1. Midpoint rule with \( n = 3 \) for \( \int_0^3 \exp(x + x^2/2)dx \)

\[
h = (3 - 0)/3 = 1
\]
\[
x_0 = 0
\]
\[
x_1 = 1
\]
\[
x_2 = 2
\]
\[
x_3 = 3
\]

Find the midpoints:

\[
m_1 = (x_0 + x_1)/2 = 0.5
\]
\[
m_2 = m_1 + h = 0.5 + 1 = 1.5
\]
\[
m_3 = m_2 + h = 1.5 + 1 = 2.5
\]

Evaluate \( \exp(x + x^2/2) \) at the midpoints:

\[
\exp(m_1 + m_2^2/2) = 1.86825
\]
\[
\exp(m_2 + m_2^2/2) = 13.80457
\]
\[
\exp(m_3 + m_3^2/2) = 277.27228
\]

Approximate the integral:

\[
\int_0^3 \exp(x + x^2/2)dx \approx h \cdot (1.86825 + 13.80457 + 277.27228)
\]
\[
= 292.9451
\]

Error analysis: Find the minimal number of rectangles \( n \) such that the error is less than \( 10^{-3} \).

\[
e_n^{mid} \leq \max_{\xi \in [a,b]} |f''(\xi)|(b - a)^3/(24n^2)
\]

Substitute in the equation \( a = 0, b = 3 \), we have

\[
n \geq \sqrt{\max_{\xi \in [0,3]} |f''(\xi)|(3 - 0)^3/(24 \times 10^{-3})}
\]

Because \( f''(x) = (2 + 2x + x^2) \exp(x + x^2/2) \) is monotonically increasing on \([0, 3]\), the maximum is attained at \( \xi = 3 \). Therefore,

\[
n \geq \sqrt{|f''(3)|(3 - 0)^3/(24 \times 10^{-3})} = 5881
\]

\( \Rightarrow \) The minimal \( n \) such that the error is below the required bound is \( n = 5881 \).
2. Trapezoid rule with \( h = 1/4 \) for \( \int_1^2 e^{-x^2} \, dx \)

\[
\begin{align*}
n &= (2 - 1)/h = 4 \\
x_0 &= 1 \\
x_1 &= 5/4 \\
x_2 &= 3/2 \\
x_3 &= 7/4 \\
x_4 &= 2
\end{align*}
\]

Evaluate \( e^{-x^2} \) at each node:

\[
\begin{align*}
e^{-x_0^2} &= 0.3679 \\
e^{-x_1^2} &= 0.2096 \\
e^{-x_2^2} &= 0.1054 \\
e^{-x_3^2} &= 0.0468 \\
e^{-x_4^2} &= 0.0183
\end{align*}
\]

Trapezoid rule approximation:

\[
\int_1^2 e^{-x^2} \, dx \approx \frac{h}{2} \cdot (0.3679 + 2 \cdot 0.2096 + 2 \cdot 0.1054 + 2 \cdot 0.0468 + 0.0183)
\]

\[= 0.1387\]

Corrected trapezoid rule approximation:

\[
\int_1^2 e^{-x^2} \, dx \approx 0.1387 - \frac{h}{24} \cdot (3 \cdot e^{-x_4^2} - 4 \cdot e^{-x_3^2} + e^{-x_2^2} + 3 \cdot e^{-x_1^2} - 4 \cdot e^{-x_0^2} + e^{-x_0^2})
\]

\[= 0.1351\]