Integrated Estimator/Guidance Law Design for Improved Ballistic Missile Defense

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Outline

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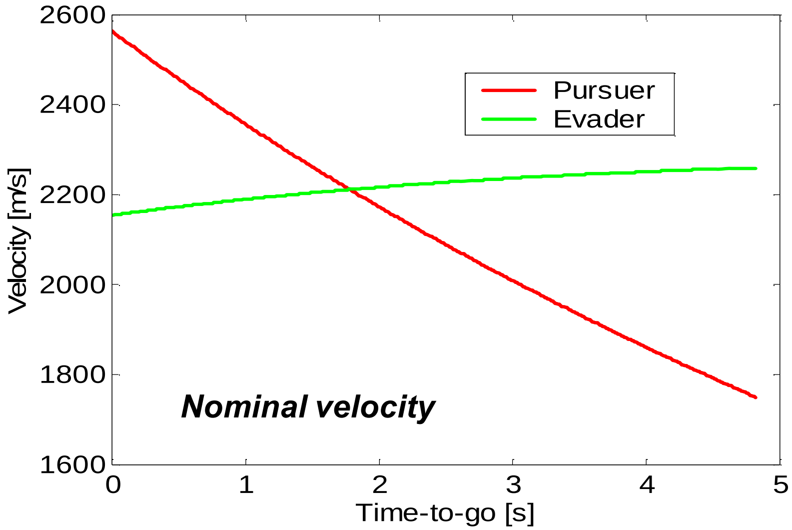
- Introduction
- Background
- Intercept Scenario
- Deterministic Guidance
- Estimation for Homing Guidance
- New Approach: 2-D Scenario
- New Approach: 3-D Scenario

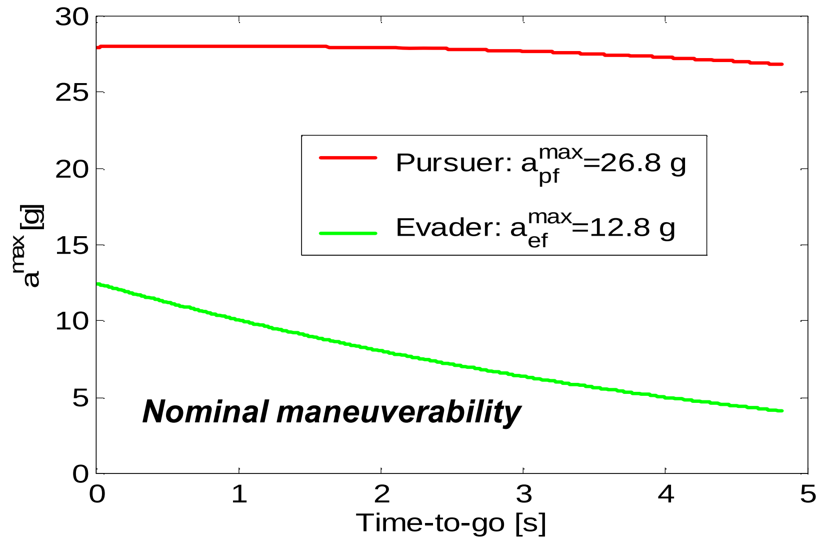
Introduction

- New interceptors (e.g., Arrow, PAC-3, THAAD, Navy Area Wide) have excellent homing performance against non-maneuvering targets
- TBMs have a substantial maneuverability potential
- Classical guidance and estimation methods are unable to guarantee hit-to-kill accuracy against highly maneuvering targets:
  - Insufficient maneuver advantage
  - Inherent estimation error
- *Development of guidance laws for interception of high-maneuverability TBMs remains a yet unsolved challenge*

Background

Hit-to-Kill Feasibility





- **Maneuverability advantage required for hit-to-kill** ***($\mu = a_{max_P}/a_{max_E}$)***

- **Air-to-air & surface-to-air experience**

PN	***μ > 5***
APN	***μ = 3-4***
OGL	***μ = 2-3***

- ***Against high maneuverability targets (μ < 2) a new guidance law is needed***

Background

Guidance Law Design

- ***Classical guidance based on PRONAV***
  - **LOS measurements (e.g., *R, Rdot,* σ_{el}, σ_{az})**
  - **Compensate for interceptor dynamics**
  - **Estimate target acceleration**
- ***Optimal guidance based on <u>Certainty Equivalence Principle</u> and associated <u>Separation Theorem</u>***
  - **Linear Quadratic guidance algorithm: infinite horizon, unbounded control**
  - **Extended Kalman Filter (EKF): assumed target acceleration, noise models**
- ***Differential game formulation based on zero-sum pursuit-evasion game***
  - **Optimal strategy for pursuer**
  - **"Worst case" target maneuver**
  - **Guaranteed miss distance**

Background

Estimator Design

- ***Linear systems with Gaussian noise***
  - **Kalman Filter is optimal (i.e., min variance, max likelihood)**
  - **Estimation error depends on discrepancy between actual and modeled dynamics, noise**
  - **Estimation latency (τ_{est}) depends on dynamics**
- ***Homing guidance problems***
  - **Nonlinear system: zero-mean, white, Gaussian <u>measurement noise</u>; bounded, discontinuous, non-Gaussian <u>process noise</u>**
  - **EKF: approximately linearizes system about estimate**
  - **Actually a nonlinear H_2/H_∞ problem: only approximate suboptimal solutions can be found**
- ***Witsenhausen conjecture (1971)***
  - **Nonlinear, non-white, non-Gaussian noise precludes application of Certainty Equivalence**
  - ***<u>A form of Separation applies</u>: Estimator can be designed independently; control law depends on conditional probability density of the estimate***

Background

Guidance System Challenge

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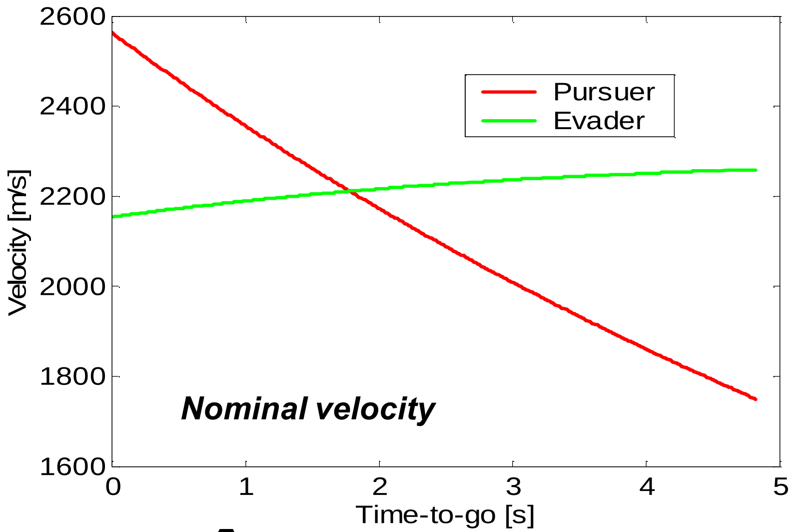
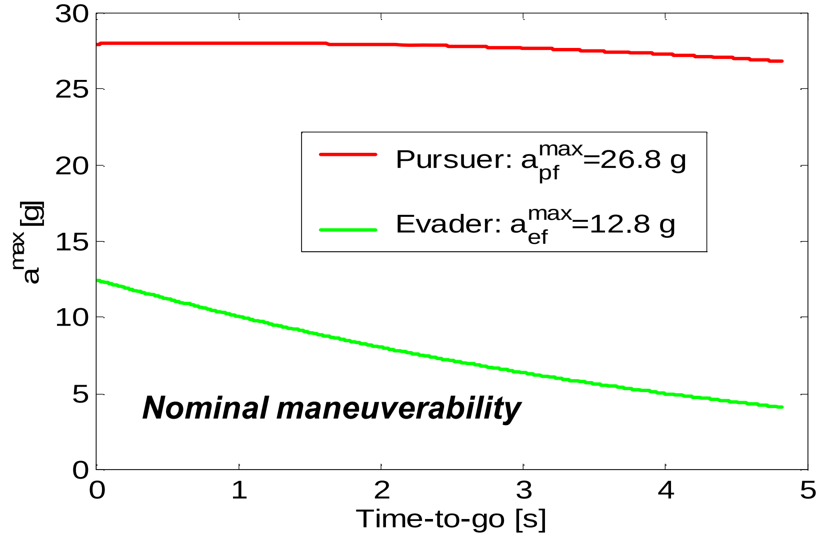
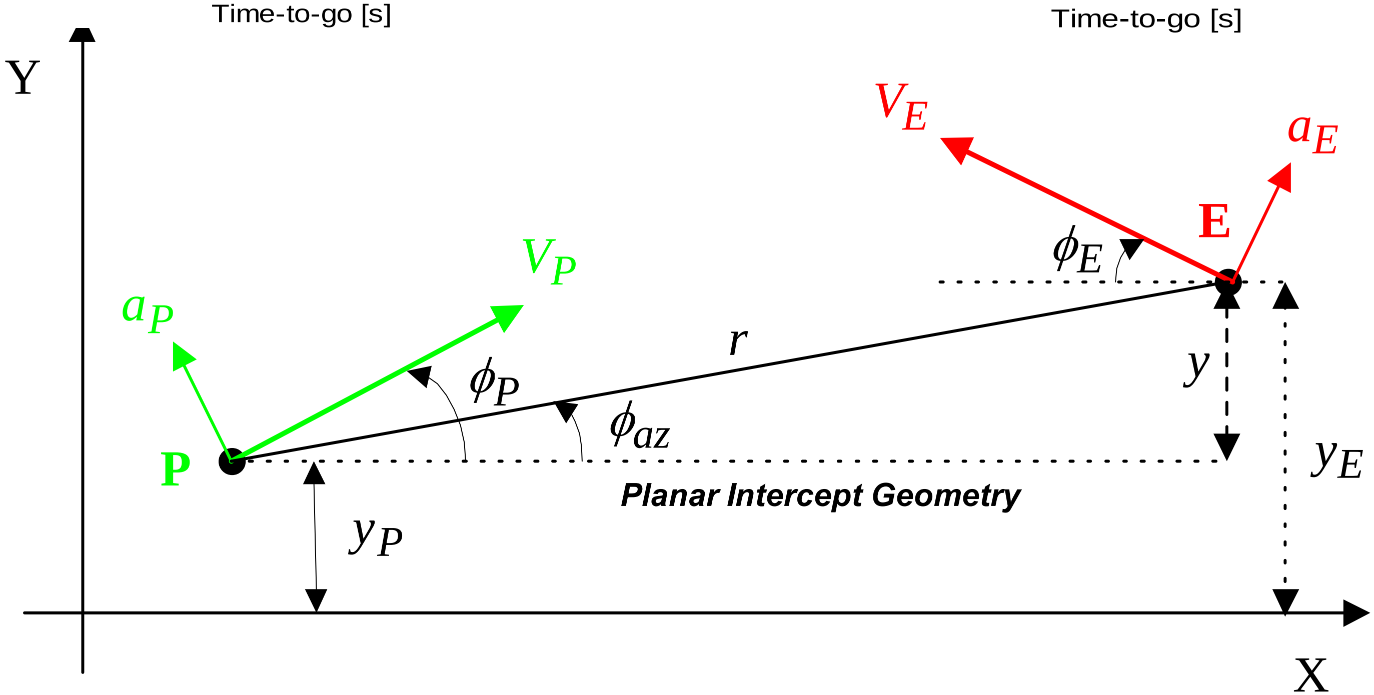
- ***Hit-to-Kill performance depends on uncertainty:***
  - **Actual target maneuver capability (intentional or not)**
  - **Discrepancy between modeled and actual target maneuver**
  - **Limitations of translating theory to practice**
- ***Guidance system design considerations***
  - **Interactions among seeker, estimator, guidance algorithm, interceptor dynamics, sensors as important as (probably more than!) particular components or algorithms**
  - **Higher fidelity target models are not panacea**
  - **Tuning for performance robustness against maneuver inevitably degrades nominal performance (i.e., hit-to-kill degrades to distribution of miss-distance)**

Intercept Scenario

Endo-atmospheric Endgame







Deterministic Guidance Law

Modeling Assumptions

(A-1) Perfect information structure

(A-2) Point-mass kinematics with linear control dynamics

(A-3) Relative endgame trajectory can be linearized around the initial (nominal) collision course geometry

(A-4) Profiles of the interceptor's and the target's nominal velocities and maximum lateral accelerations can be expressed as functions of time

(A-5) Interceptor and target have first order maneuvering dynamics

Linearization (A-3) allows the decoupling of the original 3-D scenario into two *planar* engagements in perpendicular planes, significantly simplifying the mathematical analysis

Deterministic Guidance Law

Interceptor Guidance Principle

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Universal formulation for interceptor guidance laws

$(a_P)^c$ = missile acceleration command

$$(a_P)^c = \mathcal{G}\{Z\}$$

Z = zero-effort miss distance (model dependent)

$\mathcal{G}$ = generalized operator

$\mathcal{G}(t)$; linear time varying gain

$\mathcal{G}\{\,.\,\}$ = nonlinear operator [sign { . }; sat { . }]

$$Z = Z_{PN} + \Delta Z_E - \Delta Z_P$$

Z_{PN} (kinematics); ΔZ_E (evader maneuver); ΔZ_P (own dynamics)

Deterministic Guidance Law

Perfect Information Game (DGL/1)

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Perfect information game with bounded controls

$$(a_P)^c = (a_P)_{max}\ \mathrm{sign}\ \{(Z)_{DGL/1}\}$$

with

$$(Z)_{DGL/1} = (Z)_{PN} + (\Delta Z_E)^1 - (\Delta Z_P)^1$$

where

$$(Z)_{PN} = y + (dy/dt)\ t_{go} = V_C\ t_{go}^2\ (d\lambda/dt)$$

$$(\Delta Z_E)^1 = a_E \tau_E^2\ [\exp(-t_{go}/\tau_E) + (t_{go}/\tau_E) - 1]$$

$$(\Delta Z_P)^1 = a_P \tau_P^2\ [\exp(-t_{go}/\tau_P) + (t_{go}/\tau_P) - 1]$$

Solution published in 1981 [1]; an extensive simulation study also published in 1981 [2]

1, Shinar, J., "Solution Techniques for Realistic Pursuit-Evasion Games" in *Advances in Control and Dynamic Systems,* C. T. Leondes, Ed., Vol. 17, Academic Press, NY 1981, pp.63-124.

2, Anderson, G. M., "Comparison of Optimal Control and Differential Game Intercept Missile Guidance Laws", *Journal of Guidance and Control*, Vol. 4. No. 2, 1981, pp. 109-115.

Deterministic Guidance Law

Optimality of DGL/1

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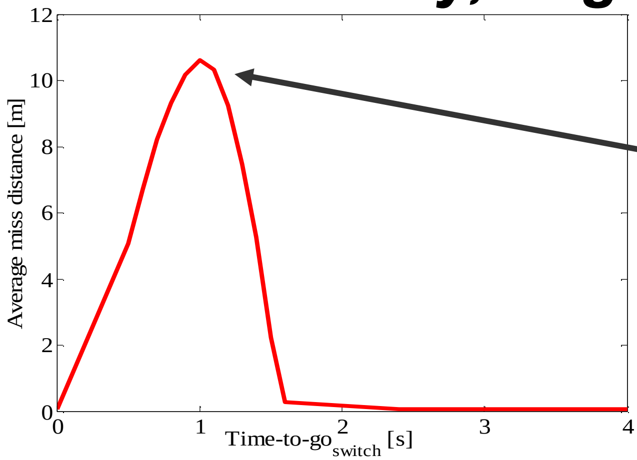
- ***DGL/1, with perfect information, guarantees zero miss distance***
  - **If $\mu = a_{max_P}/a_{max_E} > 1$ *(maneuverability advantage)***
  - **If $\varepsilon = \tau_P/\tau_E \leq 1$ *(no agility disadvantage)***
- ***Perfect information requires knowledge of current target acceleration***
  - **Not directly measurable**
  - **Estimate is scenario/model dependent**
- ***DGL/1 requires estimate of t_{go}***
  - **Implies need for active seeker**
  - **Estimate is scenario/model dependent**

Deterministic Guidance Law

Impact of Noisy Measurements

- ***DGL/1 with target state estimator (TSE)***
  - **Estimation errors induce guidance errors**
  - **Greatest estimation error source is *estimation latency***
- ***Target state estimation latency***
  - **Inherent in the dynamics (actual and modeled)**
  - **Inherent in the convergence dynamics of the TSE**
- ***<u>Target can exploit estimation latency</u>, intentionally or unintentionally, to generate large miss distances***



Large miss distance if target maneuvers at appropriate t_{go}

Homing performance of DGL/1 with EKF against "bang-bang" target maneuver commands

Deterministic Guidance Law

Correction for Information Delay (DGL/C)

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Perfect information game with delayed information

$$(a_P)^c = (a_P)_{max}\, sign\{(Z)_{DGL/C}\}$$

with

$$(Z)_{DGL/C} = (Z)_{PN} + (\Delta Z_E)^C - (\Delta Z_P)^1$$

where

$$(Z)_{PN} = y + (dy/dt)\, t_{go} = V_C\, t_{go}^2\, (d\lambda/dt)$$

$$(\Delta Z_E)^C = a_E \tau_E^2\, [\exp(-t_{go}/\tau_E) + (t_{go}/\tau_E) - 1]\, \exp(-\Delta t_{est}/\tau_E)$$

$$(\Delta Z_P)^1 = a_P \tau_P^2\, [\exp(-t_{go}/\tau_P) + (t_{go}/\tau_P) - 1]$$

Correction term for estimation delay

Solution published in 1999 [1]; a simulation study published in 2000 [2]

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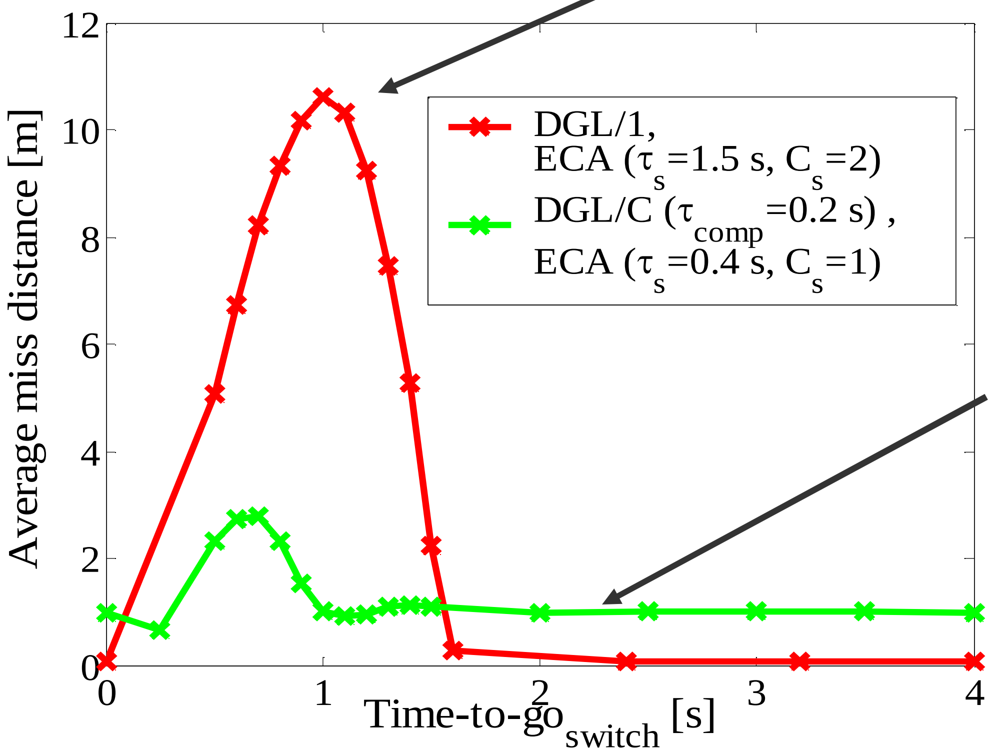
1 Shinar, J. and Glizer, V. Y. "Solution of a Delayed Information Linear Pursuit- Evasion Game with Bounded Controls" *International Game Theory Review*, Vol. 1, No. 3 & 4, 1999, pp. 197-218.

2. Shinar, J. and Shima, T., "Non-orthodox Guidance Law Development Approach for the Interception of Maneuvering Anti-Surface Missiles" AIAA paper 2000-4273, *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Denver, CO, August 2000.

Deterministic Guidance Law

Impact of Noisy Measurements

DGL/1 shows large miss distance if target maneuvers at appropriate t_{go}



DGL/C shows improved miss distance at critical t_{go}, at the expense of overall homing performance

Homing performance of DGL/1 and DLG/C with EKF against "bang-bang" target maneuver commands

Neither guarantees hit-to-kill!

Estimation for Homing Guidance

Estimator Design

- ***Contradictory Design Requirements***
  - **Convergence time for identifying a target maneuver includes *maneuver detection time* plus *estimator response time***
  - **Minimizing *maneuver detection time* increases false alarm rate**
  - **Minimizing *estimator response time* requires high bandwidth filter, increasing estimation error**
- ***Does an optimal guidance algorithm/TSE exist?***
  - **No!**
  - **Theory is incomplete**
  - **Guidance algorithm/TSE requires Monte Carlo tuning**
- ***Implications?***
  - **No guidance algorithm/TSE exists for all target maneuvers**
  - ***New guidance system design approach required!***

New Approach

Consistent Guidance System Design Philosophy

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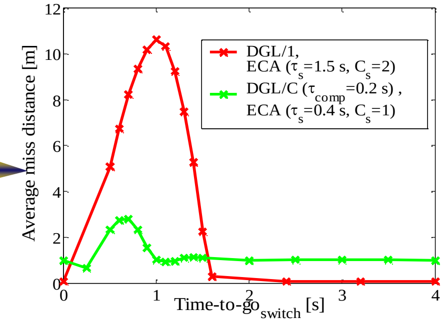
- ***Witsenhausen's conjecture of partial separation***
  - **Optimal estimator doesn't exist; design *suboptimal TSE***
  - **Derive guidance algorithm compatible with suboptimal TSE**
  - ***NOTE: novelty is estimator THEN guidance algorithm design***
- ***Time-to-go (t_{go})***
  - **Time-to-go is "Achille's heel" of endgame guidance**
  - **Estimator must be designed for short time-to-go not infinite horizon performance ➔ "tuned" for critical time-to-go = $(t_{go})_{switch}$**
  - **Guidance algorithm/TSE system must be tuned for the endgame**
- ***Why focus on endgame?***
  - **Hit-to-Kill**
  - **Target maneuvers outside of a narrow time-to-go window can be accommodated by any stable homing law, given sufficient interceptor capability**
  - ***Appropriate target maneuvers inside of a narrow time-to-go window can defeat ANY conventional guidance law***

New Approach

Logic-based Guidance



- ***Model Identification*:** $t_{go} > (t_{go})_{crit} = 1.6$ **sec**
  - – **Nominal guidance law DGL/0 (DGL/1 with $\Delta Z_E = 0$) with narrow bandwidth estimator ➔ insensitive to model errors**
  - – **Wide bandwidth multi-model estimator ➔ maneuver model identification**
- ***Model Identified*:** $t_{go} > (t_{go})_{crit} = 1.6$ **sec**
  - – **Endgame guidance law DGL/1 with narrow bandwidth TSE tuned to identified maneuver ➔hit-to-kill guidance**
  - – **Wide bandwidth multi-model estimator {$(t_{go})_{switch}$ = *1.6, 1.0, 0.5 sec*} ➔ maneuver change of direction ("jump")**
- ***"Jump" Detection*:** $t_{go} \leq (t_{go})_{crit} = 1.6$ **sec**
  - – **No maneuver "jump" detected: DGL/1 with narrow bandwidth TSE ➔ sufficient time to counter maneuver**
  - – **Maneuver "jump" detected: DGL/1 with nearest wide-bandwidth tuned estimator ➔ best response against late maneuver**

New Approach

Logic-based Guidance

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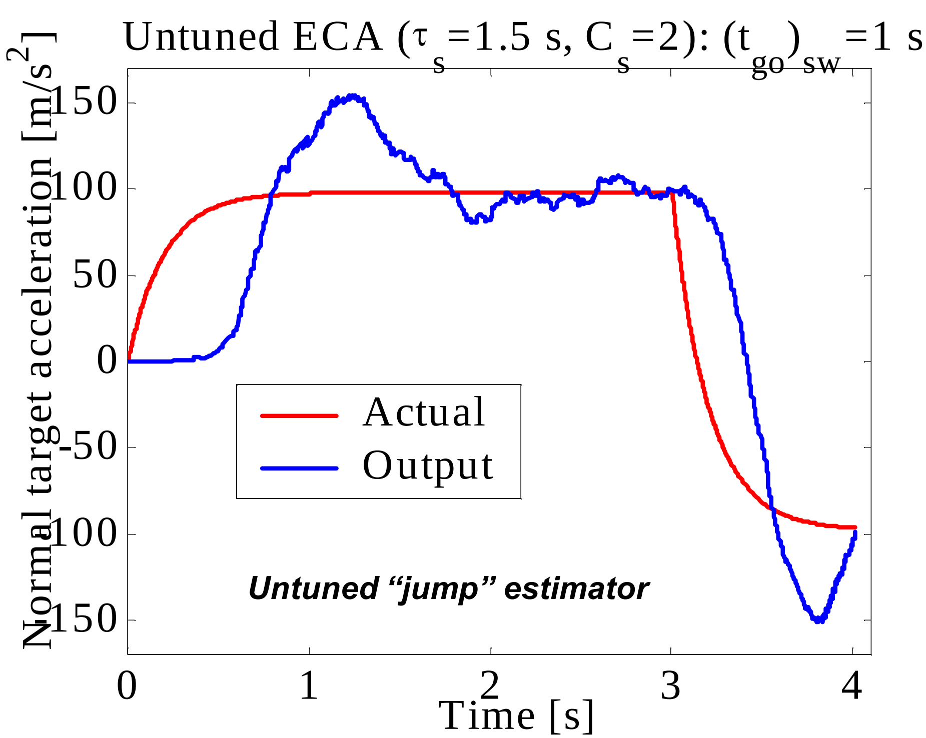
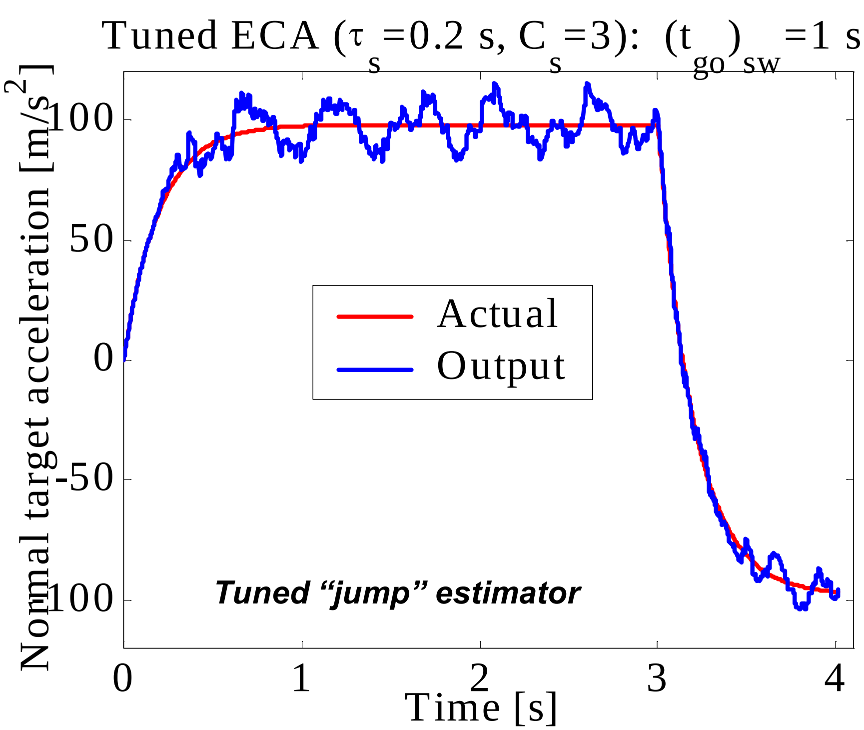
- ***Model Identified: $t_{go} > (t_{go})_{crit}$ = 1.6 sec***
  - Endgame guidance law DGL/1 with narrow bandwidth TSE tuned to identified maneuver ➔ hit-to-kill guidance
  - **Wide bandwidth multi-model estimator {($t_{go})_{switch}$ = *1.6, 1.0, 0.5 sec}* ➔ maneuver change of direction ("jump")**

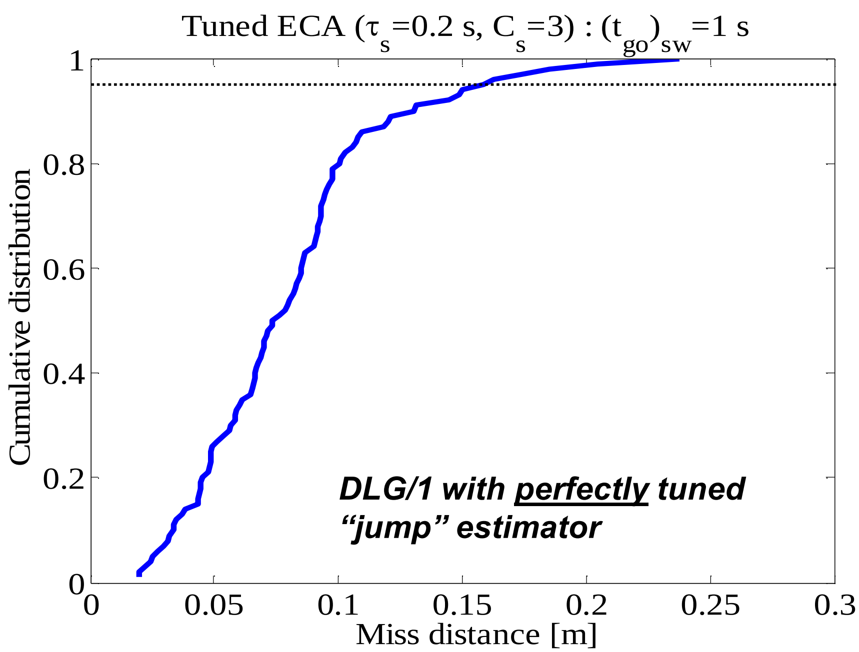


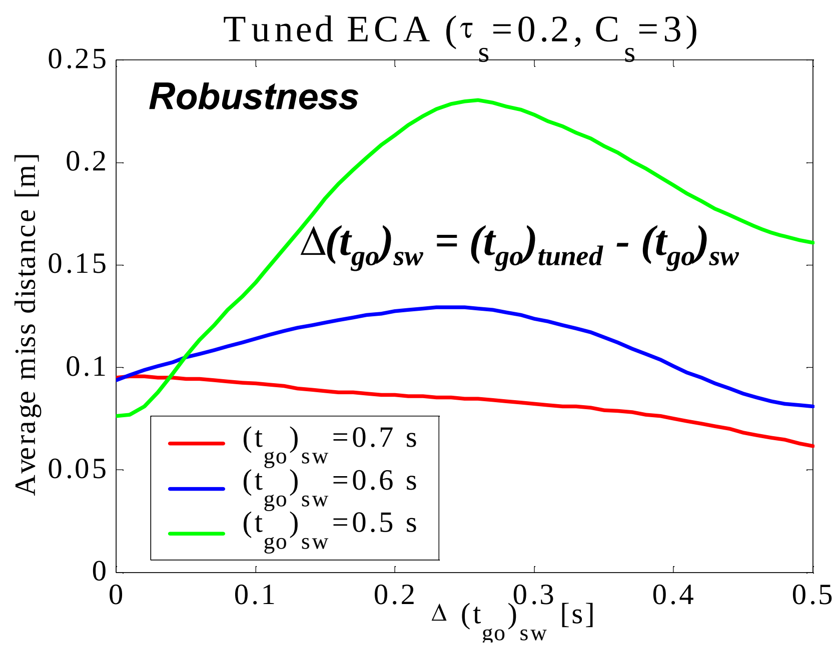


New Approach

Logic-based Guidance

- ***"Jump Detection: $t_{go} \leq (t_{go})_{crit}$ = 1.6 sec***
  - No maneuver "jump" detected: DGL/1 with narrow bandwidth TSE ➔ sufficient time to counter maneuver
  - **Maneuver "jump" detected: DGL/1 with nearest wide-bandwidth tuned estimator ➔ best response against late maneuver**





New Approach

Logic-based Guidance

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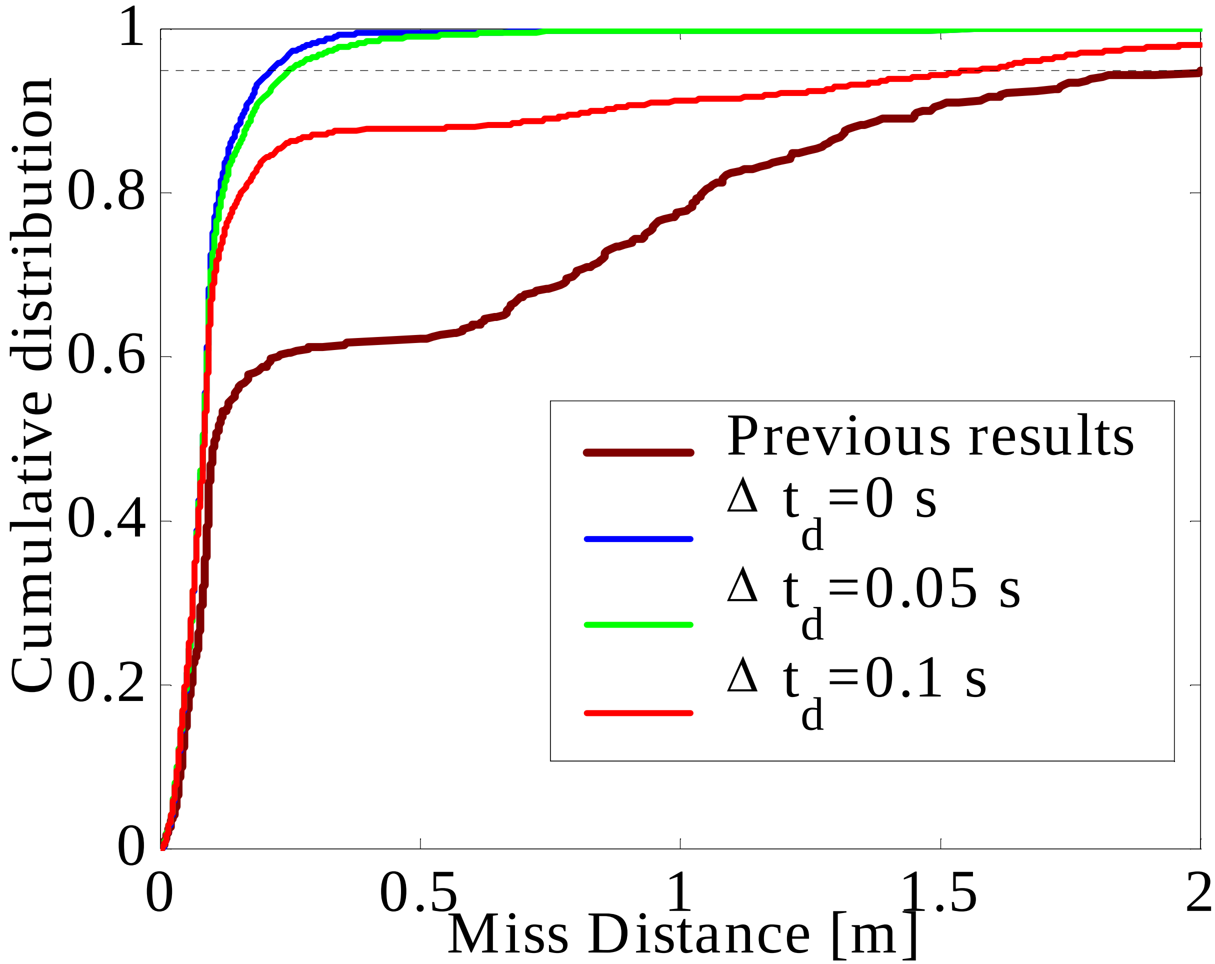

- ***Δ t_d is the maneuver detection delay***
- ***100 Monte Carlo runs for $(t_{go})_{sw}$= [0:0.1:4]sec***

New Approach

Guidance Modifications

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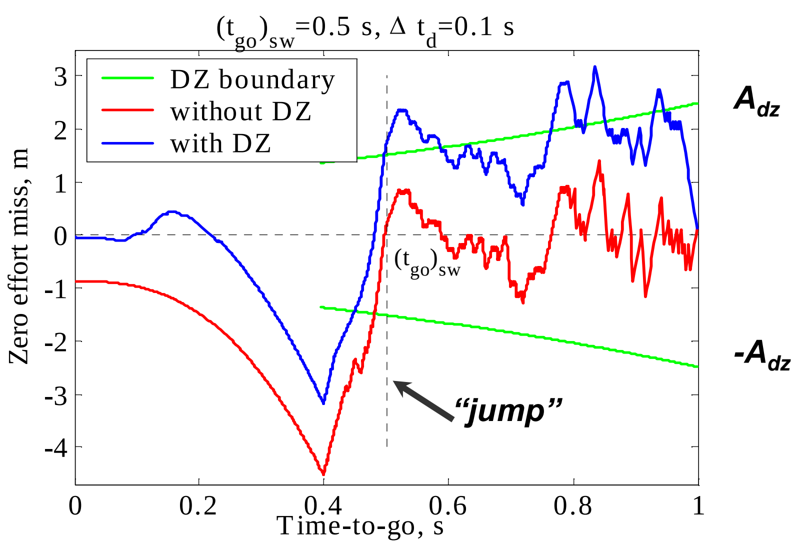
- **Insufficient lateral acceleration in endgame**
  - **Due to short t_{go} and detection delay**
  - **Increase acceleration gain for $t_{go} \leq (t_{go})_{sw}$**

$$a_p^c = a_p^c(t_{go}, k) = \frac{a_p^{max}\ \mathrm{sign}\, Z}{1 - k \exp\left(-\, t_{go} / \tau_P\right)}$$

where *k* satisfies $|\, a_p(t_f, k) \,| = a_P^{max}$

- **Time-varying zero-effort miss deadzone <u>before</u> "jump" detection for *1.6 s> t_{go} >0.2***

$$sign_{dz}(Z) = \begin{cases} 1.0, & Z > A_{dz} \exp(-b_{dz} t) \\ 0.0, & |Z| \leq A_{dz} \exp(-b_{dz} t) \\ -1.0, & Z < -A_{dz} \exp(-b_{dz} t) \end{cases}$$



New Approach

Logic-based Guidance with Modifications

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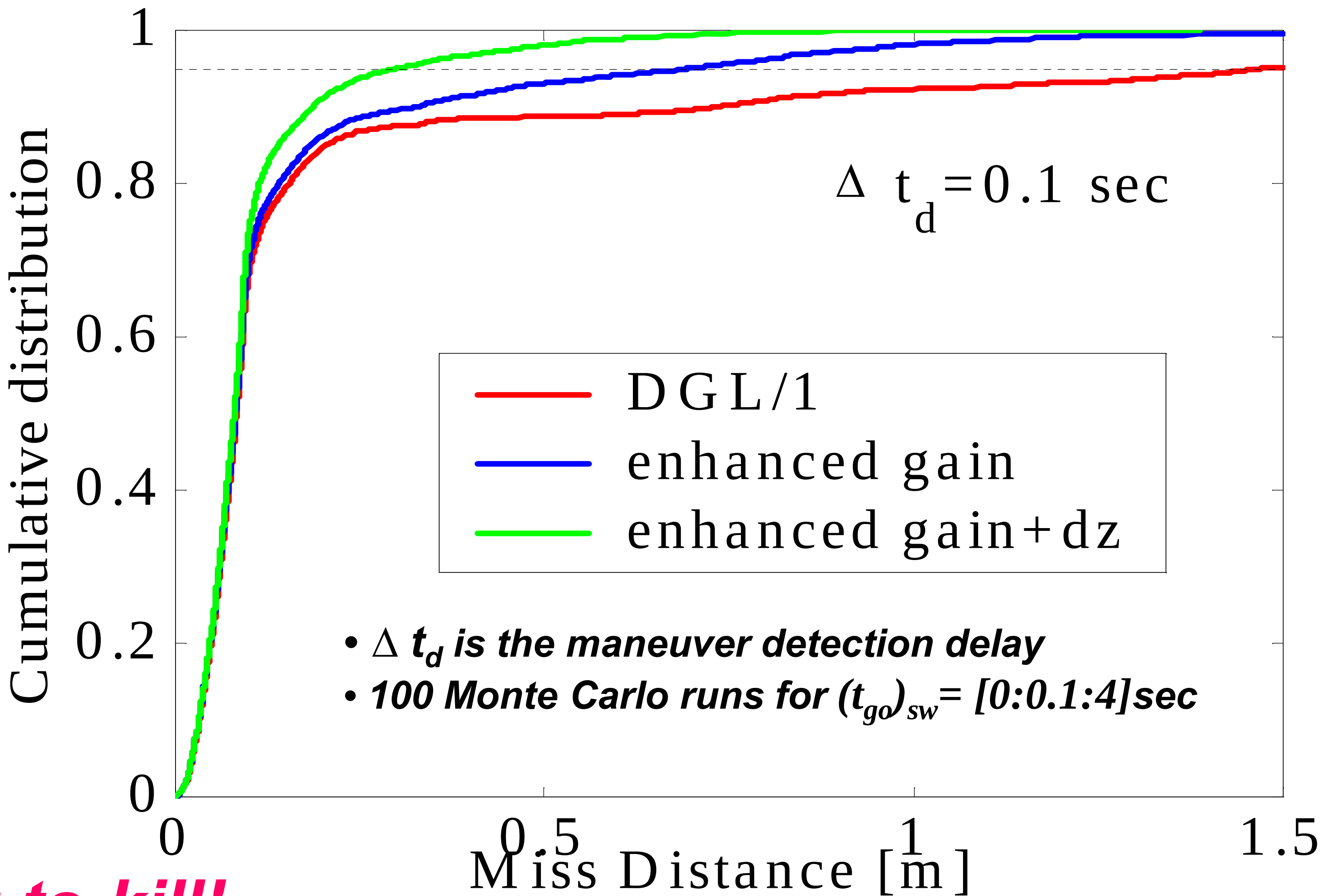

Hit-to-kill!

But these are 2D intercepts. What about 3D?

New Approach

Generic 3-D BMD endgame scenario

- **Nominal point defense scenario**
- **Desired altitude for the interception 20 km.**
- **Cruciform, aerodynamically controllable TBM (pitch and roll), with a given *ballistic coefficient* β = 5 ton/m^2 and a *lift to drag ratio* of Λ = 2.6 that can perform either horizontal or "spiral" maneuvers**
- **Cruciform interceptor with solid rocket propulsion of two stages (with 3 seconds delay between them); aerodynamically controlled and roll-stabilized. Maneuverability is limited by the maximum lift coefficient**
- **Homing endgame starts at a slant range of 20 km**
- **Time varying velocity, maneuverability and roll rate profiles**
- **Guidance laws adapted to time-varying endgames**

New Approach

Generic 3-D BMD endgame scenario

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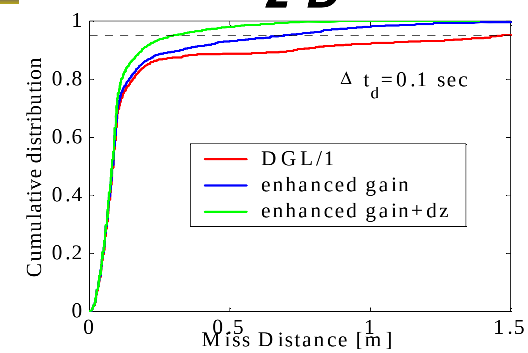
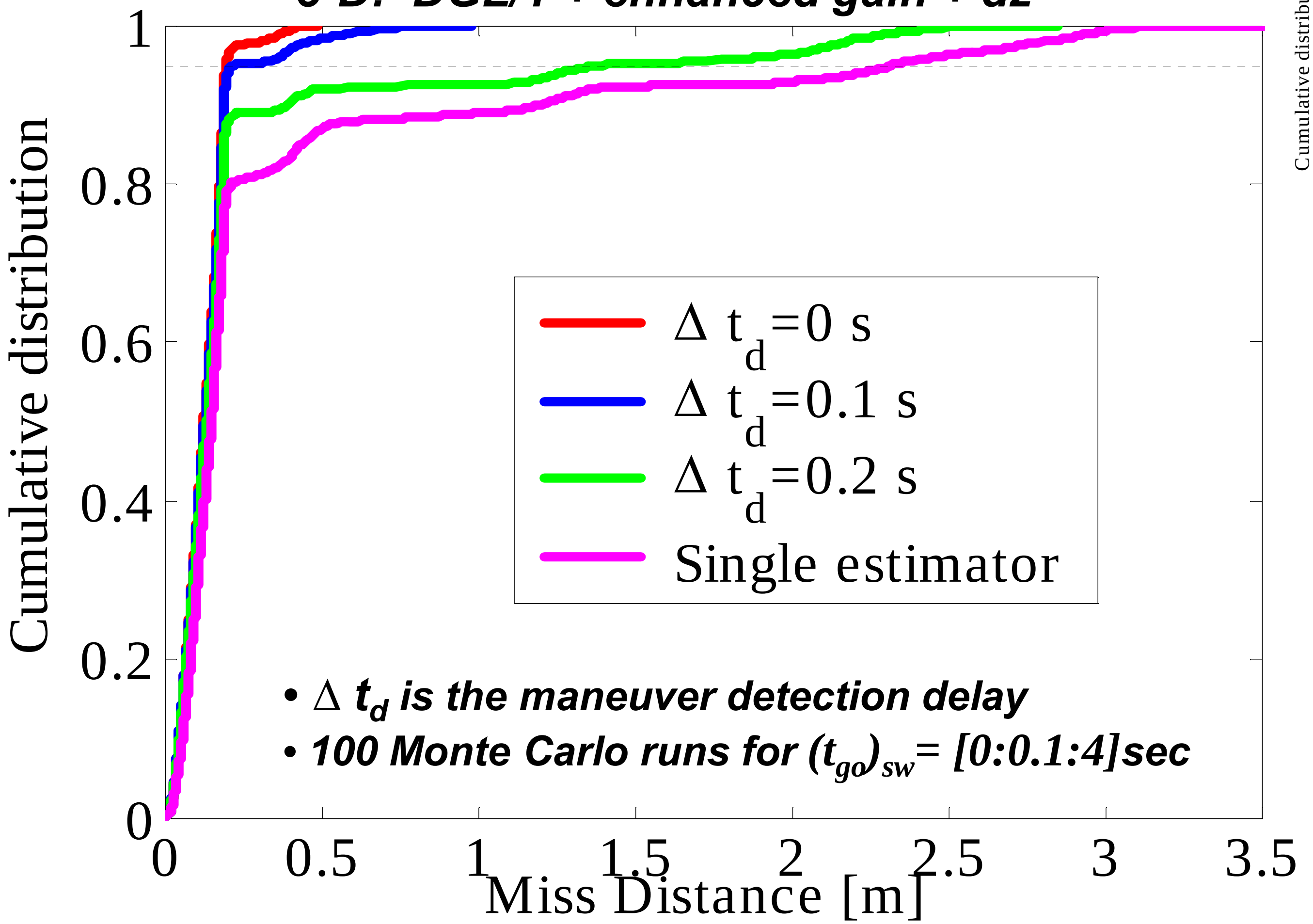
2-D



3-D: DGL/1 + enhanced gain + dz



Note impact of increased Δ t_d ! Need fast detection!

Self-Protection Scenario

Tal Shima, Technion (2009)

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Goal: Three player game solution space

1. *Maneuver strategy for E*
2. *Guidance strategy for D*
3. *Given assumptions on P guidance strategy*

Approach:

1. *D&E cooperate, share perfect info on P*
2. *D&E cooperate, share imperfect info on P*