Note: You must show all your work in order to get credit!

Problems to hand in (Not all problems may be graded.)

Notes:

• For the asymptotic-order problems below, you need to show why the order is what you claim it to be. Simply stating the order is insufficient for full credit.

• Also, keep in mind that for all of the asymptotic-order problems, when we ask for the order, we want the tightest order. (Unless otherwise requested, you do not need to show that the asserted order is indeed the tightest.)

1. The Taylor polynomials of order 4 around $x_0 = -1$ for $f(x)$ and $g(x)$ are $p_f^4(x) = (3/2)(x+1) - 3(x+1)^3$ and $p_g^4(x) = 8 + 2(x+1)^2 + (x+1)^4$, respectively. What is the Taylor polynomial of order 4 around $x_0 = -1$ for $h(x) = 2f(x) - 3g(x)$?

2. What is the asymptotic order of $f(x) = 3x + 11x^2 - 2x^4$ as $x \downarrow 0$? Show this using the definition given in class, or similarly equation (1.8) from the textbook. What are $C$, $\beta(x)$ and $\delta$? (Recall $f(x) = O(\beta(x))$ as $x \downarrow 0$ if there exist $C < \infty$ and $\delta > 0$ such that $|f(x)| \leq C\beta(x)$ for all $x \in (0, \delta)$.) Draw a figure indicating the function and the asymptotic envelope $\pm C\beta(x)$, including $\delta$.

3. Show that the asymptotic order of $f(x) = 5x^3$ as $x \downarrow 0$ is not $O(x^4)$. In particular, use an argument regarding $C$ and $\delta$.

4. What is the asymptotic order as $x \downarrow 0$ of $g(x) = \tan(x^2)$? Show this, using an appropriate Taylor polynomial and remainder. (In any Taylor remainder expression, you might want to assume $x \in (0, \pi/4)$, or a similar interval with upper bound strictly less than $\pi/2$, in which case the secant function stays bounded.)
5. What is the asymptotic order as $x \to \infty$ of $f(x) = \frac{3}{x^2} - \frac{5}{x^4}$? What are $C$, $\beta(x)$ and $R$? (Recall that we require $|f(x)| \leq C\beta(x)$ for all $x \geq R$.) Draw a figure indicating the function and the asymptotic envelope $\pm C\beta(x)$, including $R$.

6. What is the asymptotic order as $x \to \infty$ of $f(x) = \frac{7}{x^2+x^3} + \frac{3}{x^4+x^5}$?

7. Suppose $y(x) = O(x^5)$ as $x \downarrow 0$. What is the order of $f(x) = 3x^3y(x)$ as $x \downarrow 0$? Show your work.

The first and last problems are worth 5 points each. The other problems are worth 10 points each.