motion?

5. Consider a vehicle on a hyperbolic trajectory relative to a planet with parameter $p = 12,000$ km, and eccentricity, $e = 1.5$. By hand, sketch the trajectory (in the plane of the trajectory), including the location of the planet center. Indicate the asymptotes, and the angles they make with the horizontal axis. What is the minimal distance of the trajectory from the planet center?

6. Suppose that our spacecraft goes into orbit about an unknown moon. Suppose that the *altitude* (above the surface) at periapsis is 1000 km, and at apoapsis the altitude (above the surface) is 5000 km, where we estimate that the radius of the moon is approximately 1000 km. The period of our spacecraft's orbit about the moon is 9.5 hours. Would

you estimate that the composition of this moon is largely rock or largely iron?