1. \( f(x) = xe^x \), \( x_0 = 0 \) \( \Rightarrow \) \( f(x_0) = 0 \). \( f'(x) = e^x + xe^x \) \( \Rightarrow \) \( f'(x_0) = 1 \). \( f''(x) = 2e^x + xe^x \) \( \Rightarrow \) \( f''(x_0) = 2 \). The Taylor polynomials are \( p_0(x) = f(x_0) = 0 \), \( p_1(x) = f_0(x) + f'(x_0)(x - x_0) = x \), \( p_2(x) = p_1(x) + (f''(x_0)/2)(x-x_0)^2 = x + x^2 \).

2. \( f(x) = e^x \), \( x \in [0, 1] \). We must choose \( x_0 = 1/2 \) in order to minimize the upper bound on the error for all \( x \in [0, 1] \). We have \( R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1} \) with \( \xi \in (x_0, x) \). So \( |R_n(x)| = \frac{e^\xi}{(n+1)!}|x-1/2|^{n+1} \leq \frac{e(1/2)^{n+1}}{(n+1)!} \leq 0.1 \) \( \Rightarrow \) \( (n+1)! \geq 10e(1/2)^{n+1} \approx 27(1/2)^{n+1} \).

\[
\begin{array}{|c|c|c|}
\hline
n & (n+1)! & 27(1/2)^{n+1} \\
\hline
0 & 1 & 13.5 \\
1 & 2 & 6.75 \\
2 & 6 & 3.375 \\
\hline
\end{array}
\]

Therefore, \( n \geq 2 \) and \( P_2(x) = \sqrt{e}\left(1 + (x - 1/2) + \frac{1}{2}(x - 1/2)^2\right) \) and the bound on the error is \( e(1/2)^{2+1}/3! = e/48 \).

3. \( f(x) = x^6 + 3x^4 - 2x^2 \). \( |f(x)| = |x^6 + 3x^4 - 2x^2| \leq |x^6| + 3|x^4| + 2|x^2| \). For \( 0 < x < 1 = \delta \) we have \( x^6 < x^4 < x^2 \) so \( |f(x)| \leq |x^2| + 3|x^2| + 2|x^2| = 6|x^2| = C|\beta(x)| \), \( C = 6 \) and \( \beta(x) = x^2 \) and therefore \( f(x) = O(x^2) \).

4. \( y = 3 + 2x + 4x^2 + 5x^3 - 7x^4 = 3 + x(2 + 4x + 5x^2 - 7x^3) = 3 + x(2 + x(4 + 5x - 7x^2)) = 3 + x(2 + x(4 + x(5 - 7x))) \). the number of additions and multiplications required is 4 and 4 respectively. The pseudo code should look like
5. \( f(x), \ x_0 = 0, \ f'(x) = e^x, \ h = 1/2. \)

\[
D_f^f(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} = \frac{e^{1/2} - e^0}{1/2} \approx 1.7 - \frac{1}{2} = 1.4
\]

The error is \( e_h = |f'(x_0) - D_f^f(x_0)| = |1 - 1.4| = 0.4. \)

\[
\hat{D}_h^f(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h} = \frac{e^{1/2} - e^{-1/2}}{2 \cdot 1/2} \approx \frac{1.7 - 0.6}{1} = 1.1
\]

The error is \( \hat{e}_h = |f'(x_0) - \hat{D}_h^f(x_0)| = |1 - 1.1| = 0.1. \)

From Taylor’s theorem \( e_h = |f'(x_0) - D_h^f(x_0)| = |(h/2)f''(\xi)| \) where \( \xi \in (x_0, x_0 + h) \) so \( e_h \leq (h/2)f''(x_0 + h) = (1/4)\sqrt{e} = 0.425 \) (since \( f''(x) = e^x \) is increasing).