MAE 107
Assignment 8
Due Tuesday, 28 Nov., 2017

Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

1. Consider attempting to solve
   \[ \arctan\left(\frac{x}{2} + \frac{x^3}{4}\right) = 0.5 \]
   via Newton’s method. Convert this problem into two different root
   finding problems, such that for the first one, Newton’s method con-
   verges when starting from \( x_0 = 2.5\pi \), while for the second one, New-
   ton’s method does not converge when starting from \( x_0 = 2.5\pi \). You
   may do this by writing matlab code, or by using a calculator. In either
   case indicate all the iterates (i.e., the \( x_k \) and \( f(x_k) \) values for each \( k \)).
   You need to run the methods for enough steps so that the convergence
   of lack thereof is obvious, say four to ten iterations depending on how
   your method is doing. In the case where the method converges, dis-
   cuss the errors in the \( x_k \) iterates in the context of the digit-doubling
   concept.

2. Apply Newton’s method for \( n > 1 \) dimensions to find an approximate
   solution of the ("simultaneous") pair of equations \( x^2 + y^2 = 7/4 \)
   and \( 3y = x^2 \). Start from \( (x, y) = (2, 1) \). (You might note that there are two
   solutions given by \( (x, y) = (\sqrt{3/2}, 1/2) \) and \( (x, y) = (-\sqrt{3/2}, 1/2) \).
   In particular, write a matlab function to perform the iteration. You may
   choose to stop based on the number of iterations, or on a more complex
   condition, but the solution approximation should be reasonably close
   to one of the true solutions. (For this simple problem, one could use
   substitution to convert the pair into a single equation, but that is not
   what is being requested here. The system of equations is simple so as
   to lower your workload and chance of error.)

3. Suppose you will be using the fixed point method to solve \( x = g(x, p) \)
   for \( x \), where \( p \) will be a given parameter. Suppose
   \[ g(x, p) = p\left[\frac{\pi}{2}x - \arctan(x)\right] - \frac{3}{2}. \]
Using the analysis from class (or the textbook), rather than by exhaustively running a fixed-point code, determine for what range of \( p \) values you could expect the code to converge from any starting \( x_0 \).

4. Employ the secant method to solve \( \arctan(x/2) + x = -1 \), starting with \( x_0 = 2 \) and \( x_1 = 4 \), continuing on to obtain \( x_7 \) and \( f(x_7) \). Draw a graph illustrating how \( x_2 \) was obtained from \( x_0 \) and \( x_1 \) and, similarly, \( x_3 \) from \( x_1 \) and \( x_2 \). Also discuss the rate of convergence in the light of the digit-doubling aspect of Newton’s method. (You may do the computations with the aid of a calculator or matlab, but make sure to indicate the values you obtain at each step.)

5. Write a matlab function which performs the fixed-point iteration for solution of \( n \) equations in \( n \) unknowns, where \( n \) may be greater than one. This function should call another function that evaluates the right-hand sides of the equations, which must be in the correct form for the fixed-point method. Apply your code to find a solution of the set of equations

\[
x_1 = \exp[-(x_1^2 + x_2^2)/3] + 1 \quad \text{and} \quad [\sin(x_1) + \cos(x_2)]/4 - x_2 = 0.
\]

starting from \( \vec{x}^0 = (x_1^0, x_2^0)^T = (2, -4)^T \), and continuing up to \( \vec{x}^{11} \).

6. Suppose you are running the fixed point method to solve \( x = g(x) \), where you know \( |g'(x)| \leq 0.5 \) for all \( x \). Starting with \( x_0 = 1 \), you obtain \( x_1 = 2.2 \). What is the minimum \( n \) such that you can guarantee the error in \( x_n \) will be less than \( 10^{-5} \).

7. Use Lagrange interpolation to find the second-order polynomial through \( (x_0, y_0) = (2, 2), (x_1, y_1) = (3, 2), (x_2, y_2) = (5, 1) \). Reduce your solution to the form \( y = c_0 + c_1 x + c_2 x^2 \).

8. Use Lagrange interpolation to find the second-order polynomial through \( (x_0, y_0) = (2, \pi), (x_1, y_1) = (3, \pi/2), (x_2, y_2) = (5, 2\pi) \). Reduce your solution to the form \( y = c_0 + c_1 x + c_2 x^2 \). (You may re-use computations from the previous problem.)

Problems 1–4 are worth 10 points each. Problems 5–8 are worth 5 points each.