MAE142 Air Vehicle Systems
Assignment 2
Due 9pm, Friday, 1 Nov.

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

Complete list of problems.

1. (5) Let the $(\hat{I}^1, \hat{I}^2, \hat{I}^3)$ coordinate frame be obtained from the (I^1, I^2, I^3) coordinate frame by a rotation of $\theta = \pi/3$ radians in the I^1, I^2 plane. Suppose this rotation is instantaneously followed by a rotation of $\omega = \pi/4$ radians in the $\hat{I}^2, \hat{I}^3$ plane, generating the $(\tilde{I}^1, \tilde{I}^2, \tilde{I}^3)$ frame. What is the position of the point $x = (1, 2, 3)'$ in our original frame relative to this $(\tilde{I}^1, \tilde{I}^2, \tilde{I}^3)$ frame?

2. (2) For $e \in \Re^3$ with $|e| = 1$, consider the relations assertions. For all $v \in \Re^3$, $S(e)v = e \times v$, $(I - ee^T)v = [S(e)]^2 v$, $[S(e)]^3 v = S(e)v$ and $e^T S(e) = S(e)e = 0$. Which ones are incorrect?

3. (3) Suppose we have a rotation matrix defined by its Euler axis (say, e) and principal angle (say, ϕ), which we denote as $\bar{G}(e, \phi)$. Recall that $[\bar{G}(e, \phi)]^{-1} = \bar{G}(e, -\phi)$ **Using the expression**

$$\bar{G}(e, \phi) = \cos(\phi)[I - ee^T] + ee^T - \sin(\phi)S(e),$$
$$\text{where } S(e) = \begin{pmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{pmatrix},$$

show that $[\bar{G}(e, \phi)]^T = \bar{G}(e, -\phi)$.

4. (10) Suppose we are at longitude $\lambda = 0$ and latitude $\phi = \pi/6$ radians and elevation (altitude) zero. Suppose the Greenwich meridian is aligned with the first basis vector in our Earth-centered inertial (ECI) system at time $t = 0$. For simplicity, assume the Earth is a sphere of radius 6378.0 km. Obtain our position in the ECI system one hour past $t = 0$.

5. (5) Let the position in Problem 4 be the origin of a local "UEN" (up/east/north) coordinate system. Suppose that our vehicle is at $X^{uen} = (1, 10, 5)^T$ km. What is its position in the ECI system? What is its position in the local "ENU" (east/north/up) system?

6. (5) For the same situation as in Problem 4, letting $\bar{G}$ denote the combined rotation, what are the Euler axis and principal angle?

7. (10) Consider rotation matrix defined by its Euler axis (say, e) and principal angle (say, $\phi(t)$), i.e., $\bar{G}(e, \phi(t))$. Let $e = (1/\sqrt{14})(1, 2, 3)'$, $\phi(0) = 0$ and $\dot{\phi} = 0.01$ radians per second (constant rotation rate). Let the vehicle trajectory in the original frame be given by $x(t) = (2, -4, 6)' + (0.1, 0, -0.2)t$ km for all $t \geq 0$. Let $\bar{t} = 20$ sec. By a simple first-order differencing with $\delta_t = 1 \times 10^{-8}$, obtain an approximate value of the vehicle velocity in the new frame. Compare this with what we obtain from $\frac{d}{dt}\{\bar{G}(e, \phi(t))\}|_{t=\bar{t}}\, x(\bar{t}) + \bar{G}(e, \phi(\bar{t}))\dot{x}(\bar{t})$ (where $\frac{d}{dt}\{\bar{G}(e, \phi(t))\}$ is obtained analytically), $\bar{G}(e, \phi(\bar{t}))[\dot{x}(\bar{t}) - \dot{\phi}(\bar{t})S(e)x(\bar{t})]$ and $\bar{G}(e, \phi(\bar{t}))[\dot{x}(\bar{t}) - \omega \times x(\bar{t})]$, where $\omega = \dot{\phi}\, e$.

Study Problems (not to hand in)

1. Consider rotation matrix defined by its Euler axis (say, e) and principal angle (say, $\phi(t)$), i.e., $\bar{G}(e, \phi(t))$. Let $e = (1/\sqrt{14})(1, 2, 3)'$, $\phi(0) = 0$ and $\dot{\phi} = 0.01$ radians per second (constant rotation rate). Let the vehicle trajectory in the rotated frame be given by $\hat{x}(t) = (3, 7, -5)' + (0.1, 0.3, -0.2)t$ km for all $t \geq 0$. Let $\bar{t} = 20$ sec. Obtain the position in the original frame, $x(\bar{t})$. By a simple first-order differencing with $\delta_t = 1 \times 10^{-8}$, obtain an approximate value of the vehicle velocity in the original frame. Compare this with what we obtain from $[\bar{G}(e, \phi(\bar{t}))]^T[\dot{\hat{x}}(\bar{t}) + \omega \times \hat{x}(\bar{t})]$ and $[\bar{G}(e, \phi(\bar{t}))]^T\dot{\hat{x}}(\bar{t}) + \omega \times x(\bar{t})$, where $\omega = \dot{\phi}\, e$.

2. For the same situation as in Problem 4, what is our current velocity in the ECI system?

3. Consider the situation in Problem 4. Suppose we have a vehicle with position and velocity in the ENU system given by $X^{enu}(\hat{t}) = (10, 0, 5)$

km and $V^{enu}(\hat{t}) = (0.5, 0, 0.3)$ km/sec, where $\hat{t} = 3600$ seconds. What are the position and velocity of the vehicle relative to the ECI system?