Last lecture

Geodetic coordinates (conversion from and to ECEF)

ENU coordinates

The East-North-Up coordinate system is a local coordinate system with coordinate axes in the direction of east, north, and (geodetic) upward direction. Suppose the coordinate system is centered at some point $\vec{R}_g^{\mathrm{EF}}$ on the Earth with longitude λ and (geodetic) latitude ϕ, which we refer to as the ground station.
Suppose there is a spacecraft at $\vec{R}^{\mathrm{EF}}$. Let $\vec{R}_\Delta^{\mathrm{EF}} = \vec{R}^{\mathrm{EF}} - \vec{R}_g^{\mathrm{EF}}$. This is the relative position from the ground station to the spacecraft. The conversion from ECEF coordinates to ENU coordinates is as follows:

$$\vec{R}^{\mathrm{ENU}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} G_2^{-\phi} G_3^{\lambda} \vec{R}_\Delta^{\mathrm{EF}}.$$

Ground track

We shall assume that the Earth is spherical for this part.
Consider an orbit with $e = 0, i = 0, \omega = 0, \Omega = 0$. Let $a = 42\,164$ km so that $n = \sqrt{\frac{\mu}{a^3}} = \omega_e$.
Suppose at $t = 0$, the true anomaly $\nu(0) = \nu_0$, then $\nu(t) = E(t) = M(t) = \nu_0 + \omega_e t$. Then

$$\vec{R}^{\mathrm{peri}}(t) = a \begin{pmatrix} \cos \nu(t) \\ \sin \nu(t) \\ 0 \end{pmatrix} = a \begin{pmatrix} \cos(\nu_0 + \omega_e t) \\ \sin(\nu_0 + \omega_e t) \\ 0 \end{pmatrix}.$$

Converting it into ECEF (assuming that ECEF aligned with ECI at $t = 0$), we have

$$\begin{aligned} \vec{R}^{\mathrm{EF}}(t) &= G_3^{\omega_e t} \vec{R}^{\mathrm{ECI}}(t) = G_3^{\omega_e t} [(G_3^{\Omega})^\top (G_1^{i})^\top (G_3^{\omega})^\top \vec{R}^{\mathrm{peri}}(t)] \\ &= G_3^{\omega_e t} \vec{R}^{\mathrm{peri}}(t) = a \begin{pmatrix} \cos \nu_0 \\ \sin \nu_0 \\ 0 \end{pmatrix}. \end{aligned}$$

Satellites in this orbit hover over a point on the equator.

Ground track

We shall assume that the Earth is spherical for this part.
Consider an orbit with $a = 42\,164\,\text{km}, e = 0, i \neq 0, \omega = 0, \Omega = 0$.
We also assume that $\nu(0) = 0$. Then

$$\vec{R}^{\text{peri}}(t) = a \begin{pmatrix} \cos(\omega_e t) \\ \sin(\omega_e t) \\ 0 \end{pmatrix}.$$

We have

$$\begin{aligned}
\vec{R}^{\text{EF}}(t) &= G_3^{\omega_e t} \vec{R}^{\text{ECI}}(t) = G_3^{\omega_e t}[(G_3^{\Omega})^\top (G_1^{i})^\top (G_3^{\omega})^\top \vec{R}^{\text{peri}}(t)] \\
&= G_3^{\omega_e t}(G_1^{i})^\top \vec{R}^{\text{peri}}(t) \\
&= G_3^{\omega_e t} \begin{pmatrix} \cos(\omega_e t) \\ \cos(i)\sin(\omega_e t) \\ \sin(i)\sin(\omega_e t) \end{pmatrix} = \begin{pmatrix} \cos^2(\omega_e t) + \cos i \sin^2(\omega_e t) \\ \cos(\omega_e t)\sin(\omega_e t)[\cos(i) - 1] \\ \sin(i)\sin(\omega_e t) \end{pmatrix}.
\end{aligned}$$

Ground track

We shall assume that the Earth is spherical for this part.
The plot below visualizes the ground track for several inclinations.