Last lecture

Vis-viva equation

Orbit in 3D (ECI coordinate system)

Rotation Matrices

Consider a Cartesian coordinate system with axes $I^{1,2,3}$.
Suppose we rotate the axes about the I^1 axis by θ, to obtain a set of new coordinate axes $\hat{I}^{1,2,3}$.
We see that the unit vectors of the new coordinates are $\hat{I}^1 = I^1$, and

$$\hat{I}^2 = \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix}, \quad \hat{I}^3 = \begin{bmatrix} 0 \\ -\sin\theta \\ \cos\theta \end{bmatrix}.$$

A vector $\vec{R}$ in original coordinates must now be written as

$$\vec{\hat{R}} = \begin{bmatrix} \vec{R}\cdot\hat{I}^1 \\ \vec{R}\cdot\hat{I}^2 \\ \vec{R}\cdot\hat{I}^3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}}_{G_1^\theta} \vec{R}$$

in new coordinates, and we denote the corresponding rotation matrix by G_1^θ.

Rotation Matrices

Suppose, instead, we rotate the axes about the I^2 axis by θ, to obtain a set of new coordinate axes $\hat{I}^{1,2,3}$.
The unit vectors of the new coordinates are $\hat{I}^2 = I^2$, and

$$\hat{I}^1 = \begin{bmatrix} \cos\theta \\ 0 \\ -\sin\theta \end{bmatrix}, \quad \hat{I}^3 = \begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix}.$$

A vector $\vec{R}$ in original coordinates must now be written as

$$\hat{\vec{R}} = \begin{bmatrix} \vec{R}\cdot\hat{I}^1 \\ \vec{R}\cdot\hat{I}^2 \\ \vec{R}\cdot\hat{I}^3 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos\theta \end{bmatrix}}_{G_2^\theta} \vec{R}$$

in new coordinates, and we denote the corresponding rotation matrix by G_2^θ.

Rotation Matrices

Finally, rotating the axes about the I^3 axis by θ, we shall find that

$$G_3^\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

For all rotation matrices G, $G^{-1} = G^\top$.
For any G_k^θ $(k = 1, 2, 3)$, $(G_k^\theta)^{-1} = G_k^{-\theta} = (G_k^\theta)^\top$.

Rotating ECI to align with orbit plane

Using the procedure introduced at the end of last lecture, the conversion between ECI and the perifocal coordinate system is as follows.

$$\vec{R}^{\mathrm{peri}} = G_3^{\omega} G_1^{i} G_3^{\Omega} \vec{R}^{\mathrm{ECI}};$$

$$\vec{R}^{\mathrm{ECI}} = (G_3^{\Omega})^{\top} (G_1^{i})^{\top} (G_3^{\omega})^{\top} \vec{R}^{\mathrm{peri}}.$$