

MAE142 Air Vehicle Systems
Assignment 3
Due 9pm, Friday, 15 Nov.

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

Only study problems (if any) will be added to the assignment.

1. (10) Consider an aircraft making the constant-speed, constant-altitude turn discussed in class, or equivalently, Tewari Sec. 4.3. (Any sign differences between the class notes and the book do not affect your answer.) Suppose the (constant) airspeed is 260 m/sec, the wind speed, v_0 , is 30 m/sec and the aircraft lift to weight ratio is held at $L/(mg) = 1.07$, What would the range of angular rates be in radians/sec? What would the required aircraft bank angle be (in radians)? Take the gravitational acceleration to be $g \simeq 9.80 \frac{\text{m}}{\text{sec}^2}$.
2. (5) Consider the simple rocket equation given in class. Suppose the propellant specific impulse is $I_{sp} = 250$ sec, where the propellant exhaust "speed" is $v_e = g I_{sp}$. Assume the initial speed is zero, and that there is no gravitational acceleration(!). Using matlab or python, plot the terminal speed as a function of the ratio of the initial mass to the terminal mass down to the point where the terminal mass is only one percent of the initial. You must show your math, and include your code, for full credit.
3. (5) Suppose we toss a fair coin (probability of heads being 0.5) three times, and that starting from eight dollars, we gain two dollars for each time it comes up heads, and lose two dollars for each time it comes up tails. Consider the random variable, X , given by our funds after the three tosses. By hand, plot the distribution function corresponding to X . Make sure to clearly indicate the value of the distribution function at each discontinuity.

4. (5) Suppose random variable X has density

$$f(x) = \begin{cases} \frac{x}{4} & \text{if } x \in [0, 2] \\ \frac{1}{2} - \frac{x-2}{4} & \text{if } x \in (2, 4] \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that $X \in [1, 5]$? What are the mean, variance and standard deviation of X ?

5. (5) Suppose r.v. X is normal with mean, $m_x = 3$, and variance, $\sigma_x^2 = 4$, i.e., $X \sim \mathcal{N}(3, 4)$. Suppose r.v. $Y \sim \mathcal{N}(2, 9)$. Let r.v. $Z = 7X - 2Y$. What are the mean and covariance of (normal r.v.) Z ?
6. (5) Suppose $X \sim \mathcal{N}(1, 0.25)$. Estimate $\mathbb{E}[\sin(X)]$ by numerical integration. You may restrict the interval to a $3\text{-}\sigma$ region around the mean. You may use matlab or python (including any built-in integrators if you like), but make sure to include the code, even if it is only a few lines. How much does the result change if you extend the region to $4\text{-}\sigma$ around the mean?
7. (5) Suppose $X \sim \mathcal{N}(m_x, C_x)$. Suppose $Y \sim \mathcal{N}(m_y, C_y)$. Let $Z = AX + BY$. Obtain the mean and covariance [matrix] of Z for the case where

$$m_x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C_x = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}, \quad m_y = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad C_y = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & -4 \\ 0 & -4 & 16 \end{pmatrix},$$
$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0.5 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Study Problems (not to hand in)

1. Let Y be a random variable. Let $Z = 5Y + 2$. Show that $P(Z \in (7, 12)) = P(Y \in (1, 2))$. Also, suppose Y has density function $f_y(y)$. Find an expression for the density of Z , say $f_z(z)$.

2. Suppose $X \sim \mathcal{N}(m_x, C_x)$. Suppose $Y \sim \mathcal{N}(m_y, C_y)$. Let

$$m_x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C_x = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}, \quad m_y = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C_y = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}.$$

By hand, sketch a couple level curves for each of the two density functions. (The level curves for normal random variables are ellipses in the two-dimensional case, and ellipsoids in higher dimensional cases.)