You must show all your work in order to get credit. Also, you must hand in your matlab codes along with the results. These codes must be commented in accordance with the examples appearing on the website.

Problems to hand in (Not all problems may be graded.)

1. By hand, apply Euler’s method with $n = 3$ to solve $\dot{x} = 2x$ over $t \in [0, 1.5]$ with initial condition $x(0) = 0.5$.

2. Write matlab code for Euler’s method. Apply your code to solve $\dot{x} = 2x$ over $t \in [0, 1.5]$ with initial condition $x(0) = 0.5$. Do this for $n = 10, 10^2, 10^3, 10^4$ and $10^5$ steps. Also obtain the true solution analytically. Plot the true solution and your approximate solutions for the various $n$ values all on the same graph. Compute the error at the final time for each of the approximations. On a separate graph, plot $\log_{10}$ of the error versus $\log_{10}$ of the step size. Discuss the slope of this plot.

3. Apply the above codes to the problem of solving the IVP,

\[
\dot{x} = [0.5 + \cos(t)][0.5\sin(\arctan(x))] + x],
\]

\[x(0) = 1,
\]

over $t \in [0, 3]$. In this case, solve the problem with $n = 5, 50, 5 \times 10^2, 5 \times 10^3, 5 \times 10^4$ and $5 \times 10^5$ steps. Proceed as in the above problem, but in the error computations, instead of using the analytical solution as the true solution, treat the approximate solution with $5 \times 10^5$ steps as the true solution. (That is, the errors with $5 \times 10^5$ are computed by comparison with the case of $5 \times 10^5$.)

4. Suppose you would use Euler’s method to solve an initial value problem. At step two, you employ the approximate state propagation $x_2 = x_1 + f(t_1, x_1)h$, where $h$ is the step size. What are the sources of error in each of the two terms on the right-hand side of this expression. What is the additional source of error?
5. Obtain a linear interpolation for \( f(x) = \cos(x) - x/\pi \) over the interval from \( x = 0 \) to \( x = \pi/2 \). What is the bound on the error according to the formula from class (also given as the rightmost bound in the displayed math of Theorem 2.1 from the section)? What are the actual errors at \( x = \pi/6 \), \( x = \pi/4 \) and \( x = \pi/3 \)? Is the error bound satisfied at these points?

6. Now obtain a piecewise linear interpolation for the same function over the same total interval (from \( x = 0 \) to \( x = \pi/2 \)). Use exactly two linear segments, each of length \( \pi/4 \). What are the bounds on the error over each segment according to the formula from class (also given as the rightmost bound in the displayed math of Theorem 2.1 from the section)? What are the actual errors at \( x = \pi/6 \), \( x = \pi/4 \) and \( x = \pi/3 \)?

Problems 2 and 3 are each worth 10 points. The other problems are worth 5 points each.