6-30 Show that the RC OP AMP circuit in Figure P6-30 is a noninverting integrator whose input-output relationship is

\[ v_o(t) = \frac{1}{RC} \int_0^t v_s(x) \, dx + v_o(0) \]

\[ v_o(t) = \frac{1}{RC} \int_0^t v_s(x) \, dx + v_o(0) \]

6-31 Design an RC OP AMP circuit to implement the block diagram in Figure P6-31.

6-32 Repeat problem 6-31 except use an RL OP AMP circuit.

6-33 Design an RC OP AMP circuit to implement the input-output relationship

\[ v_o(t) = -10v_s(t) + 50 \int_0^t v_s(x) \, dx \]

6-34 Design an OP AMP circuit to solve the following differential equation:

\[ v_o(t) = 5v_s(t) + \frac{1}{20} \frac{dv_s(t)}{dt} \]

6-35 Using only one OP AMP and one capacitor, design an RC circuit that implements the input-output relationship

\[ v_o(t) = -5 \int_0^t v_s1(x) \, dx - 10 \int_0^t v_s2(x) \, dx \]

**Objective 6-3 Equivalent Inductance and Capacitance (Sect. 6-4)**

(a) Derive equivalence properties of inductors and capacitors or use equivalence properties to simplify LC circuits.

(b) Solve for currents and voltages in RLC circuits with dc input signals.

See Examples 6-14, 6-15 and Exercises 6-12, 6-13, 6-14
6-36 Find a single equivalent element for each circuit in Figure P6-36.

![Image of the circuit with capacitors and an inductor](image1)

**FIGURE P6-36**

6-37 A 100-μH inductor is connected in series with a 1-mH inductor and the combination connected in parallel with a 2-mH inductor. Find the equivalent inductance of the connection.

6-38 Verify Eqs. (6-30) and (6-31).

6-39 What are the equivalent capacitance and initial voltage of a series connection of a 1-μF capacitor with 10 V stored and a 3.3-μF capacitor with 15 V stored?

6-40 For the circuit in Figure P6-40, find an equivalent circuit consisting of one inductor and one capacitor.

![Image of the circuit with an inductor and capacitors](image2)

**FIGURE P6-40**

6-41 Figure P6-41 is the equivalent circuit of a two-wire feed-through capacitor.

(a) What is the capacitance between terminal 1 and ground when terminal 2 is open?
(b) What is the capacitance between terminal 1 and ground when terminal 2 is grounded?

![Image of the two-wire feed-through capacitor circuit](image3)

**FIGURE P6-41**

6-42 A capacitor bank is required that can be charged to 500 kV and store at least 250 J of energy. Design a series/parallel combination that meets the voltage and energy requirements using 20-μF capacitors each rated at 2 kV max.

6-43 A switching power supply requires an inductor that can store at least 1 mJ of energy. A list of available inductors is shown below. Select the inductor that best meets the requirement. Consider both how well it meets the specifications and cost. Explain your choice.

<table>
<thead>
<tr>
<th>L(μH)</th>
<th>Imax(A)</th>
<th>Cost (each)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9.3</td>
<td>$2.75</td>
</tr>
<tr>
<td>20</td>
<td>7.2</td>
<td>$3.00</td>
</tr>
<tr>
<td>50</td>
<td>5.5</td>
<td>$2.50</td>
</tr>
<tr>
<td>100</td>
<td>4.5</td>
<td>$2.75</td>
</tr>
<tr>
<td>150</td>
<td>3.5</td>
<td>$2.50</td>
</tr>
<tr>
<td>250</td>
<td>2.6</td>
<td>$2.75</td>
</tr>
<tr>
<td>500</td>
<td>1.8</td>
<td>$3.00</td>
</tr>
</tbody>
</table>

6-44 The circuits in Figure P6-44 are driven by dc sources. Find the current through the 100-Ω resistor under dc conditions.

![Image of the circuit with a capacitor and resistor](image4)

**FIGURE P6-44**

6-45 The circuit in Figure P6-45 is driven by a 15-V dc source. Find the energy stored in the capacitor and inductor under dc conditions.

![Image of the circuit with a capacitor, inductor, and resistor](image5)

**FIGURE P6-45**

6-48 A. L.C. circuit: $L = \frac{1}{	ext{At}}$ is the voltage at time $t$.

(a) Use the initial conditions $v(t_1) = 0$, $i(t_1) = 0$ for $t < 0$. Use MATLAB to determine $v(t)$ and $i(t)$ for $t > 0$.
The OP AMP circuit in Figure P6-46 has a capacitor in its feedback loop. Determine the circuit gain at dc and as the frequency approaches \( \infty \) \text{rad/s}.

\[ \text{FIGURE P6-46} \]

**6-49** Supercapacitor

Supercapacitors have very large capacitances (typically from 0.1 to 50 F), very long charge holding times, and small sizes, making them useful in non-battery backup power applications. To measure its capacitance, a supercapacitor is charged to an initial voltage \( V_0 = 5.5 \text{ V} \). At \( t = 0 \) the device undergoes a constant current discharge of \( i_0 = 2 \text{ mA} \). At \( t = 2500 \text{ s} \) the voltage remaining on the capacitor is 3 V. Find the device capacitance.

\[ \text{FIGURE P6-48} \]

**6-50** Analog Computer Solution

Design an OP AMP circuit that solves the following second-order differential equation for \( v_0(t) \). Solve for the response for \( v_0(t) \) using OrCAD. Cautions: Avoid saturating the OP AMPs by distributing the gain across several OP AMPs.

\[ 10^{-6} \frac{d^2 v_0(t)}{dt^2} + 1 \times \frac{d v_0(t)}{dt} + v_0(t) = 2u(t) \]

**6-51** RC OP AMP Circuit Design

An upgrade to one of your company’s robotics products requires a proportional plus integral compensator that implements the input-output relationship

\[ v_0(t) = v_s(t) + 50 \int_0^t v_s(x)dx \]

The input voltage \( v_s(t) \) comes from an OP AMP, and the output voltage \( v_0(t) \) drives a 10-k\( \Omega \) resistive load. Two competing designs are shown in Figure P6-51. As the project engineer, you are responsible for recommending one of these designs for production. Which design would you recommend and why?

(Your mentor, a wise senior engineer, suggests that you first check that both designs implement the required signal-processing function.)
SUMMARY

- Circuits containing linear resistors and the equivalent of one capacitor or one inductor are described by first-order differential equations in which the unknown is the circuit state variable.
- The zero-input response in a first-order circuit is an exponential whose time constant depends on circuit parameters. The amplitude of the exponential is equal to the initial value of the state variable.
- For linear circuits the total response is the sum of the forced and natural responses. The natural response is the general solution of the homogeneous differential equation obtained by setting the input to zero. The forced response is a particular solution of the differential equation for the given input.
- For linear circuits the total response is the sum of the zero-input and zero-state responses. The zero-input response is caused by the initial energy stored in capacitors or inductors. The zero-state response results from the input driving forces.
- The initial and final values of the step response of a first- and second-order circuit can be found by replacing capacitors by open circuits and inductors by short circuits and then using resistance circuit analysis methods.
- The transient response to a first-order circuit when the input is other than a step requires that the forced response solution be of the same form as the input. Hence, an exponential input suggests an exponential forced response; a sinusoidal input suggests a sinusoidal forced response, and so forth.
- For a sinusoidal input the forced response is called the sinusoidal steady-state response, or the ac response. The ac response is a sinusoid with the same frequency as the input but with a different amplitude and phase angle. The ac response can be found from the circuit differential equation using the method of undetermined coefficients.
- Circuits containing linear resistors and the equivalent of two energy storage elements are described by second-order differential equations in which the dependent variable is one of the state variables. The initial conditions are the values of the two variables at \( t = 0 \).
- The zero-input response of a second-order circuit takes different forms depending on the roots of the characteristic equation. Unequal real roots produce the overdamped response, equal real roots produce the critically damped response, and complex conjugate roots produce underdamped responses.
- The circuit damping ratio \( \zeta \) and undamped natural frequency \( \omega_0 \) determine the form of the zero-input natural responses of any second-order circuit. The response is overdamped if \( \zeta > 1 \), critically damped if \( \zeta = 1 \), and underdamped if \( \zeta < 1 \). Active circuits produce undamped \(( \zeta = 0 \) and unstable \(( \zeta < 0 \)) responses.
- Computer-aided circuit analysis programs can generate numerical solutions for circuit transient responses. Some knowledge of analytical methods and an estimate of the general form of the expected response is necessary to use these analysis tools.

PROBLEMS

OBJECTIVE 7–1 FIRST-ORDER CIRCUIT ANALYSIS (SECTS. 7–1, 7–2, 7–3, 7–4)

Given a first-order RC or RL circuit:
(a) Find the circuit differential equation, the circuit characteristic equation, the circuit time constant, and the initial conditions (if not given).
(b) Find the zero-input response.
(c) Find the complete response for step function, exponential, and sinusoidal inputs.
See Examples 7–1, 7–2, 7–3, 7–4, 7–5, 7–6, 7–7, 7–8, 7–9, 7–10, 7–12, 7–13, 7–14, 7–15, 7–16, 7–17, 7–18, 7–19

7–1 Find the function \( i(t) \) that satisfies the following differential equation and initial condition:

\[
500 \frac{di(t)}{dt} + 2500i(t) = 0, \quad i(0) = 10 \text{ mA}
\]

7–2 Find the function \( v(t) \) that satisfies the following differential equation and initial condition:

\[
10^{-2} \frac{dv(t)}{dt} + v(t) = 0, \quad v(0) = 100 \text{ V}
\]

7–3 Find the time constants of the circuits in Figure P7–1.
This page contains problems related to circuits. The problems involve analyzing and solving for various circuit responses, including finding time constants, analyzing on-off switches, and solving for voltages and currents in different circuit configurations.

**Problem 7-7**
The switch in Figure P7-7 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_C(t)$ for $t \geq 0$. Repeat the problem for the switch being in position B for a long time and moving to position A at $t = 0$.

**Problem 7-8**
The switch in Figure P7-8 has been open for a long time and is closed at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

**Problem 7-9**
The circuit in Figure P7-9 is in the zero state. (a) Find the voltage $v_C(t)$ for $t \geq 0$ when an input of $i_S(t) = I_A e^{-\alpha t} A$ is applied. Identify the forced and natural components in the output. (b) Repeat when the input is $i_S(t) = I_A e^{-\alpha t} A$.

**Problem 7-10**
The circuit in Figure P7-10 is in the zero state when the input $v_S(t) = V_A u(t) V$ is applied. Find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output.
7-11 The circuit in Figure P7-11 is in the zero state when the input $v_S(t) = 10 \mu V$ is applied. If $C = 0.1 \mu F$ and $R = 10 \Omega$, find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output.

![Figure P7-11](image)

7-12 The circuit in Figure P7-12 is in the zero state when the input $v_S(t) = 60 \mu V$ is applied. If $L = 100 \mH$ and $R = 100 \Omega$, find $v_O(t)$ for $t \geq 0$. Identify the forced and natural components in the output. On a single set of axes, use MATLAB to plot the forced response, the natural response, and the complete response.

![Figure P7-12](image)

7-13 The switch in Figure P7-13 has been in position A for a long time and is moved to position B at $t = 0$. Find $v_C(t)$ for $t \geq 0$. Identify the forced and natural components in the response.

![Figure P7-13](image)

7-17 The switch in Figure P7-17 has been open long enough for $i_S(0)$ to reach 0 A and is closed at $t = 0$.
(a) Use OrCAD to find $v_S(t)$ for $t \geq 0$ if the input is $v_S(t) = 10e^{-100t} V$.
(b) Use OrCAD to find $v_S(t)$ for $t \geq 0$ if the input is $v_S(t) = 10 \cos(100t) V$.

![Figure P7-17](image)

7-18 The switch in Figure P7-18 has been in position A for a long time and is moved to position B at $t = 0$. Find $i_S(t)$ for $t \geq 0$ and use MATLAB to plot the waveform.

![Figure P7-18](image)

7-20 Switches 1 and 2 in Figure P7-20 have both been in position A for a long time. Switch 1 is moved to position B at $t = 0$ and Switch 2 is moved to position B at $t = 20$ ms. Find the voltage across the 0.1-$\mu F$ capacitor for $t \geq 0$ and sketch its waveform.

![Figure P7-19](image)