Sensor Attack Avoidance and Asset Protection: Risk-Averse and Linear-Quadratic Multi-Agent Differential Game Approach

AFOSR Workshop on Adversarial and Stochastic Elements in Autonomous Control

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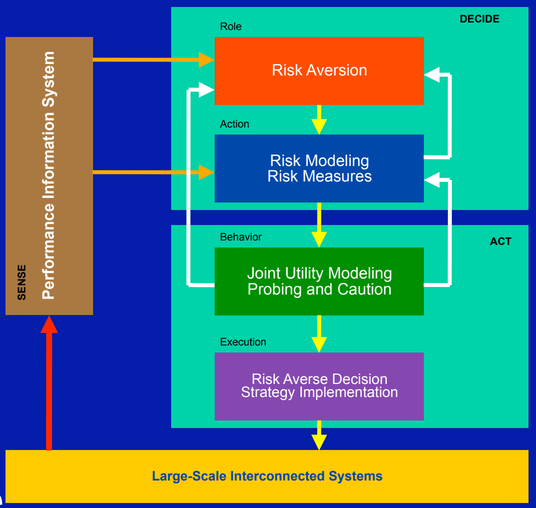
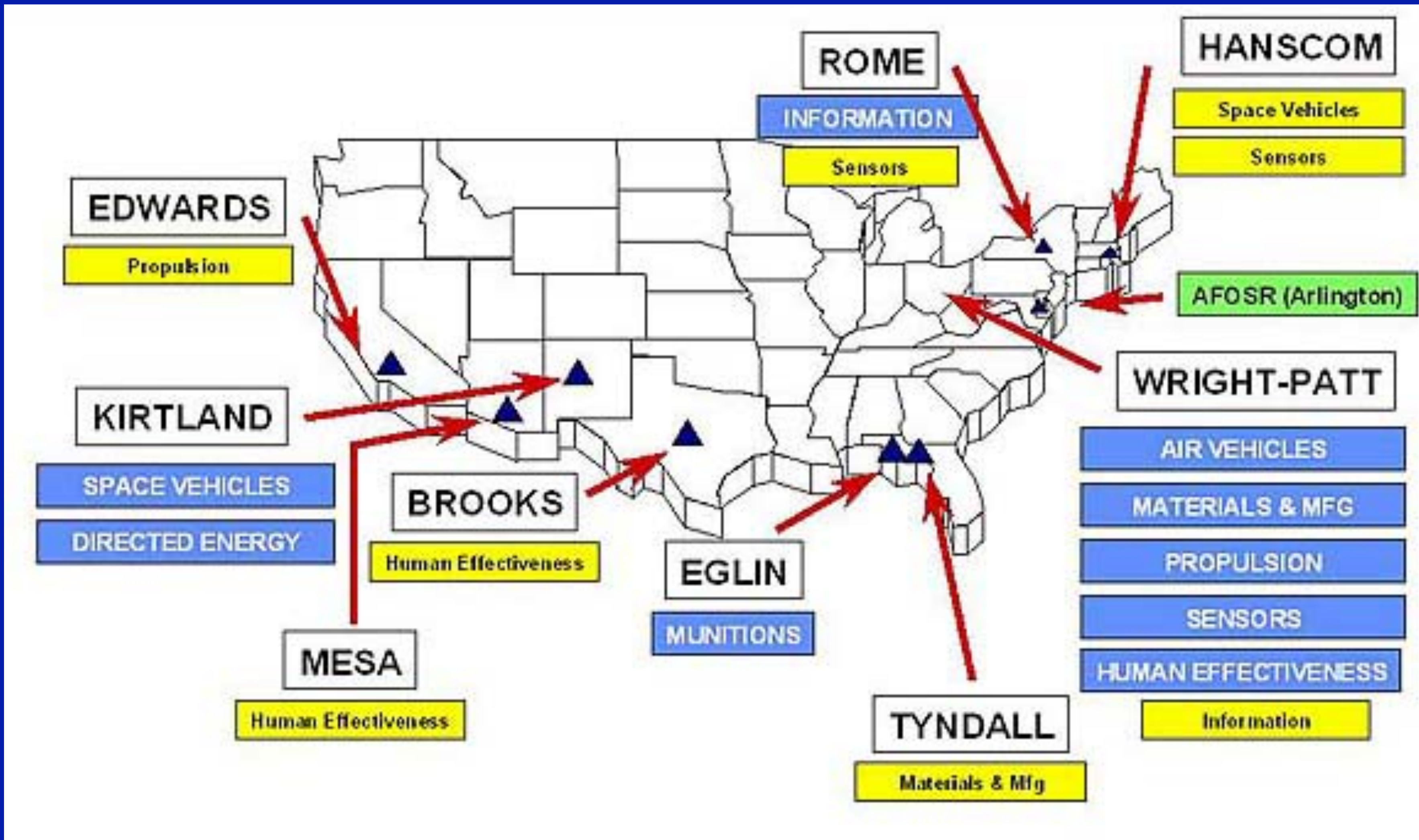


Introduction: What is this AFRL?



Introduction:
So what is Space Vehicles Directorate then?

http://www.vs.afrl.af.mil/

Flight Experiments

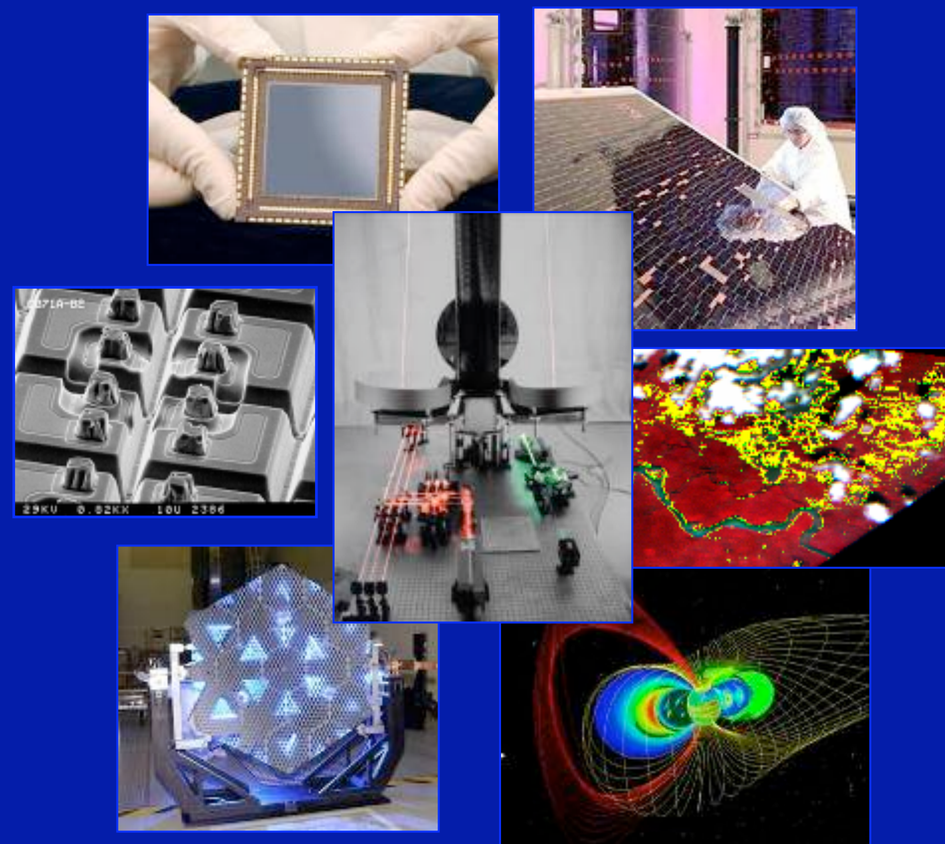
- ***C/NOFS***
- ***DSX***
- ***XSS-11***

Research

- ***Power & Photovoltaics***
- ***Detectors & FPA's***
- ***Radiation Hard Electronics***
- ***Structures***
- ***Dynamics & Controls***
- ***Ionosphere & effects***
- ***Space Environment***

Tech Development

- ***Laser Communications***
- ***Remote Sensing***
- ***Precision Navigation***

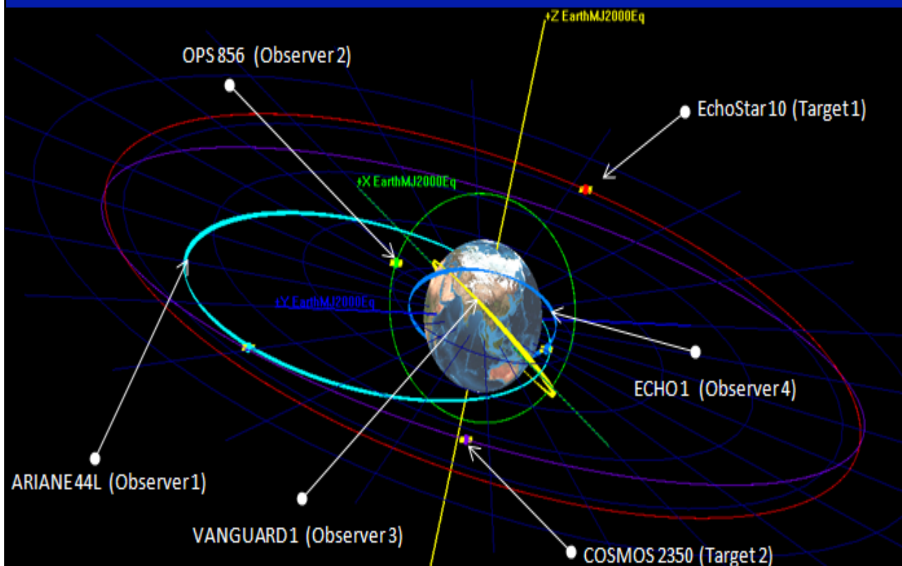


C/NOFS: http://www.spacewar.com/news/milspace-comms-05zzzu.html

DSX: http://www.spacewar.com/news/milspace-05zy.html

XSS-11: http://www.spacewar.com/news/launchers-05zd.html

Motivations: Autonomous Reporting System



Operational Capability

- System design guidelines on *whether to actively sense, what* sensing actions to take, and *how much gain to expect in terms of* enhancing the situation awareness
- Network scalability wherein each space sensor compete only for its own performance without attempting on others' behalf
- Autonomous event reporting via multi-agent frameworks; adversarial strategies and models

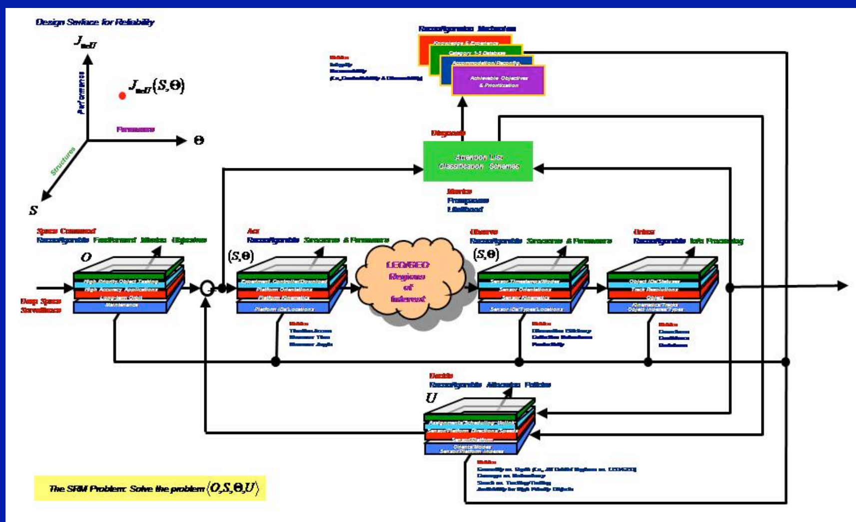
Technology Challenges

- Detection, tracking, and identification of large numbers of intelligent objects with all intervention/communication/detection latencies
- Adaptive coordination of sensor dwells and motions, in accordance with power, weather conditions, aerodynamic, earth blockages, obscurations and other constraints
- Situation awareness metrics guiding sensor and network resources allocation to data acquisition, interpretation & decision making

Far Term Experiment/Demo

- Persistent awareness demonstration of dynamic spatial-temporal regions
- Opportune and autonomous data acquisition using compressive sensing and active sensing
- Prototype of accurate tracking of maneuvers (e.g., changes in altitudes, inclinations, and phases) and time critical objects in preference with neutral and protected objects
- Demonstration of autonomous multi-level sequential surveillance strategy & visualization

Motivations: Autonomous Reporting System



Assumptions and Boundaries

- Time critical object capability: kinematics and dynamics of platforms; traverse speeds and times; levels of intelligence and awareness
- Capabilities of of neutral objects and assets being protected (e.g., stationary, orbit transfer)
- Effects based sensor modeling: reaction time; probability of detection; probability of acquisition, signature values; effective radius; max SBV and Radar ranges & resolutions

Performance Metrics

- Responsive Measurants: Timeliness access, maneuver time, detection time, gap time, ignorance cost, probability of occurrence
- Operational Measurants: Availability for high priority task; coverage vs. redundancy, usefulness, confidence, and probability of correct classification
- Affordable Measurants: Observation efficiency, collection robustness, mean time between sorties, endurance, and productivity

Mission CONOPS

- Uncorrelated objects in the mist of high density environments; Objects' tactics favor use of protection from uncertain vicinity
- All of altitude, inclination, and phase changes are important for catalog updates for abnormal maneuvers→ affording opportune surveillance
- Maintenance of transient behaviors of time critical objects against all facets of numerous short transitions and followed by deception

Motivations: Mission Impacts and Benefits

- Coordination for Effects and Tactical Advantage
  - Cooperative Sensor Attack Avoidance
  - High-Value Asset Protection
- Training, Exercise, and Supportability
  - Early Missile Warning
  - On-Orbit Maneuvers
  - Anomaly Resolution and Prediction
- Multi-Agent Differential Game Theory → A Natural Framework
  - Mathematical models for offense and defense engagement
  - Approaches for autonomous offensive and defensive tactics
  - Performance robustness beyond statistical averaging

Enabling Capabilities: 1) Know How; 2) Know-to-Cooperate; and 3) Need-to-Cooperate

Adversarial Hierarchies: Coalition-Conscious Interaction Dynamics

Let

$$x^X \triangleq \left[\left(x_1^X\right)^T, \ldots, \left(x_{m^X}^X\right)^T\right]^T \qquad A^X \triangleq \operatorname{diag}\left(A_1^X, \ldots, A_{m^X}^X\right) \qquad G^X \triangleq \operatorname{diag}\left(G_1^X, \ldots, G_{m^X}^X\right)$$

$$u^X \triangleq \left[\left(u_1^X\right)^T, \ldots, \left(u_{m^X}^X\right)^T\right]^T \qquad B_u^X \triangleq \operatorname{diag}\left(B_{u_1}^X, \ldots, B_{u_{m^X}}^X\right) \qquad X = A, S, T$$

$$dx^X(t) = \left(A^X x^X(t) + B_u^X u^X(t)\right)dt + G^X dw^X(t); \qquad x^X(t_0) = x_0^X; \qquad t \in \left[t_0, t_f\right];$$

Team vs. team confrontations

$$dx(t) = \left(Ax(t) + B^A u^A(t) + B^S u^S(t) + B^T u^T(t)\right)dt + G(t)dw(t); \qquad x(t_0) = x_0; \qquad t \in \left[t_0, t_f\right]$$

where

$$x \triangleq \left[\left(x^A\right)^T, \left(x^S\right)^T, \left(x^T\right)^T\right]^T \qquad B^A \triangleq \left[\left(B_u^A\right)^T, 0, 0\right]^T \qquad B^T \triangleq \left[0, 0, \left(B_u^T\right)^T\right]^T \qquad G \triangleq \operatorname{diag}\left(G^A, G^S, G^T\right)$$

$$A \triangleq \operatorname{diag}\left(A^A, A^S, A^T\right) \qquad B^S \triangleq \left[0, \left(B_u^S\right)^T, 0\right]^T \qquad w \triangleq \left[\left(w^A\right)^T, \left(w^S\right)^T, \left(w^T\right)^T\right]^T$$

Adversarial Hierarchies: Encountered Environments with Basic Elements Known

Uncertain environments consisted of un-modeled nonlinearities & disturbances with mixed random realizations defined on $\left(\Omega_i^X, F_i^X, \{F_i^X\}_{t_f \geq t \geq t_0}, P_i^X\right)$

$$w_i^X(t) \triangleq w_i^X(t, \omega_i^X) : [t_0, t_f] \times \Omega_i^X \mapsto \mathbb{R}^{p_i^X}; \qquad X = A, S, T \text{ and } i = 1, \ldots, m_X$$

$$E\left\{[w_i^X(\tau) - w_i^X(\sigma)][w_i^X(\tau) - w_i^X(\sigma)]^T\right\} = W_i^X |\tau - \sigma|; \quad \forall \tau, \sigma \in [t_0, t_f]$$

Thus, aggregate uncertain environments with a-priori characteristics become

$$w(t) \triangleq w(t, \omega) : [t_0, t_f] \times \Omega \mapsto \mathbb{R}^p$$

$$E\left\{[w(\tau) - w(\sigma)][w(\tau) - w(\sigma)]^T\right\} = W |\tau - \sigma|; \quad \forall \tau, \sigma \in [t_0, t_f]$$

where

$$W \triangleq \operatorname{diag}\left(W_1^A, \ldots, W_{m_A}^A; W_1^S, \ldots, W_{m_S}^S; W_1^T, \ldots, W_{m_T}^T\right)$$

Risk and Uncertainty Being Injected by Additive Stochastic Disturbances

Adversarial Hierarchies: Respective Tradeoff Strategies and Preferences

$$u_i^X \in U_i^X \subseteq L^2_{F_{it}^X}\left(t_0, t_f; \mathbb{R}^{m_i^X}\right); \qquad u^X \in U^X \triangleq \times_{i=1}^{m_X} U_i^X$$

Associated with an admissible 4-tuple $\left(x(\cdot); u^A(\cdot); u^S(\cdot); u^T(\cdot)\right)$

$$J_i^r\left(x_0; u^A(\cdot), u^S(\cdot), u^T(\cdot)\right) = w_{ir}^S \left\| P\left(x_i^S(t_f)\right) - P\left(x_r^T(t_f)\right) \right\|^2 - \sum_{j=1}^{m_A} w_{ij}^A \left\| P\left(x_j^A(t_f)\right) - P\left(x_i^S(t_f)\right) \right\|^2$$

$$+ \int_{t_0}^{t_f} \left[\sum_{j=1}^{m_S} \left(u_j^S\right)^T(\tau) R_{ij}^{SS} u_j^S(\tau) - \sum_{j=1}^{m_A} \left(u_j^A\right)^T(\tau) R_{ij}^{SA} u_j^A(\tau) - \sum_{j=1}^{m_T} \left(u_j^T\right)^T(\tau) R_{ij}^{ST} u_j^T(\tau) \right] d\tau$$

$$+ \int_{t_0}^{t_f} \left[\overline{w}_{ir}^S \left\| P\left(x_i^S(\tau)\right) - P\left(x_r^T(\tau)\right) \right\|^2 - \sum_{j=1}^{m_A} \overline{w}_{ij}^A \left\| P\left(x_j^A(\tau)\right) - P\left(x_i^S(\tau)\right) \right\|^2 \right] d\tau \quad ; \qquad r = 1, \ldots, m_T; \quad i = 1, \ldots, m_S$$

$$P : \mathbb{R}^{n_i^X} \mapsto N \text{ with } P\left(x_i^X\right) = \left[x_{i1}^X, \ldots, x_{in_0}^X\right]^T \in N \subseteq \mathbb{R}^{n_0}$$

$R_{ij}^{SA} \in \mathbb{R}^{m_j^A \times m_j^A}$; $R_{ij}^{SS} \in \mathbb{R}^{m_j^S \times m_j^S}$; $R_{ij}^{ST} \in \mathbb{R}^{m_j^T \times m_j^T}$ $\qquad$ $\overline{w}_{ir}^S$, $\overline{w}_{ij}^A$, w_{ir}^S, w_{ij}^A -- weighting scalars

Implementation of Stochastic Tracking Preferences

Adversarial Hierarchies: Simultaneous One-on-One Engagements

$$J_i\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right)=\begin{bmatrix} J_i^1\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right) \\ J_i^2\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right) \\ \vdots \\ J_i^{m_T}\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right) \end{bmatrix}; \qquad i=1,\ldots,m_S$$

For a given set of $\alpha_r^S \geq 0$ and $\sum_{r=1}^{m_T}\alpha_r^S = 1,$ the set of Pareto measures is given by

$$\bar{J}_i\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right)=\sum_{r=1}^{m_T}\alpha_r^S J_i^r\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right); \qquad i=1,\ldots,m_S$$

$$\bar{J}_i\left(x_0;u^A(\cdot),u^S(\cdot),u^T(\cdot)\right)=x^T\left(t_f\right)Q_{if}^{SS}x\left(t_f\right)+\int_{t_0}^{t_f}x^T(\tau)Q_i^{SS}x(\tau)d\tau$$

$$+\int_{t_0}^{t_f}\left[\sum_{j=1}^{m_S}\left(u_j^S\right)^T(\tau)R_{ij}^{SS}u_j^S(\tau)-\sum_{j=1}^{m_A}\left(u_j^A\right)^T(\tau)R_{ij}^{SA}u_j^A(\tau)-\sum_{j=1}^{m_T}\left(u_j^T\right)^T(\tau)R_{ij}^{ST}u_j^T(\tau)\right]d\tau$$

Adversarial Hierarchies: Pareto Coordination for Structuring Coalition Interactions

Consider a ***Pareto team*** defined by

1. A finite set of team members, $\bar{S}^S \triangleq \{1, 2, \ldots, S^S\} \subseteq \bar{T}^S$
2. A collection of decision sets indexed on $\bar{S}^S; \{U_j^S\}_{j \in \bar{S}^S}$
3. A payoff functional $J : \times_{j=1}^{m_A} U_j^A \times \times_{j=1}^{S^S} U_j^S \times \times_{j=1}^{m_T} U_j^T \mapsto \mathbb{R}_+$

Finding *efficient and collective decisions* such that

$$U^A \triangleq \times_{j=1}^{m_A} U_j^A; \qquad U_C^S \triangleq \times_{i=1}^{S^S} U_i^S; \qquad U^T \triangleq \times_{j=1}^{m_T} U_j^T$$

$$\operatorname*{argmin}_{u^A \in U^A, u_C^S \in U_C^S, u^T \in U^T} \left\{ \bar{J}_1\left(x_0; u^A, u_C^S, u^T\right), \ldots, \bar{J}_{S^S}\left(x_0; u^A, u_C^S, u^T\right) \right\}$$

Coalitive Pareto decision parameterization

For given $u \in U^A$ and $u^T \in U^T$, $\hat{u}_C^S \in U_C^S$ is efficient $\Leftrightarrow$ There exists $\xi^S \in \mathbb{R}^{S^S}$ and $\xi^S > 0$ such that $\hat{u}_C^S$ is a Pareto optimal solution of the single-objective problem

$$\operatorname*{arg\,min}_{u^A \in U^A, u_C^S \in U_C^S, u^T \in U^T} \left\{ J\left(x_0; u^A, u_C^S, u^T\right) \triangleq \sum_{i=1}^{S^S} \xi_i^S \bar{J}_i\left(x_0; u^A, u_C^S, u^T\right) \right\}$$

Know-How-to-Cooperate: Managing Interference & Facilitating Between Goals

Balance of Intra-Team and Inter-Team Objectives

Let $Q_f \triangleq \sum_{i=1}^{S^S} \xi_i^S Q_{if}$; $Q \triangleq \sum_{i=1}^{S^S} \xi_i^S Q_i$; $R_j^{SA} \triangleq \sum_{i=1}^{S^S} \xi_i^S R_{ij}^{SA}$; $R_j^{SS} \triangleq \sum_{i=1}^{S^S} \xi_i^S R_{ij}^{SS}$; $R_j^{ST} \triangleq \sum_{i=1}^{S^S} \xi_i^S R_{ij}^{ST}$

➔

$$J\left(x_0; u^A(\cdot), u_C^S(\cdot), u^T\right) = x^T(t_f) Q_f x(t_f) + \int_{t_0}^{t_f} \left[x^T(\tau) Q x(\tau)\right] d\tau$$
$$+ \int_{t_0}^{t_f} \left[\sum_{j=1}^{S^S} \left(u_{cj}^S\right)^T(\tau) R_j^{SS} u_{cj}^S(\tau) - \sum_{j=1}^{m_A} \left(u_j^A\right)^T(\tau) R_j^{SA} u_j^A(\tau) - \sum_{j=1}^{m_T} \left(u_j^T\right)^T(\tau) R_j^{ST} u_j^T(\tau) \right] d\tau$$

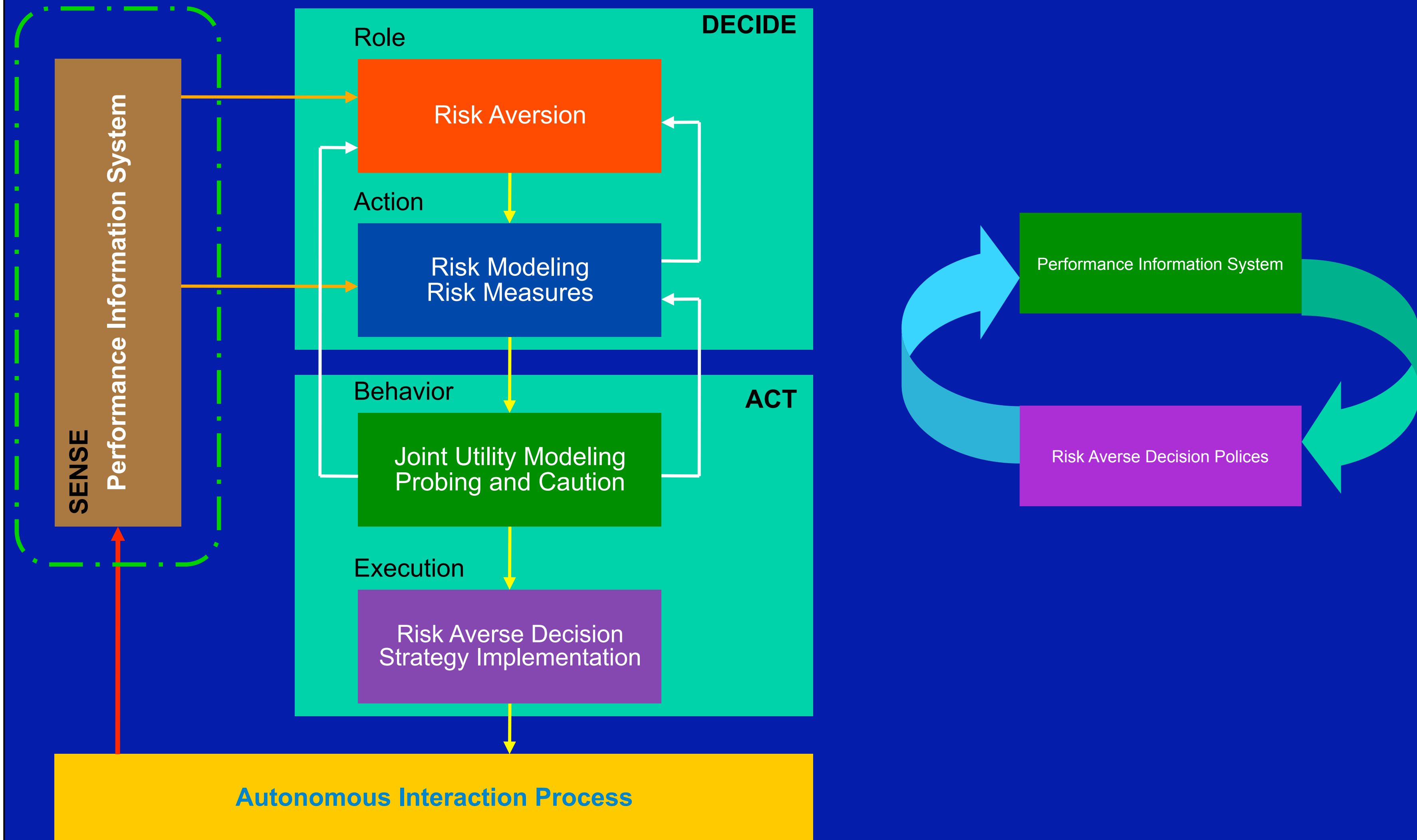
Further $R^{SS} \triangleq diag\left(R_1^{SS}, \ldots, R_{S^S}^{SS}\right)$; $R^{SA} \triangleq diag\left(R_1^{SA}, \ldots, R_{m_A}^{SA}\right)$; $R^{ST} \triangleq diag\left(R_1^{ST}, \ldots, R_{m_T}^{ST}\right)$

➔

$$J\left(x_0; u^A(\cdot), u_C^S(\cdot), u^T\right) = x^T(t_f) Q_f x(t_f) + \int_{t_0}^{t_f} \left[x^T(\tau) Q x(\tau)\right] d\tau$$
$$+ \int_{t_0}^{t_f} \left[\left(u_C^S\right)^T(\tau) R^{SS} u_C^S(\tau) - \left(u^A\right)^T(\tau) R^{SA} u^A(\tau) - \left(u^T\right)^T(\tau) R^{ST} u^T(\tau) \right] d\tau$$

Know-How-to-Cooperate: Inducing Pareto Coordination

A Decision Architecture for Risk Averse based Multi-Agent Differential Game of A Kind



Performance Information: Perception of Elements in Current Situation

Status

- Interactive teams A, S, and T of members
- End game engagement near assets being either tracked or protected

Attributes

- Decision horizon $t \in [t_0, t_f]$
- Default information of dynamical features of interaction management
  $A; B^A; B^S; B^T; C$ and $Q_f \geq 0; Q \geq 0; R^{SS}, R^{SA}, R^{ST} > 0$
- Pairs (A, B^A), (A, B^S) and (A, B^T) stabilizable

Environmental Features

- Stationary Wiener process $\left(\Omega, F, \{F_t\}_{t \geq t_0 \geq 0}, P\right)$ for the uncertain environment which affects the outcomes via its mixed random sample path realizations

$$E\left\{[w(\tau) - w(\sigma)][w(\tau) - w(\sigma)]^T\right\} = W|\tau - \sigma|; \;\; \tau, \sigma \in [t_0, t_f].$$

Sample-Path Realizations from Environment Leading to Riskier Performance

Performance Information: Comprehension of the Current Situation

Multi-Agent Differential Game

- forms action-outcome pictures

$$dx(t) = \left(Ax(t) + B^A u^A(t) + B^S u_C^S(t) + B^T u^T(t)\right)dt + G\,dw(t)$$

$$x(t_0) = x_0$$

$$J\left(x_0; u^A(\cdot), u_C^S(\cdot), u^T\right) = x^T(t_f)Q_f x(t_f) + \int_{t_0}^{t_f} \left[x^T(\tau)Qx(\tau)\right] d\tau$$

$$+\int_{t_0}^{t_f} \left[\left(u_C^S\right)^T(\tau)R^{SS}u_C^S(\tau) - \left(u^A\right)^T(\tau)R^{SA}u^A(\tau) - \left(u^T\right)^T(\tau)R^{ST}u^T(\tau)\right] d\tau$$

- comprehend the significance of linear-quadratic nature of interaction dynamics and integrate this characteristic property to the performance-measure

→ $J\left(x_0; u^A(\cdot), u_C^S(\cdot), u^T\right)$ is a random variable with Chi-squared type !!

Stochastic Variable → Environment's Sample Realizations → Performance Risk & Uncertainty

Performance Information: Information System and Its Value

Multi-Agent Differential Game

- determines which cues are relevant to performance uncertainty and risk, e.g.,

Moment- and Cumulant-Generating Functions (parameterized by θ)

$$\varphi(s, x_s; \theta) \triangleq E\left\{\exp\left\{\theta J(s, x_s)\right\}\right\}$$

$$\psi(s, x_s; \theta) \triangleq \ln\left\{\varphi(s, x_s; \theta)\right\}$$

$\varphi(s, x_s; \theta)$ denotes information system, φ

$\psi(s, x_s; \theta)$ denotes the value of information system, ψ

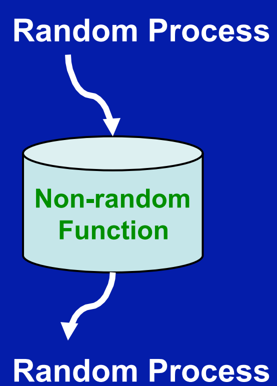
$\theta \in \Theta$ denotes the set of parameters for the information system, φ

Ω denotes the set of uncertain states of the uncertain environment

Transferring Data to States of Knowledge

Performance Information:
Information Value Determinant → Action Flexibility

U^A, U_C^S, and U^T denote the sets of actions via state feedback strategies to maintain a fair degree of accuracy on team behaviors and the security level utility



$$\gamma^A : \Gamma^A \mapsto U^A$$
$$\gamma^S : \Gamma^S \mapsto U_C^S$$
$$\gamma^T : \Gamma^T \mapsto U^T$$
$$u^A = \gamma^A(\eta)$$
$$u_C^S = \gamma^S(\eta)$$
$$u^T = \gamma^T(\eta)$$
$$\eta = (t, x(t))$$

$$u^A(t) \triangleq K^A(t)x(t)$$
$$u_C^S(t) \triangleq K^S(t)x(t)$$
$$u^T(t) \triangleq K^T(t)x(t)$$

Closed-Loop Decisions

Decision Variables Controlled by Autonomous Teams

Performance Information:
Information Value Determinant → Outcome Function

f denotes the outcome function mapping outcome-action-uncertain state triplets into outcomes, e.g., $x \in L^2_{\{F_t\}_{s \le t \le t_f}}\left(\Omega, C\left(\left[s, t_f\right]; R^n\right)\right)$ of R^n – valued, square integrable processes on $\left[s, t_f\right]$ that are adapted to the sigma field F_t generated by $w(t)$, e.g.,

$$dx(t) = f\left(x; u^A, u^S_C, u^T; w\right); \quad x(s)$$
$$= \left(A + B^A K^A + B^S K^S + B^T K^T\right)x(t)dt + Gdw(t); \quad x(s)$$

In a compact notation, it results in

$$dx(t) = F(t)x(t)dt + Gdw(t); \quad x(s)$$
$$F(t) \triangleq A + B^A K^A(t) + B^S K^S(t) + B^T K^T(t)$$

Technology and Environment (and thus Outcomes) for Autonomous Teams

Performance Information: Information Value Determinant → Utility Function

$J(s, x_s)$ denotes an utility function mapping outcomes into utility levels i.e.,

$$J(s,x_s) \triangleq x^T(t_f)Q_f x(t_f) + \int_s^{t_f} x^T(\tau)\left[Q + \left(K^S\right)^T(\tau)R^{SS}K^S(\tau)\right]x(\tau)d\tau$$

$$+\int_s^{t_f} x^T(\tau)\left[-\left(K^A\right)^T(\tau)R^{SA}K^A(\tau) - \left(K^T\right)^T(\tau)R^{ST}K^T(\tau)\right]x(\tau)d\tau$$

Or, equivalently

$$J(s,x_s) = x^T(t_f)Q_f x(t_f) + \int_s^{t_f} x^T(\tau)N(\tau)x(\tau)d\tau$$

$$N(t) \triangleq Q + \left(K^S\right)^T(t)R^{SS}K^S(t) - \left(K^A\right)^T(t)R^{SA}K^A(t) - \left(K^T\right)^T(t)R^{ST}K^T(t)$$

Performance Information: Attributes of Information Value →Cumulants of Performance-Measure

$$\psi\left(s,x_s;\theta\right)=\sum_{k=1}^{\infty}\frac{\partial^{(k)}}{\partial\left(\theta\right)^{(k)}}\psi\left(s,x_s;\theta\right)\bigg|_{\theta=0}\frac{\left(\theta\right)^k}{k!}\triangleq\sum_{k=1}^{\infty}\frac{\left(\theta\right)^k}{k!}\kappa_k$$

$$\kappa_k=\left(x_s\right)^T\frac{\partial^{(k)}}{\partial\left(\theta\right)^{(k)}}\Upsilon\left(s;\theta\right)\bigg|_{\theta=0}x_s+\frac{\partial^{(k)}}{\partial\left(\theta\right)^{(k)}}\upsilon\left(s;\theta\right)\bigg|_{\theta=0}$$

$$\triangleq\left(x_s\right)^T H\left(s,k\right)x_s+D\left(s,k\right)$$

$$\frac{d}{ds}\Upsilon\left(s;\theta\right)=-\left[A+B^AK^A\left(s\right)+B^SK^S\left(s\right)+B^TK^T\left(s\right)\right]^T\Upsilon\left(s;\theta\right)$$

$$-\Upsilon\left(s;\theta\right)\left[A+B^AK^A\left(s\right)+B^SK^S\left(s\right)+B^TK^T\left(s\right)\right]-2\Upsilon\left(s;\theta\right)GWG^T\Upsilon\left(s;\theta\right)$$

$$-\theta\left[Q+\left(K^S\right)^T\left(s\right)R^{SS}K^S\left(s\right)-\left(K^A\right)^T\left(s\right)R^{SA}K^A\left(s\right)-\left(K^T\right)^T\left(s\right)R^{ST}K^T\left(s\right)\right]$$

$$\frac{d}{ds}\upsilon\left(s;\theta\right)=-Tr\left\{\Upsilon\left(s;\theta\right)GWG^T\right\}$$

$$\Upsilon\left(t_f;\theta\right)=\theta Q_f;\qquad\upsilon\left(t_f;\theta\right)=0.$$

Performance Information: Information Dynamics of Performance-Measure

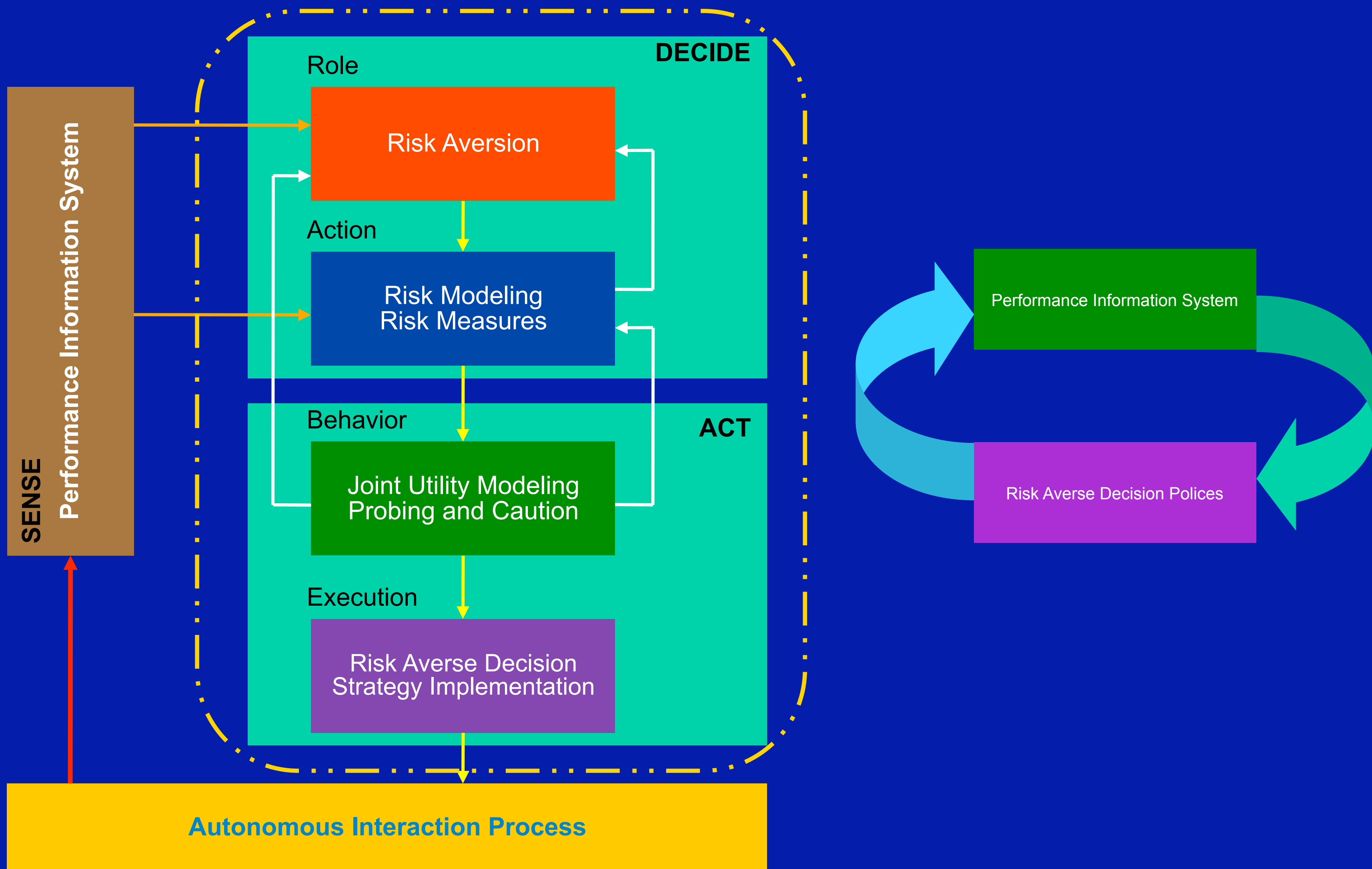
$$\frac{d}{ds}H(s,1) = -\left[A + B^A K^A(s) + B^S K^S(s) + B^T K^T(s)\right]^T H(s,1)$$
$$-H(s,1)\left[A + B^A K^A(s) + B^S K^S(s) + B^T K^T(s)\right]$$
$$-Q - \left(K^S\right)^T(s) R^{SS} K^S(s) + \left(K^A\right)^T(s) R^{SA} K^A(s) + \left(K^T\right)^T(s) R^{ST} K^T(s)$$

$$\frac{d}{ds}H(s,r) = -\left[A + B^A K^A(s) + B^S K^S(s) + B^T K^T(s)\right]^T H(s,r)$$
$$-H(s,r)\left[A + B^A K^A(s) + B^S K^S(s) + B^T K^T(s)\right]$$
$$-\sum_{v=1}^{r-1} \frac{2r!}{v!(r-v)!} H(s,v) G W G^T H(s,r-v); \qquad 2 \le r \le k$$

$$\frac{d}{ds}D(s,r) = -Tr\left\{H(s,r) G W G^T\right\}; \qquad D\left(t_f, r\right) = 0 \qquad 1 \le r \le k$$

$$H\left(t_f, 1\right) = Q_f; \quad H\left(t_f, r\right) = 0 \text{ for } 2 \le r \le k$$

A Decision Architecture for Risk Averse based Multi-Agent Differential Game of A Kind



Decision Making under Uncertainty: A Risk-Value Model

Suppose S_i and S_j are two stochastic systems. A preference comparison between S_i and S_j can be made by a risk-value model:

$$S_i \succsim S_j \quad \text{if and only if} \quad \phi\left(V\left(S_i\right), R\left(S_i\right)\right) \geq \phi\left(V\left(S_j\right), R\left(S_j\right)\right)$$

where

V measures the value of a system,

R measures its riskiness, and

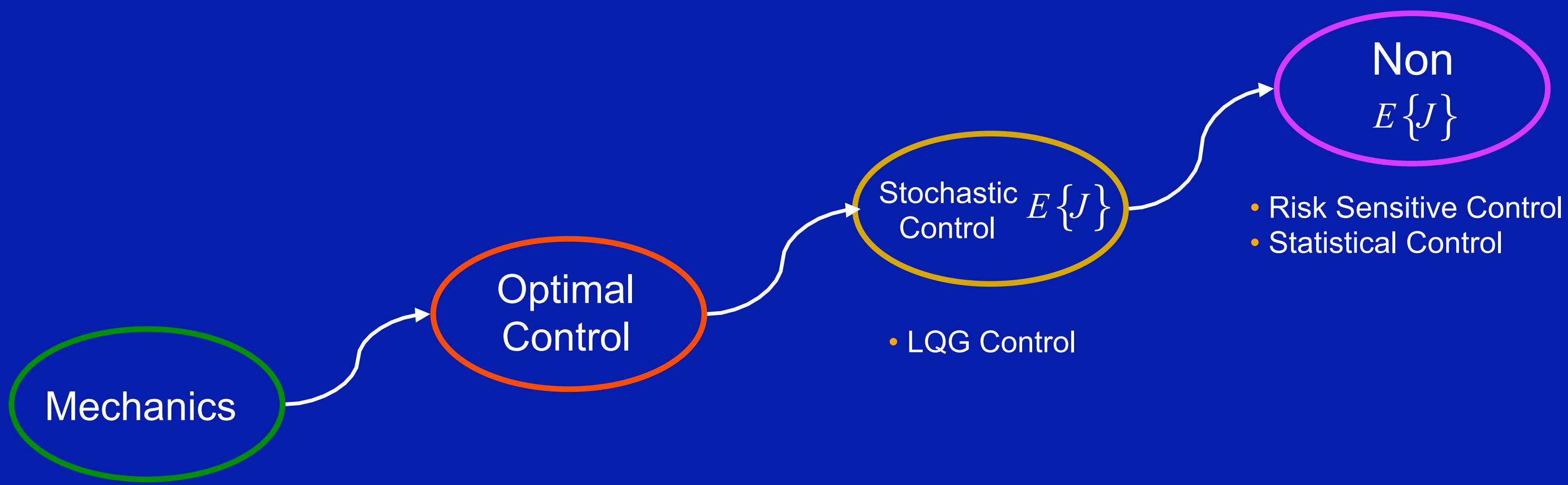
ϕ reflects the trade-off between value and riskiness.

Models of Perceived Risk:

- Finance → Variance
- Safety → Both the *probability* and the *magnitude* of adverse effects

Models of Risk Must be Specialized by Classes of Applications

Decision Making under Uncertainty: Measures of Performance Risk



Some Possible Interpretations

- LQG Control → An expected utility minimizer with risk defined as expected values
- Risk Sensitive Control → A risk sensitivity minimizer with all centralized moments weighted in a specific way to yield the riskiness

Decision Making under Uncertainty:
A General Measure of Performance Risk

J--physical /social
→ fuel; energy; dollars

$$E_w\{\ \ \}$$

J--intermediate path
→"in-between";
"middle-man"

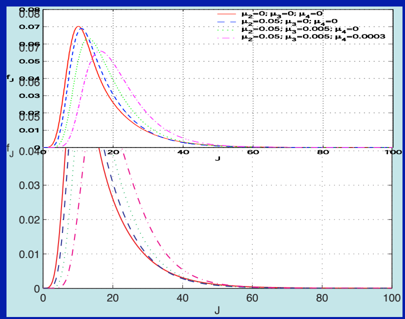
→ LQG Control

Idea:

$$\min\ E\{J\} < E\{J\} < \infty$$

→Pursue additional goals…

→ Risk Sensitivity & Beyond…



In Statistical Control: Weightings for Mean, Variance, Skewness, Flatness,…→ Design Freedom

Decision Making under Uncertainty: Preference Model for Decision Making

Fundamental Issue: Drive some state $x\left(x_0; w; u_C^A, u_C^S, u_C^T\right)$ to zero

New Model of Preference: *Statistical Control*

$$\min_{K^A, K^S, K^T} \left\{ \sum_{r=1}^{k} \mu_r \kappa_r \left(K^A, K^S, K^T\right) \right\}$$

Emphases are on different competing cumulants and team prioritizations which are determined by $\mu \triangleq \{\mu_r \geq 0\}_{r=1}^{k}$ with $\mu_1 > 0$.

Classical Preference: *LQG Control*

$$\min_{\not{K}^A, K^S, \not{K}^T} \left\{ \kappa_1 \left(\not{K}^A, K^S, \not{K}^T\right) \right\}$$

Fairly New Preference: *Risk-Sensitive Control*

$$\min_{\not{K}^A, K^S, \not{K}^T} \left\{ \sum_{r=1}^{\infty} \frac{\theta}{r!} \kappa_r \left(\not{K}^A, K^S, \not{K}^T\right) \right\}$$

Decision Making under Uncertainty: Performance Uncertainty Probing & Cautioning

$$\phi_0 : \{t_0\} \times \left(\mathbb{R}^{n\times n}\right)^k \times \mathbb{R}^k \mapsto \mathbb{R}_+$$

$$F \triangleq F_1 \times \cdots \times F_k$$
$$G \triangleq G_1 \times \cdots \times G_k$$

$$H_r \equiv H_r\left(\cdot; K^A, K^S, K^T\right)$$
$$D_r \equiv D_r\left(\cdot; K^A, K^S, K^T\right)$$

$$H_f \triangleq Q_f \times 0 \times \cdots \times 0$$
$$D_f \triangleq 0 \times \cdots \times 0$$

$$\frac{d}{ds}H(s) = F\left(s; H(s); K^A(s), K^S(s), K^T(s)\right); \qquad H(t_f) \equiv H_f$$
$$\frac{d}{ds}D(s) = G\left(s; H(s)\right); \qquad D(t_f) \equiv D_f$$

Value Measure

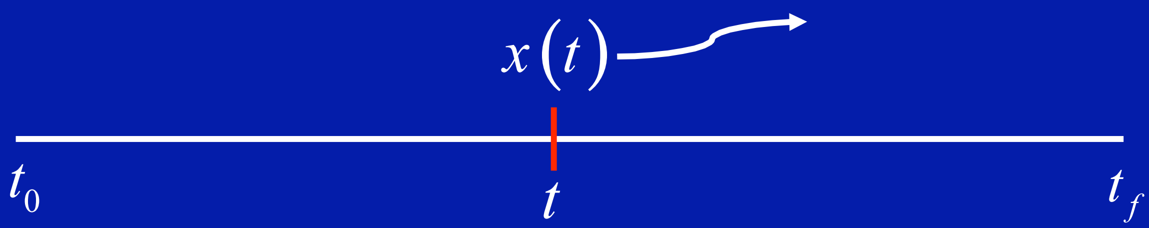
Risk Measure

$$\phi_0(t_0; H, D) \triangleq \mu_1\kappa_1 + \mu_2\kappa_2 + \mu_3\kappa_3 + \mu_4\kappa_4 + \cdots + \mu_k\kappa_k$$
$$= \mu_1\left[x_0^T H_1(t_0) x_0 + D_1(t_0)\right] + \cdots + \mu_k\left[x_0^T H_k(t_0) x_0 + D_k(t_0)\right]$$

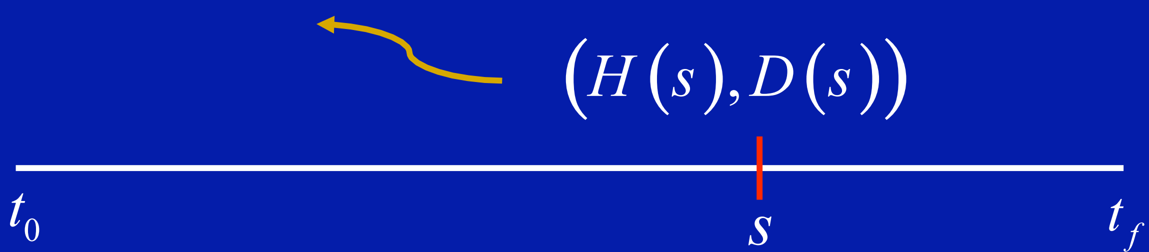
Decision Making under Uncertainty:
Principle of Optimality

An optimal strategy has the property that whatever the initial state and time, all remaining decisions must constitute an optimal strategy…

Classical Control Class ➔ Terminal Cost Problems



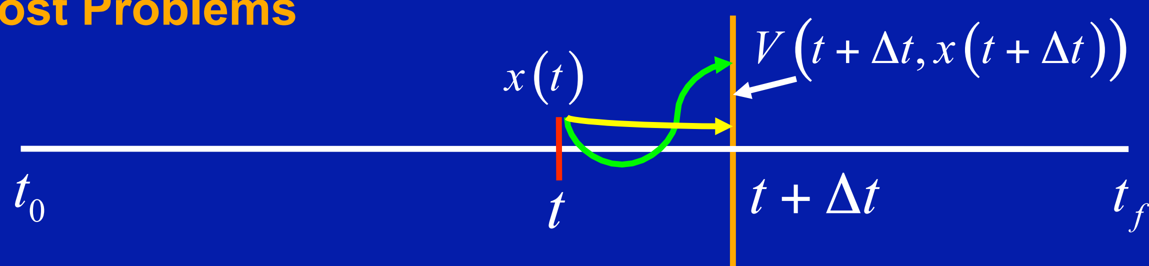
Statistical Control Class ➔ The Initial Cost Problem



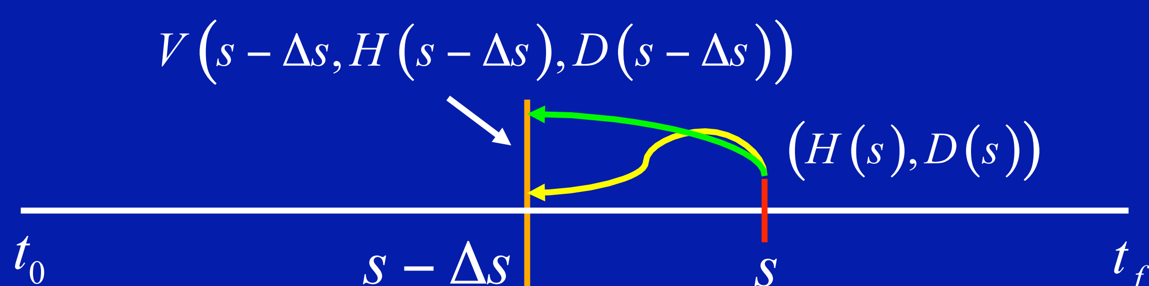
Decision Making under Uncertainty:
Tenet of Transition

"… we are dealing with a family of optimization based on different starting points. Consider an interlude of time in mid-play. At its commencement the path has reached some definitive point. Consider all possible x (H, D) *which may be reached at the end of the interlude for all possible choices of* (u^A, u_C^S, u^T) (K^A, K^S, K^T).
Suppose that for each endpoint, the optimization beginning there has already been solved (V is known there). Then the payoff resulting from each choice of $(u^A, u_C^S, u^T)(K^A, K^S, K^T)$ *will be known, and they are to be so chosen as to render it minimum. When we let the duration of the interlude approach* t_0 (t_f)*, …"*

Terminal Cost Problems



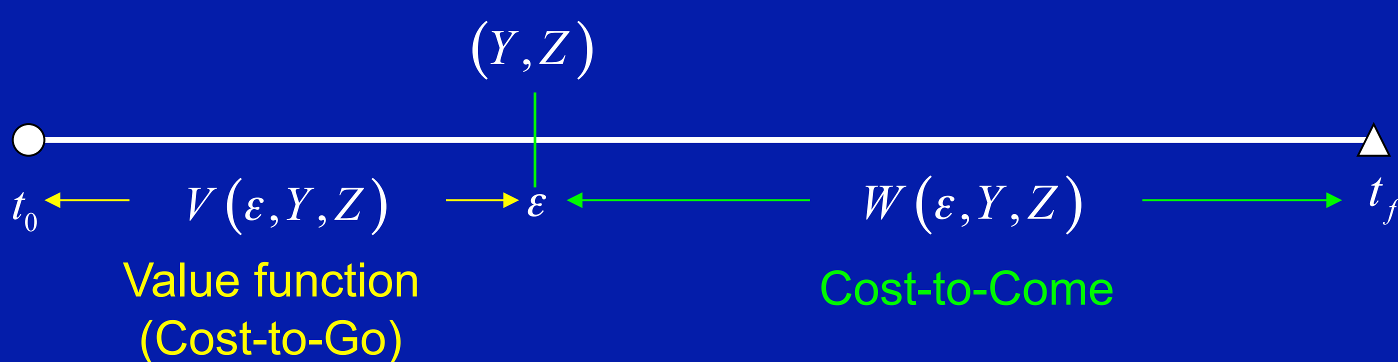
The Initial Cost Problem



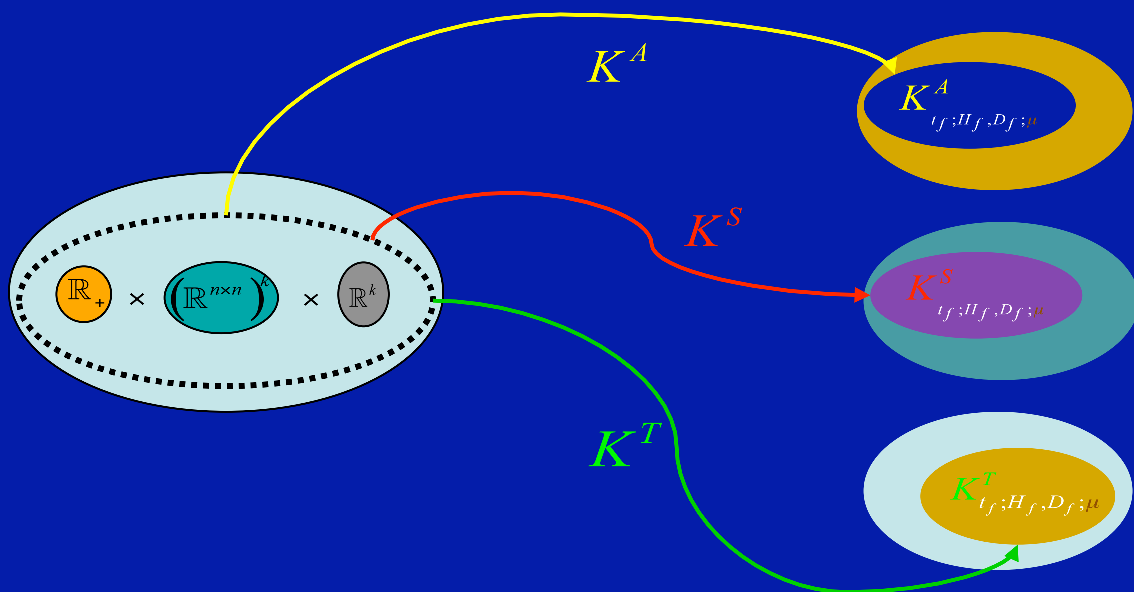
Decision Making under Uncertainty:
Tenet of Transition

“… we are dealing with a family of optimization based on different starting points. Consider an interlude of time in mid-play. At its commencement the path has reached some definitive point. Consider all possible x (H,D) *which may be reached at the end of the interlude for all possible choices of* (u^A, u_C^S, u^T) (K^A, K^S, K^T).
Suppose that for each endpoint, the optimization beginning there has already been solved (V is known there). Then the payoff resulting from each choice of $(u^A, u_C^S, u^T)(K^A, K^S, K^T)$ *will be known, and they are to be so chosen as to render it minimum. When we let the duration of the interlude approach* t_0 (t_f)*, …”*

➔ Leads to as sufficient condition to HJB equation



Decision Making under Uncertainty: HJB Equation for Mayer Problem



$$\min_{K^A\in\bar{K}^A,\,K^S\in\bar{K}^S,\,K^T\in\bar{K}^T}\left\{\begin{array}{l}\dfrac{\partial}{\partial\varepsilon}V(\varepsilon,Y,Z)+\dfrac{\partial}{\partial vec(Y)}V(\varepsilon,Y,Z)\bullet vec\left(F\left(\varepsilon,Y,K^A,K^S,K^T\right)\right)\\ +\dfrac{\partial}{\partial vec(Z)}V(\varepsilon,Y,Z)\bullet vec\left(G(\varepsilon,Y)\right)\end{array}\right\}=0$$

B.C.

Decision Making under Uncertainty: Solving the Mayer Problem

Candidate Solution for HJB equation

$$W(\varepsilon, Y, Z) = x_0^T \sum_{r=1}^{k} \mu_r \left(Y_r + E_r(\varepsilon)\right) x_0 + \sum_{r=1}^{k} \mu_r \left(Z_r + T_r(\varepsilon)\right)$$

$E \in C^1\left([t_0, t_f]; R^{n\times n}\right)$ and $T \in C^1\left([t_0, t_f]; R\right)$ yet to be determined

Associated HJB equation

$$\min_{K^A \in \bar{K}^A, K^S \in \bar{K}^S, K^T \in \bar{K}^T} \left\{ \begin{array}{l} x_0^T \left[\sum_{r=1}^{k} \mu_r \frac{d}{d\varepsilon} E_r(\varepsilon)\right] x_0 + \sum_{r=1}^{k} \mu_r \frac{d}{d\varepsilon} T_r(\varepsilon) \\ + x_0^T \left[\sum_{r=1}^{k} \mu_r F_r\left(\varepsilon, Y, K^A, K^S, K^T\right)\right] x_0 + \sum_{r=1}^{k} \mu_r G_r(\varepsilon, Y) \end{array} \right\} = 0$$

Decision Making under Uncertainty: Risk-Averse and Pareto Optimal Decision Strategies

$$K^{S*}(s) = -\left(R^{SS}\right)^{-1}\left(B^{S}\right)^{T}\sum_{r=1}^{k}\hat{\mu}_r H_r^*(s)$$

$$K^{A*}(s) = \left(R^{SA}\right)^{-1}\left(B^{A}\right)^{T}\sum_{r=1}^{k}\hat{\mu}_r H_r^*(s)$$

$$K^{T*}(s) = \left(R^{ST}\right)^{-1}\left(B^{T}\right)^{T}\sum_{r=1}^{k}\hat{\mu}_r H_r^*(s)$$

$$\hat{\mu}_r \triangleq \frac{\mu_r}{\mu_1}$$

$$x^* \triangleq \left[\left(x^{A*}\right)^T \quad \left(x^{S*}\right)^T \quad \left(x^{T*}\right)^T\right]^T$$

$$K^{A*} \triangleq \left[K_*^{AA} \quad K_*^{AS} \quad K_*^{AT}\right]; \quad K^{S*} \triangleq \left[K_*^{SA} \quad K_*^{SS} \quad K_*^{ST}\right]; \quad K^{T*} \triangleq \left[K_*^{TA} \quad K_*^{TS} \quad K_*^{TT}\right]$$

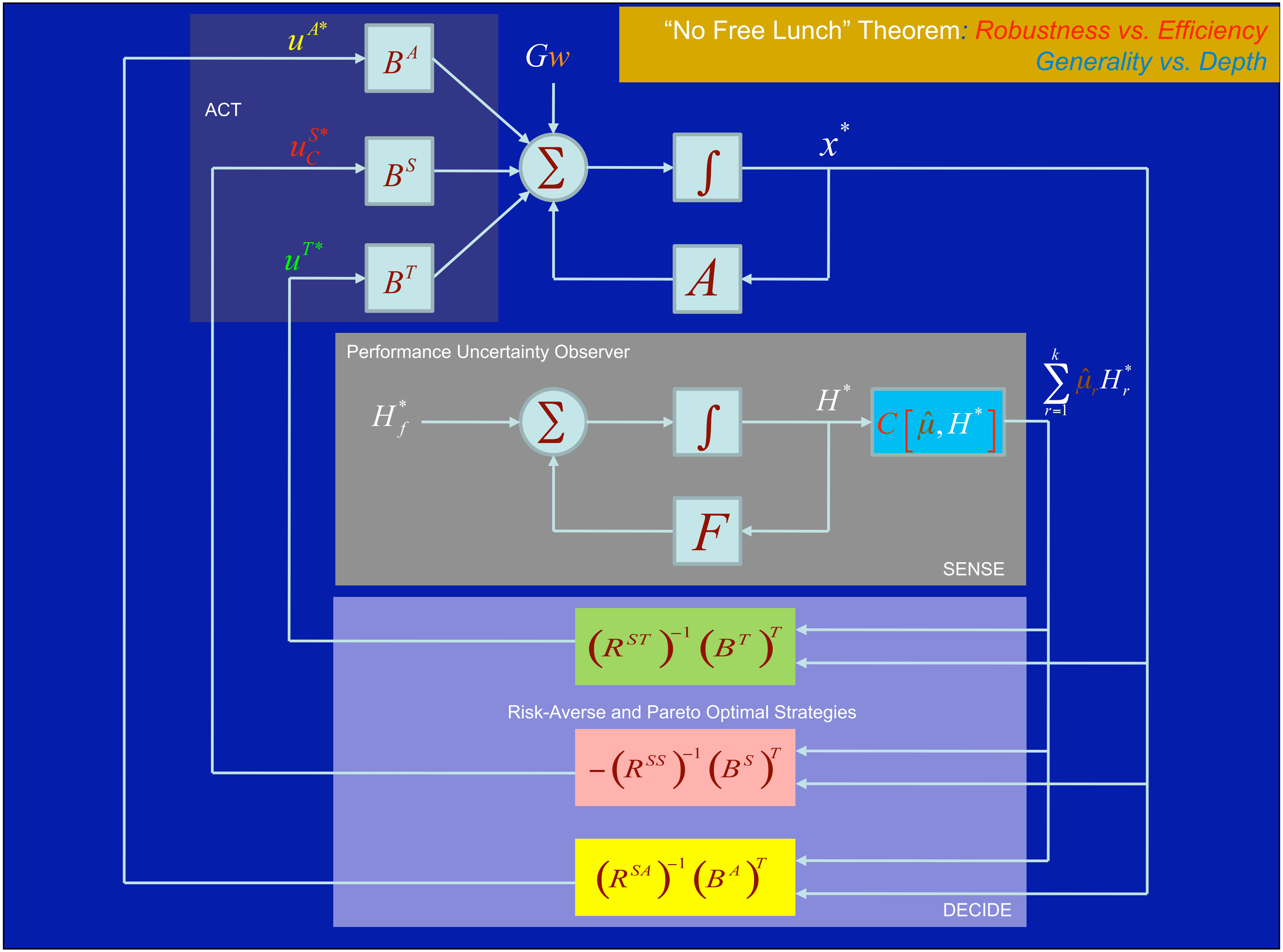
➔

$$u^{A*}(t) = K_*^{AA}(t)x^{A*}(t) + K_*^{AS}(t)x^{S*}(t) + K_*^{AT}(t)x^{T*}(t)$$

$$u_C^{S*}(t) = K_*^{SA}(t)x^{A*}(t) + K_*^{SS}(t)x^{S*}(t) + K_*^{ST}(t)x^{T*}(t)$$

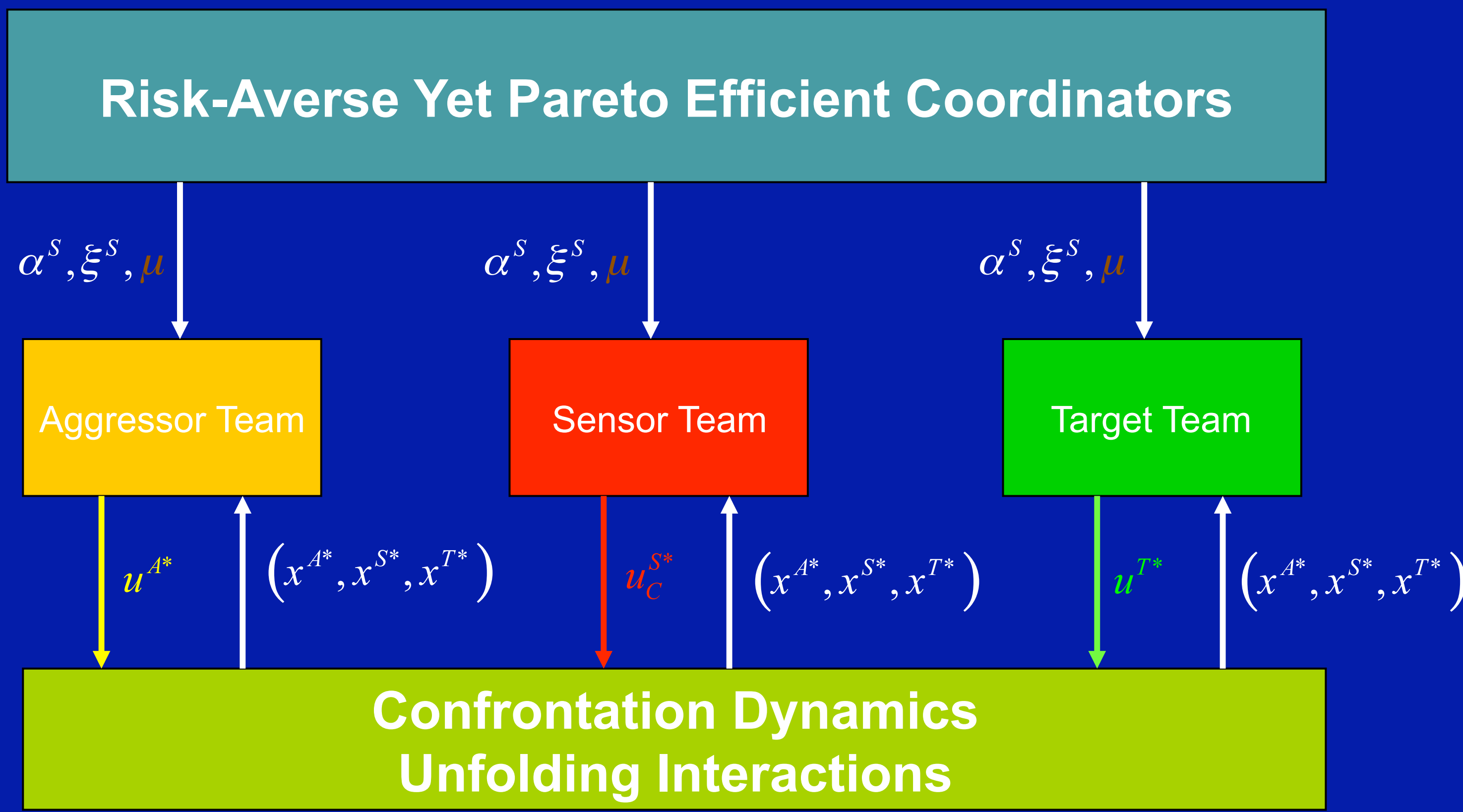
$$u^{T*}(t) = K_*^{TA}(t)x^{A*}(t) + K_*^{TS}(t)x^{S*}(t) + K_*^{TT}(t)x^{T*}(t)$$

Adaptable and *Robust* Against *Adversaries and Random Sample-Path Realizations*



"No Free Lunch" Theorem: Robustness vs. Efficiency
Generality vs. Depth
ACT
u^{A*}
u_C^{S*}
u^{T*}
B^A
B^S
B^T
Gw
Σ
$\int$
x^*
A
Performance Uncertainty Observer
H_f^*
H^*
$C[\hat{\mu}, H^*]$
$\sum_{r=1}^{k} \hat{\mu}_r H_r^*$
F
SENSE
$(R^{ST})^{-1}(B^T)^T$
Risk-Averse and Pareto Optimal Strategies
$-(R^{SS})^{-1}(B^S)^T$
$(R^{SA})^{-1}(B^A)^T$
DECIDE

Multi-Level Control and Coordination



Conclusions

Major contributions offered by statistical game theory to risk-averse and linear-quadratic multi-agent differential games, include:

- Performance information modeling
- Determinants of performance information value
- Cumulants having consistent effects on performance information value
- Attributes of decision settings and decision makers not having the same effects on performance information value
- Natural linkages to LQG theory and risk sensitive control
- In-depth knowledge of utilizing "performance-measure statistics" to shape team performance robustness
- Integration of perceived performance risk with team decision strategies, and
- Unifying framework for potential streams of research on performance risk judgments and decision making

Performance uncertainty representations and adaptive algorithms from
logic and probability
are combined to maintain performance uncertainty representations that are
compact and robust