Last Lecture

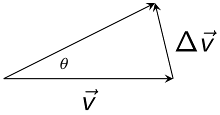
Chemical Propulsion Equations

Maneuvers

- Plane change
- Hohmann transfer
- Bi-elliptic transfer

Plane change

Suppose we'd like to change the inclination of an orbit. The current orbit plane and the new desired orbit plane would intersect at a "hinge line", and the two points where the hinge line intersects the orbits are called the ascending and descending node. The manuever may be done at either node.

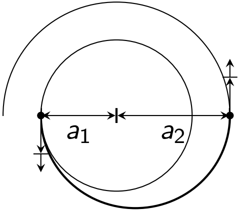


Suppose the velocity at the node is v, then

$$\Delta v = 2v\left|\sin\frac{\theta}{2}\right| \overset{\theta\ \text{small}}{\approx} v\theta.$$

Hohmann transfer

Suppose we want to transfer from a circular orbit with radius a_1 to another orbit with radius a_2.



Hohmann transfer

The initial velocity of the vehicle is $v = \sqrt{\frac{\mu}{a}}$. By vis-viva equation, we shall increase speed such that

$$\frac{v^2}{\mu} = \frac{2}{a_1} - \frac{2}{a_1 + a_2}.$$

Hence for the first burn

$$\Delta v_{\mathrm{peri}} = \sqrt{\frac{\mu}{a_1}} \left(\sqrt{\frac{2a_2}{a_1 + a_2}} - 1 \right).$$

Similarly, for the second burn,

$$\Delta v_{\mathrm{apo}} = \sqrt{\frac{\mu}{a_2}} \left(1 - \sqrt{\frac{2a_1}{a_1 + a_2}} \right).$$

Bi-elliptic transfer

The Bi-elliptic transfer requires less Δv than Hohmann transfer if $a_2 \gg a_1$ (say $a_2 > 15 a_1$).