Station keeping

Since Earth is an ellipsoid, the gravitational potential Φ around Earth cannot be exactly modelled by that of a point mass. Gravitational potential of Earth is more accurately given by (in spherical coordinates)

$$\Phi(R, \phi, \lambda) \approx -\frac{\mu}{R}\left(1 + \frac{J_2}{2}\left[\frac{3\sin^2(\hat{\phi}) - 1}{2}\right]\right)$$

where $J_2 \approx 0.00108$. These corrections induce periodic and secular perturbations to Keplerian model.

Secular perturbations

The secular perturbation induced by the J_2 term changes the longitude of node and the argument of periapsis over time and affects how mean motion changes (which grows at a constant rate in Keplerian model). In particular,

$$\dot{\Omega} = -kn\cos(i)$$
$$\dot{\omega} = kn\left(2 - \frac{5}{2}\sin^2(i)\right)$$
$$\dot{M} = n + kn\sqrt{1-e^2},$$

where n is the mean motion,

$$k = -\frac{3}{2}\frac{J_2 R_e^2}{a^2(1-e^2)},$$

and R_e is the equatorial radius of the Earth.

Launching

Due to the rotation of Earth, if we launch a spacecraft at latitude ϕ, the spacecraft is already moving at a speed $v = \frac{\omega_e R_e \cos\phi}{\sqrt{1-\tilde{e}^2 \sin^2\phi}}$.

We consider a new coordinate system called the FGH system, whose origin initially (i.e. at launch time) at the launch site, with its I^1 axis is aligned with the launch azimuth, moving at the same velocity as the launch site, which in local ENU coordinates is

$$\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \omega_e \underbrace{\frac{R_e \cos\phi}{\sqrt{1 - \tilde{e}^2 \sin^2\phi}}}_{\hat{R}} .$$

The FGH coordinate system is an inertial reference frame. During launch, the trajectory of the spacecraft lies in the I^1-I^3 plane (x-z plane).

Launching

Denote the launch time by t_e.
At launch time t_e we have $\vec{R}^{\mathrm{FGH}}(t_e) = G_3^{\frac{\pi}{2}-\alpha}\vec{R}^{\mathrm{ENU}}(t_e)$.
At time t,

$$\vec{R}^{\mathrm{ENU}}(t) = (\vec{G}_3^{\frac{\pi}{2}-\alpha})^\top \vec{R}^{\mathrm{FGH}}(t) + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \frac{\omega_e R_e \cos\phi}{\sqrt{1-\tilde{e}^2\sin^2\phi}}(t-t_e)$$

$$\vec{v}^{\mathrm{ENU}}(t) = (\vec{G}_3^{\frac{\pi}{2}-\alpha})^\top \vec{v}^{\mathrm{FGH}}(t) + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \frac{\omega_e R_e \cos\phi}{\sqrt{1-\tilde{e}^2\sin^2\phi}}.$$