MAE142 Air Vehicle Systems
Assignment 1
Due 9pm, Sunday, 20 Oct.

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

Complete list of problems.

1. (5) By hand (with the aid of a calculator etc), obtain the diagonal matrix of eigenvalues (Λ in class) and the matrix of eigenvectors (S in class) for matrix A below. Show your work.

$$A = \begin{pmatrix} 5 & -9 \\ -12 & 2 \end{pmatrix}.$$

2. (5) Using matlab, obtain Λ and S for

$$A = \begin{pmatrix} 1+7i & 6i & -6i \\ 4-4i & 5-3i & -3+3i \\ 4+4i & 4+4i & -2-4i \end{pmatrix}.$$

Based only on these results, indicate whether A is nonsingular, and why. Do the same thing for

$$A = \begin{pmatrix} 1+7i & 6i & -6i \\ -4-4i & -3-3i & 3+3i \\ -4+4i & -4+4i & 4-4i \end{pmatrix}.$$

3. (10) Obtain an analytical (i.e., math - not code) solution to $\dot{\xi}(t) = A\xi(t)$, $\xi(0) = (3,5)^T$ with

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

4. (5) Find an analytical solution to $\dot{\xi}(t) = A\xi(t) + \nu(t)b$, $\xi(0) = (3,5)^T$ with

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \nu(t) = \exp(-2t).$$

5. (8) In the notation of class, find $\tilde{S}$ and $\tilde{\Lambda}$ for matrix

$$A = \begin{pmatrix} 2 & 3 \\ -3 & 8 \end{pmatrix}.$$

6. (7) Find an expression for $\exp(\tilde{\Lambda}t)$ for your $\tilde{\Lambda}$ from Problem 5. Use that and $\tilde{S}$ (from Problem 5) to solve $\dot{\xi}(t) = A\xi(t)$, $\xi(0) = (3,5)^T$ in the case of matrix A from Problem 5.

Study Problems (not to hand in)

1. Consider vector $x = (1,2,3)^T$ in our base coordinate system (i.e., our base frame), with basis vector set (I^1, I^2, I^3). Suppose we have another coordinate system with the same origin, but with basis $(\hat{I}^1, \hat{I}^2, \hat{I}^3)$, where the latter is obtained by a right-hand rotation by angle $\theta = \pi/3$ radians in the I^1, I^2 plane. Suppose the vector denoted by x relative to the original system, is denoted by $\hat{x}$ relative to the new system. What is $\hat{x}$? Do the same thing, for the cases where the rotation is in the I^2, I^3 plane and where it is in the I^3, I^1 plane.

2. Let the $(\hat{I}^1, \hat{I}^2, \hat{I}^3)$ coordinate frame be obtained from the (I^1, I^2, I^3) coordinate frame by a rotation of $\theta = 2\pi/3$ radians in the I^1, I^2 plane. Suppose this rotation is instantaneously followed by a rotation of $\omega = -\pi/4$ radians in the $\hat{I}^2, \hat{I}^3$ plane, generating the $(\tilde{I}^1, \tilde{I}^2, \tilde{I}^3)$ frame. What is the position of the point $x = (1,2,3)'$ in our original frame relative to this $(\tilde{I}^1, \tilde{I}^2, \tilde{I}^3)$ frame?

3. Consider the compound rotation of Study Problem 2. For general angles θ and ω there, obtain the expression for the compound rotation. Also obtain an expression for the inverse operation, taking us from $(\tilde{I}^1, \tilde{I}^2, \tilde{I}^3)$ back to (I^1, I^2, I^3).

4. In the case of the compound rotation of Study Problem 2, what are the principle axis and corresponding angle of rotation? (It is easiest to obtain these through the aid of matlab or other software.)