Adjusting apoapsis

Suppose we'd like to raise the apoapsis from R_a to $\hat{R}_a$. This corresponds to changing eccentricity from e to $\hat{e}$.
Recall the vis-viva equation

$$\frac{v_p^2}{\mu} = \frac{2}{R_p} - \frac{1}{a}$$

where the periapsis $R_p = a(1-e)$. The speed at periapsis is

$$\begin{aligned} v_p^2 = \mu\left(\frac{2}{a(1-e) - \frac{1}{a}}\right) &= \frac{\mu}{a}\left(\frac{2}{1-e} - \frac{1-e}{1-e}\right) \\ &= \sqrt{\frac{\mu}{a(1-e)}}\sqrt{1+e} = \sqrt{\frac{\mu}{R_p}}\sqrt{1+e}. \end{aligned}$$

In order to change the eccentricity to $\hat{e}$, the velocity at periapsis must be changed to

$$\hat{v}_p = \sqrt{\frac{\mu}{R_p}}\sqrt{1+\hat{e}}.$$

In terms of R_a and $\hat{R}_a$, we have

$$v_p = \sqrt{\frac{\mu}{R_p}}\sqrt{\frac{2R_a}{R_a+R_p}}, \quad \hat{v}_p = \sqrt{\mu}R_p\sqrt{\frac{2\hat{R}_a}{\hat{R}_a+R_p}}.$$

Adjusting periapsis

Suppose we'd like to change the apoapsis from R_p to $\hat{R}_p$. This would again correspond to changing eccentricity from e to $\hat{e}$. We similarly have

$$v_a = \sqrt{\frac{\mu}{R_a}}\sqrt{1-e}$$

After adjusting the periapsis, the velocity at apoapsis must be

$$\hat{v}_a = \sqrt{\frac{\mu}{R_a}}\sqrt{1-\hat{e}}.$$

In terms of R_p and $\hat{R}_p$, we have

$$v_a = \sqrt{\frac{\mu}{R_a}}\sqrt{\frac{2R_p}{R_a+R_p}}, \quad \hat{v}_a = \sqrt{\frac{\mu}{R_a}}\sqrt{\frac{2R_p}{\hat{R}_a+R_p}}.$$

Increasing energy

Suppose initially the orbital energy is

$$\mathcal{E} = \frac{|\vec{v}|^2}{2} - \frac{\mu}{R}.$$

After a (quick) burn, the orbital energy is

$$\mathcal{E} = \frac{|\hat{\vec{v}}|^2}{2} - \frac{\mu}{R}.$$

Writing $\hat{\vec{v}} = \vec{v} + \Delta\vec{v}$, we have

$$\Delta\mathcal{E} = \frac{|\hat{\vec{v}}|^2}{2} - \frac{|\vec{v}|^2}{2} = \frac{1}{2}\left(2v \cdot \Delta v + |\Delta v|^2\right).$$

We see that the energy changes the most if $\Delta\vec{v}$ is aligned with $\vec{v}$.

Example: changing to parabolic orbit

Find Δv at periapsis to maneuver a parabolic orbit.
In current orbit, the velocity at periapsis is

$$v_{\mathrm{peri}} = \sqrt{\frac{\mu(1+e)}{a(1-e)}}.$$

For parabolic trajectory, the velocity at periapsis is

$$0 = \frac{v_{\mathrm{para}}^2}{2} - \frac{\mu}{R_{\mathrm{peri}}} \quad \rightsquigarrow \quad v_{\mathrm{para}} = \sqrt{\frac{2\mu}{R_{\mathrm{peri}}}}.$$

$$\Delta v = v_{\mathrm{para}} - v_{\mathrm{peri}}.$$

Phasing

Consider two spacecrafts are in the same orbit, with periapsis passage time τ^1 and τ^2. Suppose we'd like one to catch the other. As an example, we assume the orbit has period $T = 1.5\,\text{hr}$, and the most recent periapsis passages were $\tau_0^1 = 0\,\text{min}$, $\tau_0^2 = -10\,\text{min}$. Then after k revolutions, $\tau_k^1 = \tau_0^1 + kT$ and $\tau_k^2 = \tau_0^2 + kT$.
By vis-viva equation,

$$\frac{v_p^2}{\mu} = \frac{2}{R_{\text{PERI}}} - \frac{1}{a}.$$

$$\frac{\hat{v}_p^2}{\mu} = \frac{2}{R_{\text{PERI}}} - \frac{1}{\hat{a}}.$$

where we'd like $\hat{T} = T + \tau_k^1 - \tau_k^2$.
$\Delta v \downarrow 0$ if one can wait.