Deviations from the Linear-Quadratic-Gaussian Problem with Nested Information Patterns

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LQG Paradigm

- State vector propagated by linear dynamics and measurements are linear function of the state.
- Additive Gaussian noise.
  - Gauss-Markov process.
- Perfect memory of the measurement sequence.
  - Nested information pattern.
- Minimization (or minimax) of the expected value of a sum (or integral) of quadratic functions of the state and control (H_2).
  - Expected value of the exponential of that sum (H_∞).
- Any changes from this paradigm induces significant complexity.

Nonlinear System

- Controller becomes a function of the pdf conditioned on the measurement sequence.
  - Not currently implementable.
- Simplifications occur by worst case design, i.e., by large deviation theory or nonlinear H_∞, the HJI equation results.
  - Develops dissipative inequalities from quadratic storage functions.
  - Conservative approach.
  - Some effort found in *stochastic* nonlinear H_∞.
  - Not currently implementable.
- Deterministic minimax differential games.
  - Solution space dominated by singular surfaces (Isaac).
    - HJI solution are difficult to obtain.
  - Additive stochastic input tends to dissipate these surfaces, but the 2^{nd}-order HJI is challenging.

Non-Nested Information Patterns

- For a very simple LQG problem (2-stages), no analytic solution is available.
  - Witsenhausen counter example.
- For non-nested information patterns, the standard stochastic Hamilton-Jacobi-Bellman equation is no longer applicable.
- LQG Games with different information patterns.
  - Each strategy has an infinite dimensional kernel operating on its measurement sequence.
    - Seems to produces a saddle point solution.
    - Formal solution given by Warren Willman.
  - Is there a finite dimensional representation for the strategies?

Non-Quadratic Performance Criterion

- Minimum time to the origin.
  - Requires bounded control and terminal constraints.
    - Produces singular surfaces, where optimal return function is not differentiable.
    - The Bushaw problem.
- Stochastic problems with deterministic path inequality constraints or terminal constraints (fuel) is still challenging.

Non-Gaussian Noise Models

- For non-Gaussian additive noise, the pdf conditioned on the measurement sequence is infinite dimensional.
  - Minimum variance linear estimator based on the first two moments.
- Controller is optimal for state dependent noise with perfect state information.
  - No noisy partial information results available.
- Recent work in propagating the conditional pdf for impulsive measurement and process *Cauchy* noise through a scalar linear system.
  - Conditional mean and variance exist and are both functions of the measurements.

Control in the Presences of Structural Disruptions

- System disruptions, such as component or element failures, induce large structural changes in the dynamics and/or measurements.
- Develop techniques that, with a given false and miss alarm probability, detects and isolates faults in minimum time.
- Develop fault sensitive controllers, where the system may switch to any of m systems at a random time with a delay in detecting the change.

Adversarial Encounters Under Uncertainty

- Many on many encounters
  - First problem: 2 on 2
    - Special dynamics might give insights.
    - Decomposition of continuous dynamics into primitives of the motion.
  - Stochastic team on team
    - The team share information among its members, but not the other team.
    - Possible approach is to assume worst case design.
      - Not clear how to define noise inputs when considering them as adversarial or cooperative players.
      - Conservative design.
  - There are few results in this area, since it suffers from the same issues in the deviation from the LQG and the additional complexity of team on team interactions.