Last lecture

Finding orbital elements from initial conditions
Solving for trajectory given orbital elements
ECEF coordinates

Position and velocity in ECEF

$$\vec{R}^{\mathrm{EF}}(t) = G_3^{\omega_e(t-\tau_e)} \vec{R}^{ECI}(t)$$

$$\vec{v}^{\mathrm{EF}}(t) = G_3^{\omega_e(t-\tau_e)} \vec{v}^{\mathrm{ECI}}(t) + \left[\frac{\mathrm{d}}{\mathrm{d}t} G_3^{\omega_e(t-\tau_e)}\right] \vec{R}^{\mathrm{ECI}}(t)$$

$$= G_3^{\omega_e(t-\tau_e)} \left(\vec{v}^{\mathrm{ECI}}(t) - \omega_e \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \vec{R}^{\mathrm{ECI}}(t) \right),$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t} G_3^{\omega_e(t-\tau)} = \omega_e \begin{bmatrix} -\sin(t-\tau_e) & \cos(t-\tau_e) & 0 \\ -\cos(t-\tau_e) & -\sin(t-\tau_e) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Ellipsoid model of Earth

We use longitude and latitude to specify the location on Earth.

Suppose $\vec{R}^{\mathrm{EF}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is on the surface of the Earth. The longitude $\lambda \in (-\pi, \pi]$ is found by solving

$$\cos\lambda = \frac{x}{u}, \quad \sin\lambda = \frac{y}{u}.$$

The equatorial radius of Earth is about 6378.14 km. The polar radius is 6356.75 km.

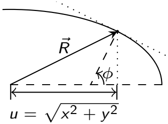
Ellipsoid model of Earth

The geocentric latitude $\hat{\phi}$ is the angle between the equatorial plane and $\vec{R}$, which is given by

$$\tan \hat{\phi} = \frac{z}{\sqrt{x^2 + y^2}}.$$

Ellipsoid model of Earth

The geodetic latitude ϕ is the angle between the equatorial plane and the normal vector (to the surface of Earth) at $\vec{R}$.



The geodetic latitude can be found using

$$\tan\phi = \frac{1}{1-\tilde{e}^2}\frac{z}{u},$$

where $\tilde{e}^2 = 1 - \frac{b^2}{a^2} \approx 0.006\,694\,38$ (b is polar radius and a is equatorial radius of Earth).

From Long/Lat to ECEF coordinates

Given (geodetic) longitude λ and latitude ϕ of some point on Earth, the ECEF coordinate of the point is given by

$$\vec{R}^{\mathrm{EF}} = \frac{a}{\sqrt{1 - \tilde{e}^2 \sin^2 \phi}} \begin{pmatrix} \cos\phi \cos\lambda \\ \cos\phi \sin\lambda \\ (1 - \tilde{e}^2) \sin\phi \end{pmatrix} .$$