Cohesive fracture: robustness issues and algorithms

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Abstract

It is suggested that the isoperimetric property of isotropic meshes for numerical discretization is a necessary condition for any possible spatial convergence of cracks constrained to propagate along mesh-cell edges in the general case that the crack path is not known in advance. Conversely it is established that if a two-dimensional isotropic mesh is used, the discrete interface first activated in a finite element model will converge, as the mesh size tends to zero, to the theoretical initial crack. Convergence takes place in three senses: position of the interface, time of initiation, and orientation of the interface [1]. Isotropic meshes are also used to produce characteristic volumes of composite materials with randomly oriented fibers.

Further, the notion of time continuity for the analysis of cohesive zone interface finite element models is introduced. The focus is on initially rigid models in which an interface is inactive until the traction across it reaches a critical level. It is argued that methods in this class are time discontinuous, unless special provision is made for the opposite. Time discontinuity leads to pitfalls in numerical implementations: oscillatory behavior, non-convergence in time and dependence on nonphysical regularization parameters [2]. These problems arise at least partly from the attempt to extend uniaxial tractiondisplacement relationships to multiaxial loading.

Previous attempts to address time discontinuity are discussed. A potential-based formulation for the traction-displacement relation of an initially rigid cohesive model is proposed [3]. The key feature of the energy functional is a term for the energy stored in the interfaces that is nondifferentiable at the origin. A consequence of this formulation is that, unlike other initially rigid cohesive models proposed in the literature, there is no need to define an activation criterion as a separate entity from the traction-displacement relationship itself. Instead, activation happens automatically when the load reaches a critical level because the minimizer of the potential no longer occurs at the 0-displacement level. Thus, the activation computation necessary in previous initially rigid formulations is now replaced by the computation of a minimizer of a nondifferentiable objective function. This immediately makes the method more amenable to implicit time stepping, since the activation criterion no longer interacts with the nonlinear solver for the next time step. The optimization problem is solved by a continuation (homotopy) method used in conjunction with an augmented Lagrangian and a trust region minimization algorithm to find the minimal energy configuration. Because the approach eliminates the need for an activation criterion, the algorithm sidesteps the complexities of time-discontinuity and traction-locking previously observed in relation to initially rigid models.

References

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