Moving layers and interfaces : a thermodynamical approach of Wear Contact

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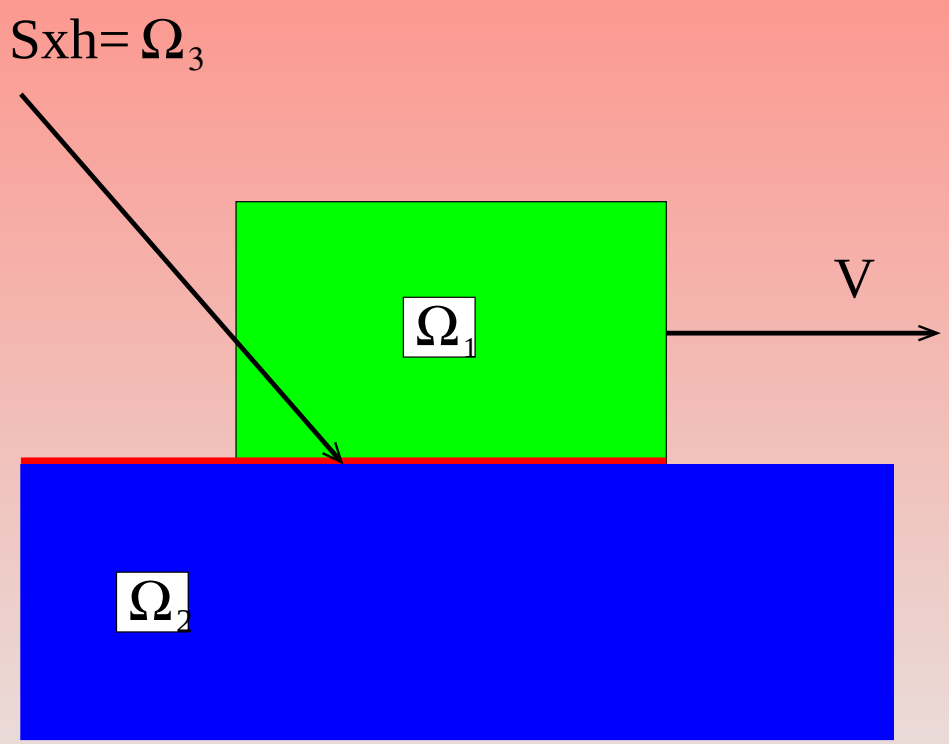
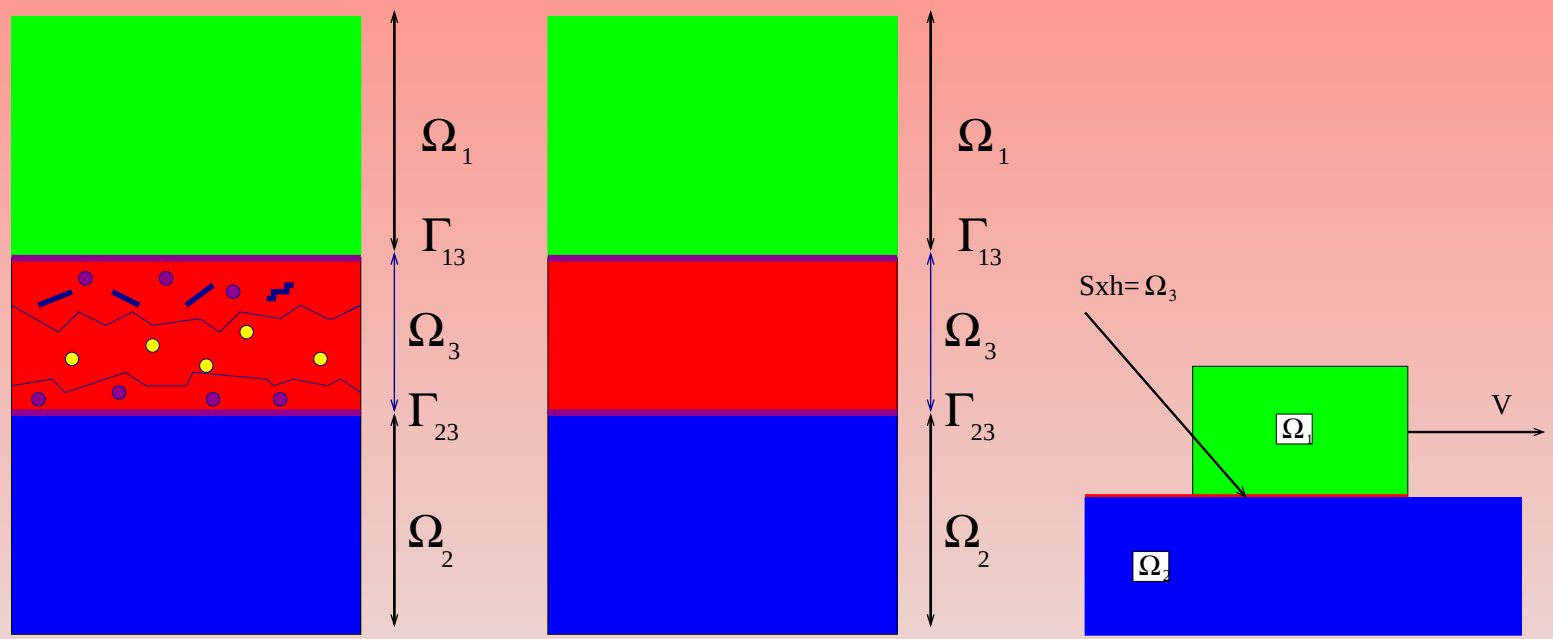


Loss of sound material is due to wear.

Friction - contact between bodies - interface propagation

Essai de video essai de vidéo



Model of damage - Dissipation - Global behaviour of the interface Ω_3

$\Omega_3 = Sxh$: detached particles, damaged media, fluids : complex behaviour

Integration over the thickness : constitutive equation for the interface S

Moving interface / moving layer

Material characteristics change :

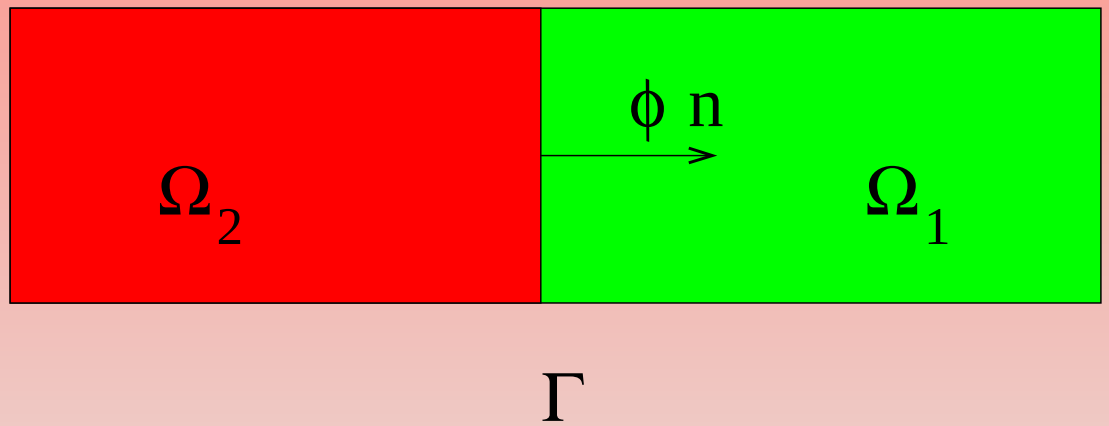
- Brutal transition : moving interface and discontinuities
- Continuous Transition : moving layer and continuous damage laws.

Moving interface and Transformation of material

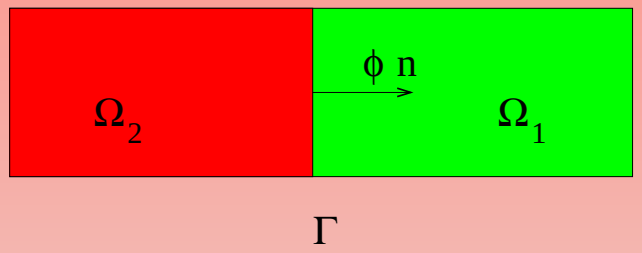
Material 1 is changed in material 2 along a moving interface

the mass flux is the loss of sound material.

Preliminaries



$$
\begin{aligned}
F &= \int_a^b f(x,t)\,dx \\
&= \int_a^{\boxed{\Gamma(t)}} f(x,t)\,dx + \int_{\boxed{\Gamma(t)}}^b f(x,t)\,dx
\end{aligned}
$$

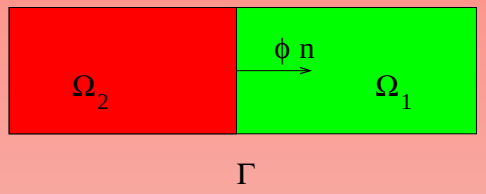


$$\dot{F} = \int_a^b \dot{f}\, dx + (\boxed{f(\Gamma^+, t)} - \boxed{f(\Gamma^-, t)})\dot{\Gamma}$$

$$[\, f \,]_\Gamma = f(\Gamma^+, t) - f(\Gamma^-, t)$$

Propagation of interface

$\Rightarrow$ jump conditions.



Each medium is linear elastic : x proportion of phase 2

Equilibrium :

$$\sigma = E_1\varepsilon_1 = E_2\varepsilon_2 = \Sigma, \quad u(0) = 0$$

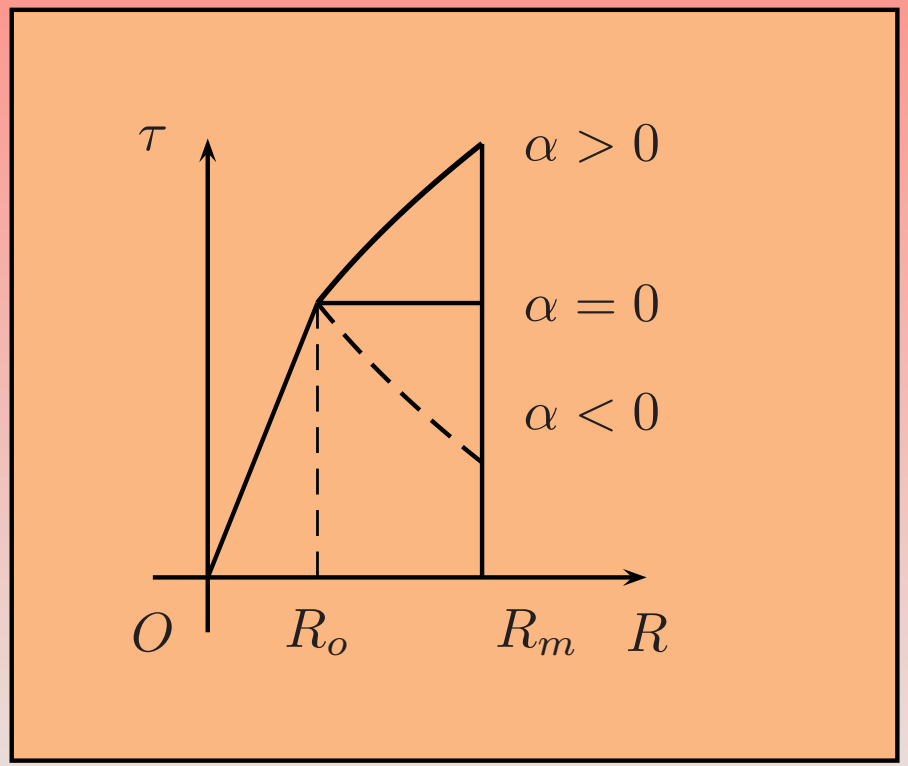
Energy :

$$W(\sigma, x) = -\frac{1}{2}\sigma^2(\frac{x}{E_2} + \frac{1-x}{E_1})$$

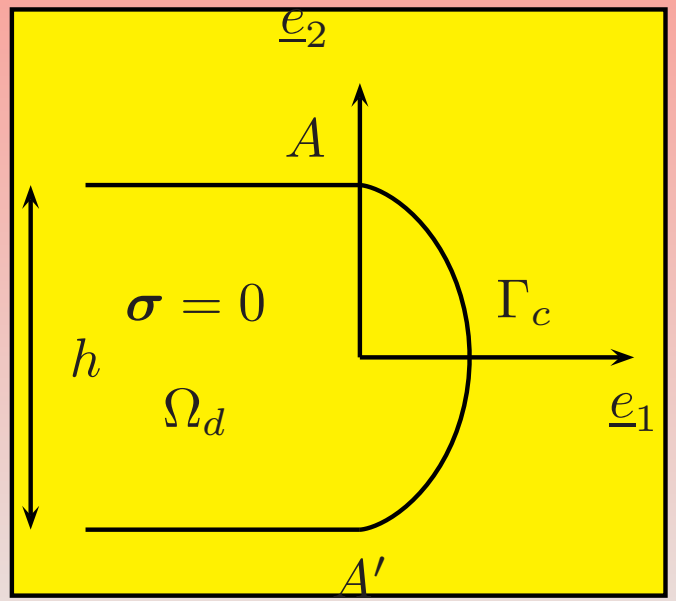
Energy release rate

$$G = -\frac{\partial W}{\partial x} = \frac{1}{2}\sigma^2(\frac{1}{E_2} - \frac{1}{E_1}); \quad D_m = G\dot{x}$$

Quasicrack : antiplane shear conditions



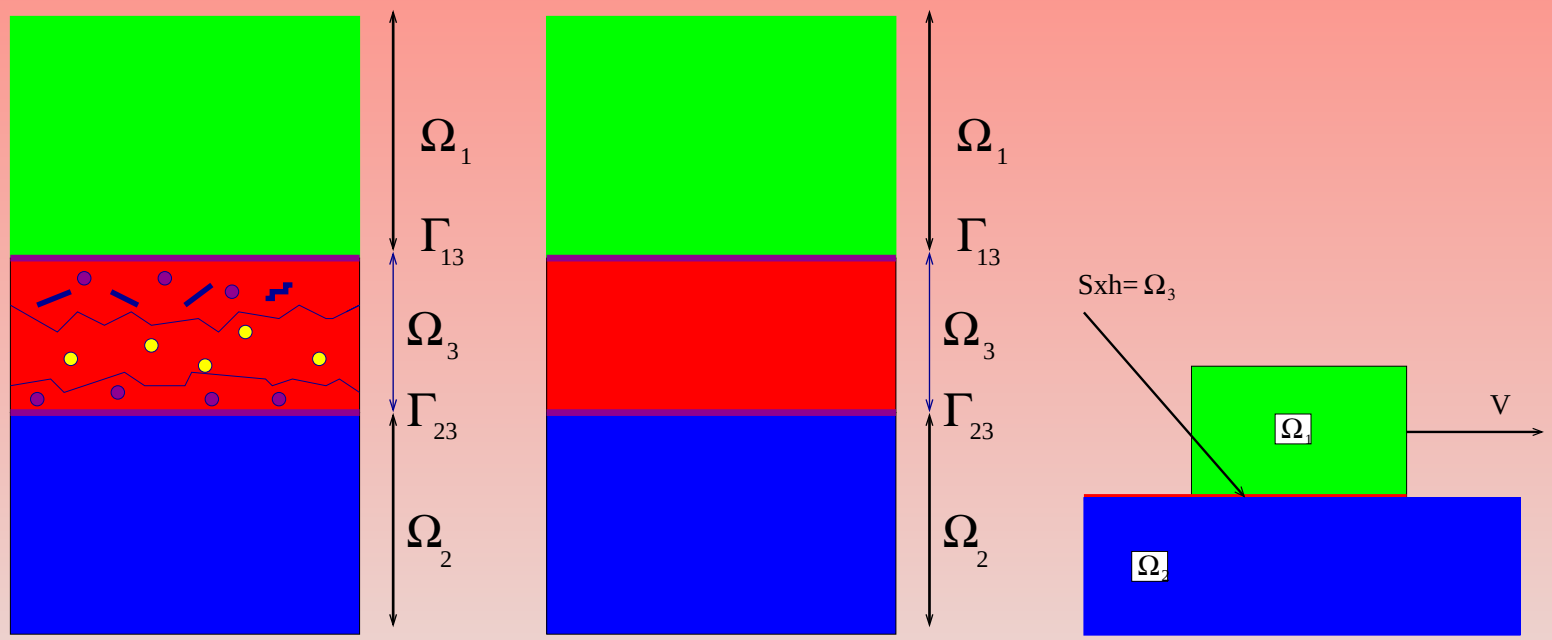
$\boldsymbol{\sigma} = K/\sqrt{r}f(\theta)$ as $r \to \infty$



$$K^2 = h\frac{\tau_o^2}{\alpha+1}\Big[(\frac{R_m}{R_o})^{\alpha+1} + \frac{\alpha-1}{2}\Big]$$

Neuber (1969), Bui-Ehrlacher (1978), Stolz (2010).

Wear description- Dissipation



$$D_m = \int_S (G_{13}\phi_1 + G_{23}\phi_2 + \int_h d_3 dz)\ \mathrm{d}S$$

$$G = [\,w\,]_\Gamma - \boldsymbol{\sigma} : [\,\boldsymbol{\varepsilon}\,]_\Gamma$$

Interpretation

$$D_\Gamma = \mathcal{G}\ \phi, \quad m = \rho\phi$$

Then

$m = 0$, no loss of sound material $\Rightarrow$ no wear

$$\boxed{\begin{cases} \mathcal{G}(X,t) < G_c, & \phi = 0 \\ \mathcal{G}(X,t) = G_c, & \phi \geq 0 \end{cases}}$$

friction $D_3 = \int_h d_m\ dz = \int_h (\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \dot{w}_3)\ dz$

Moving Layer

The material is described with continuous damage

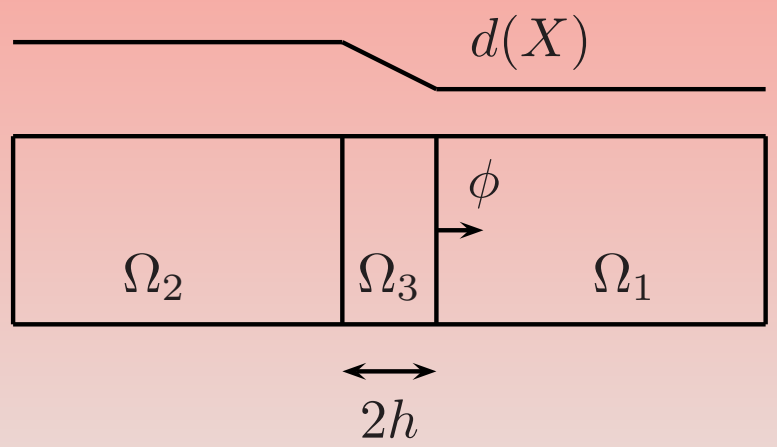
In Ω free energy : $w(\boldsymbol{\varepsilon}, d, \alpha) = (1-d)w_o(\boldsymbol{\varepsilon}, \alpha) + H(\alpha)$

$$Y = -\frac{\partial w}{\partial d}, \quad A = -\frac{\partial w}{\partial \alpha}$$

d varies from 0 to 1, no discontinuities.

$$d_m = Y\dot{d} + A\dot{\alpha}$$

Dissipation

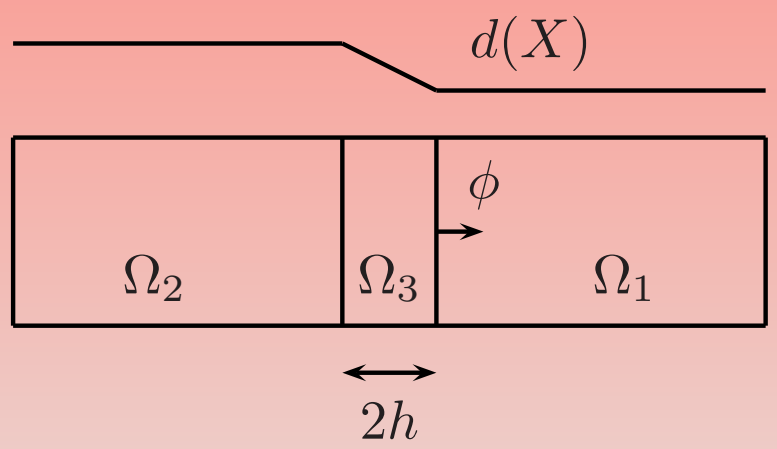


$$D_m = \int_h Y\dot{d} + A\dot{\alpha}\, dz$$

but in the moving frame

$$f(x,t) = f(x - \phi t\, \underline{n}, t) \quad \dot{f} = -\phi \nabla f.\underline{n} + D_\phi f$$

1-D Example :



Free energy : $W = \frac{1}{2}E(d)\varepsilon^2$

Matching Conditions $E(0) = E_1, E(1) = E_2$

Total Dissipation under steady-state condition $(\dot{d} + \phi\nabla d.\underline{n} = 0)$

$$D_m = \phi \int_h \frac{1}{2}\sigma^2(-\frac{E'}{E^2})\nabla d \, \mathrm{d}z = \phi\frac{1}{2}\sigma^2 \int_1^0 (-\frac{E'}{E^2})\mathrm{d}d = G\phi$$

Dissipation with moving layer

$$D_m = \phi \int_h (Y\nabla d + A\nabla\alpha)dz + \int_h (YD_\phi d + AD_\phi\alpha)\, dz$$

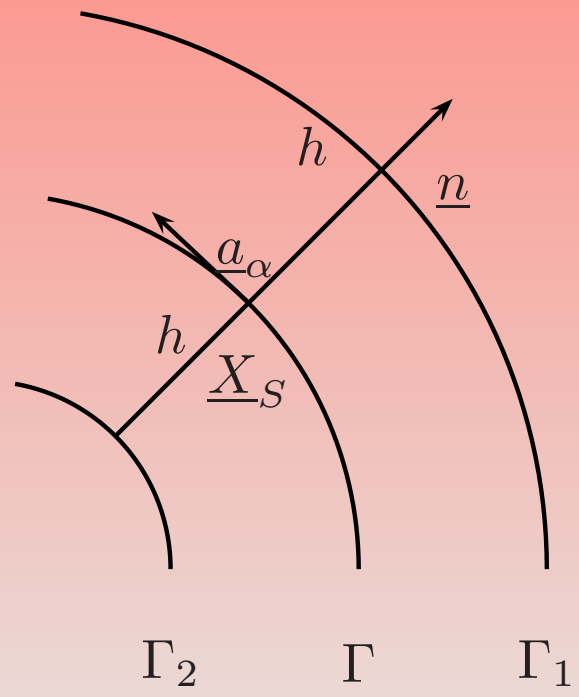
that is

$$D_m = \bar{G}\phi + \int_h d_\phi\, dz$$

Steady-state motion : the last term is zero.

$$\bar{G} = \int_h (Y\nabla d + A\nabla\alpha)dz \leq hY_c$$

Geometry of the layer, Curvature tensor : $\nabla \underline{n}.\underline{a}_\alpha = -b_\alpha^\beta \underline{a}_\beta$



$$\mathrm{d}S_i = \det(b \pm_i h\, \mathbf{I}) = j_i\, \mathrm{d}S$$

$$\int_B f\, \mathrm{d}\Omega = \int_\Gamma (\int_h f j(z) dz)\, \mathrm{d}S = \int_\Gamma f_S\, \mathrm{d}S$$

Eshelby momentum tensor

$$\boldsymbol{P} = w\,\mathbf{I} - \boldsymbol{\sigma}.\nabla\underline{u}$$

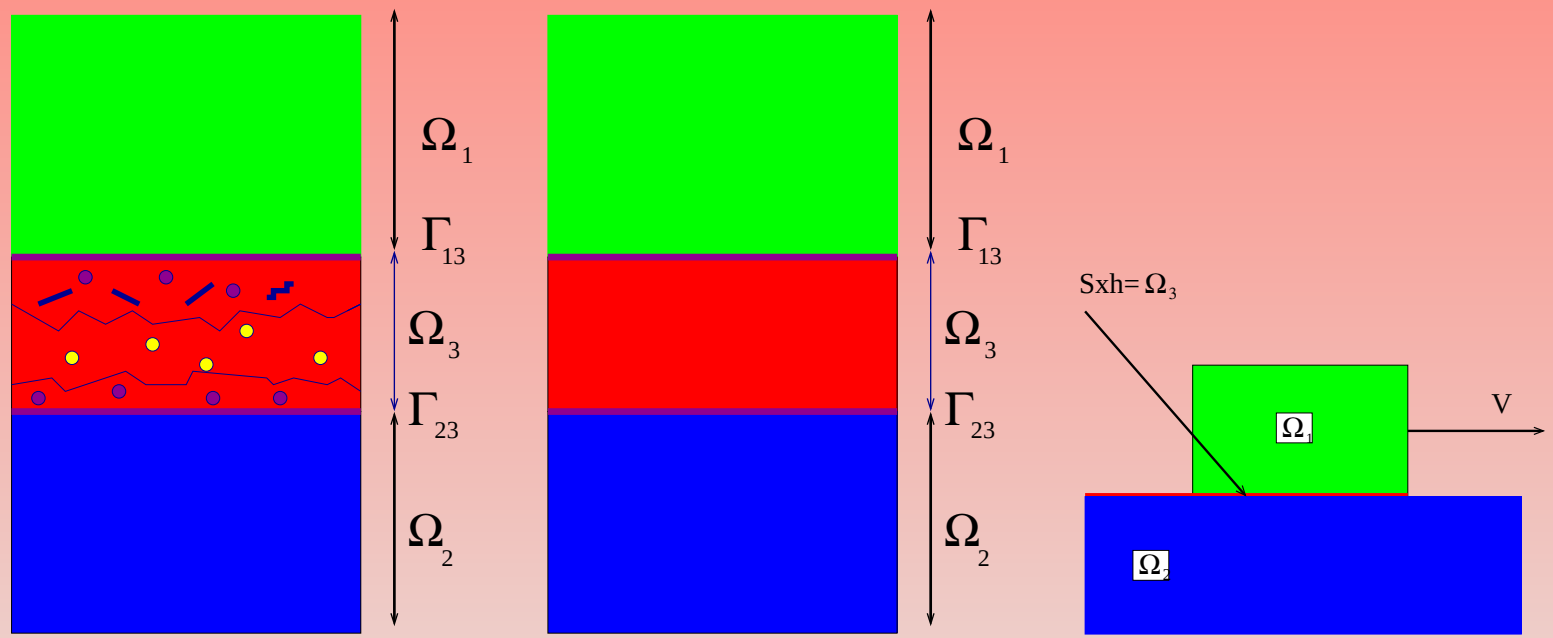
Then

$$\operatorname{div}\boldsymbol{P}^T = -Y\nabla d - A\nabla\alpha$$

Introducing the relative coordinate

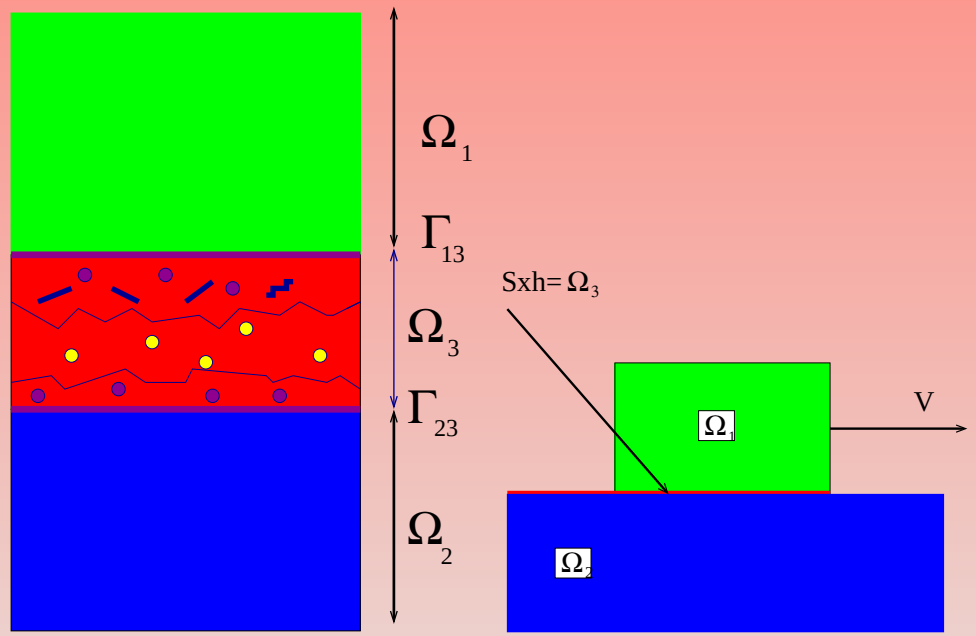
$$\underline{x} = \underline{X}_S + z\underline{n}, \quad D_\phi\,\underline{x} = \phi\underline{n} + z\nabla\phi$$

$$\int_\Gamma \bar{G}\phi(s)\,\mathrm{d}S = -\int_B \boldsymbol{P} : \nabla(D_\phi\,\underline{x})\,\mathrm{d}\Omega + \int_{\Gamma_i} \underline{n}.\boldsymbol{P}.D_\phi(\underline{x})j_i\,\mathrm{d}S$$



$$D_m = \int_S (\bar{G}_1\phi_1 + \bar{G}_2\phi_2 + \int_{h_3} d_3 dz)\ \mathrm{d}S$$

$$\bar{G}_i(s) = (\int_{h_i} (Y\nabla d + A\nabla\alpha) j\ \mathrm{d}S).\underline{n}(s)$$



$$\psi_s \rho_s = \int_h w(\varepsilon, \alpha)_z \rho_z j(z) \, \mathrm{d}z$$

$$\mathrm{D}_s \rho_s = \int_h \mathrm{D}(\dot{\varepsilon}, \dot{\alpha})_z \rho_z j(z) \, \mathrm{d}z$$

Dissipation associated with the interface

Total potential energy : $\int_\Omega w\rho \, \mathrm{d}\Omega + \int_\Gamma \psi_s \rho_s \, \mathrm{d}S$

$$
\begin{aligned}
D_m &= \int_{\partial\Omega} \underline{n}.\boldsymbol{\sigma}.\underline{v} \, \mathrm{d}\Omega - \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\Omega_i} w_i \rho_i \, \mathrm{d}\Omega + \int_\Gamma \rho_s \psi_S \, \mathrm{d}S\right) \\
&= \int_\Gamma (-\boldsymbol{\sigma}_i : [\,\boldsymbol{\varepsilon}_i\,]_\Gamma + \rho_i(w_i - \psi_S)) j_i \phi_i \, \mathrm{d}S \\
&+ \int_\Gamma \frac{\partial \mathrm{D}_S}{\partial \underline{v}_i}.\mathcal{D}_\phi \underline{u}_i + \frac{\partial \mathrm{D}_S}{\partial \alpha}.\mathcal{D}_\phi \alpha + Q.\mathcal{D}_\phi \rho_s \, \mathrm{d}S
\end{aligned}
$$

The mass density is an internal parameter for the interface.

The behaviour of the layer

Main idea : $-h \leq z \leq h$

$$\underline{u}_3(X_S, z) = \underline{u}^o(X_S) + z\underline{u}^1(X_S) + ...$$

Continuity of displacement

$$u_1 = \underline{u}_3(X_s, h), \quad u_2 = \underline{u}_3(X_s, -h)$$

Then

$$w(X_s) = u_1 - u_2 = 2h\nabla\underline{u}.\underline{n}, \quad \underline{u}^o = \frac{1}{2}(\underline{u}_1 + \underline{u}_2) + o(h^2)$$

Introducing these asymptotic expansion in global potential for the layer

Order 0

Free global energy :

$$\rho_s \psi_s(\underline{u}_1, \underline{u}_2, \alpha) = \int_h w_3(\boldsymbol{\varepsilon}(X + z\underline{n}), \alpha)\rho_3 j(z) dz$$

Thermodynamical forces

$$\underline{T}_i^r = \rho_s \frac{\partial \psi_s}{\partial \underline{u}_i}, A = -\rho_s \frac{\partial \psi_s}{\partial \alpha}, G_h = -\rho_s \frac{\partial \psi_s}{\partial h}$$

Global potential of dissipation

$$\rho_s D_s(\underline{v}_1, \underline{v}_2, \dot{\alpha}) = \int_h \rho_3 d_3(\dot{\boldsymbol{\varepsilon}}, \dot{\alpha}) dz$$

Then

$$\underline{T}_i^{ir} = \rho_s \frac{\partial D_s}{\partial \underline{v}_i}, \ldots$$

Equilibrium on interface

$$\underline{n}.\boldsymbol{\sigma} j_i = \underline{T}_i^r + \underline{T}_i^{ir}$$

More informations : order 1

The energy depends on $\nabla \underline{u}^o, \underline{w}$, so

$$\rho_s \psi_s = f(\underline{u}_i, \nabla \underline{u}_i), \quad \rho_s D_s = g(\underline{v}_i, \nabla \underline{v}_i)$$

$$\rho_s = \int_h \rho_3 dz, \text{ Conservation}$$

The equilibrium is then obtained as

$$\underline{n}.\boldsymbol{\sigma} j_i = \rho_s \frac{\partial \psi_s}{\partial \underline{u}_i} - \nabla(\rho_s \frac{\partial \psi_s}{\partial \nabla \underline{u}_i}) + \rho_s \frac{\partial D_s}{\partial \underline{v}_i} - \nabla(\rho_s \frac{\partial D_s}{\partial \nabla \underline{v}_i})$$

Example of constitutive laws : f volume fraction of “defects”

$$\psi_s = \frac{1}{2}k(f)(w_n)^2 + \frac{1}{2}\mu(f)(w_t - \alpha)^2$$

$$d(\dot{w}, \dot{\alpha}) = \frac{1}{2}\eta_t(f)\dot{w}_t^2 + \frac{1}{2}\eta_n(f)\dot{w}_n^2 + \frac{1}{2}H(f)\dot{\alpha}^2$$

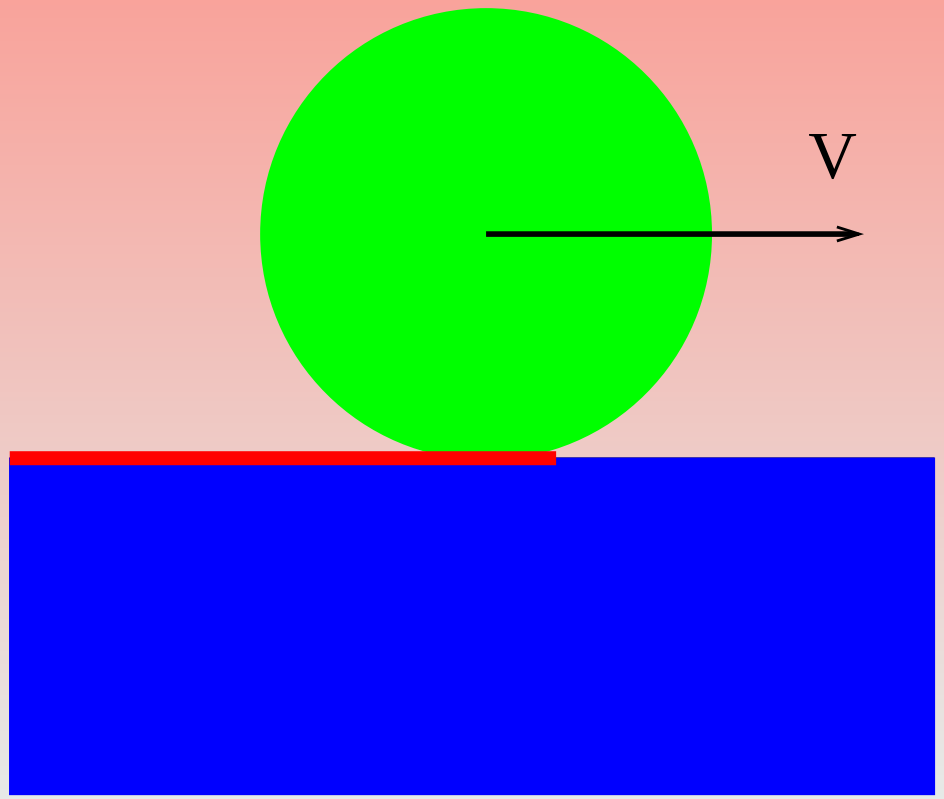
$$\underline{n}.\boldsymbol{\sigma} = (k_n w_n + \eta_n \dot{w}_n)\underline{n} + (\mu(w_t - \alpha) + \eta_t \dot{w}_t)\tau; \quad A = k_t(w_t - \alpha) = H\dot{\alpha}$$

Strömberg, IJSS 1996,

Del Piero, Eur. J. Mech (2010),

Stupkiewicz, Wear (1990),...

Moving punch on a half-space



Rigid Punch on an elastic half space. (Dragon-Louiset,2000)

The interface 3 : fluid + particles.

Suspension of particles in a viscous fluid : $\kappa(c), \quad \eta(c)$

Integral equations on an half elastic space : Galin's Equation

Displacement on the boundary due to stresses on the boundary.

$$\psi_s = \frac{1}{2}\kappa(c)w_n^2, \quad D_s = \frac{1}{2}\eta(c)\dot{w}_t^2$$

Behavior of the interface (3) viscosity in shear

$$\sigma_{xy} = \eta(c)(\dot{u}_x^1 - \dot{u}_x^2), \quad \sigma_{yy} = k(c)(u_y^1 - u_y^2)$$

Galin's Equations for the elastic half space

$$\begin{aligned} c_1\ u_{x,x}(x) &= c_2\ \sigma_{yy}(x) + Vp\frac{1}{\pi}\int_{-a}^{a}\frac{\sigma_{xy}(s)}{s-x}ds \\ c_1\ u_{y,x}(x) &= -c_2\ \sigma_{xy}(x) + Vp\frac{1}{\pi}\int_{-a}^{a}\frac{\sigma_{yy}(s)}{s-x}ds \end{aligned}$$

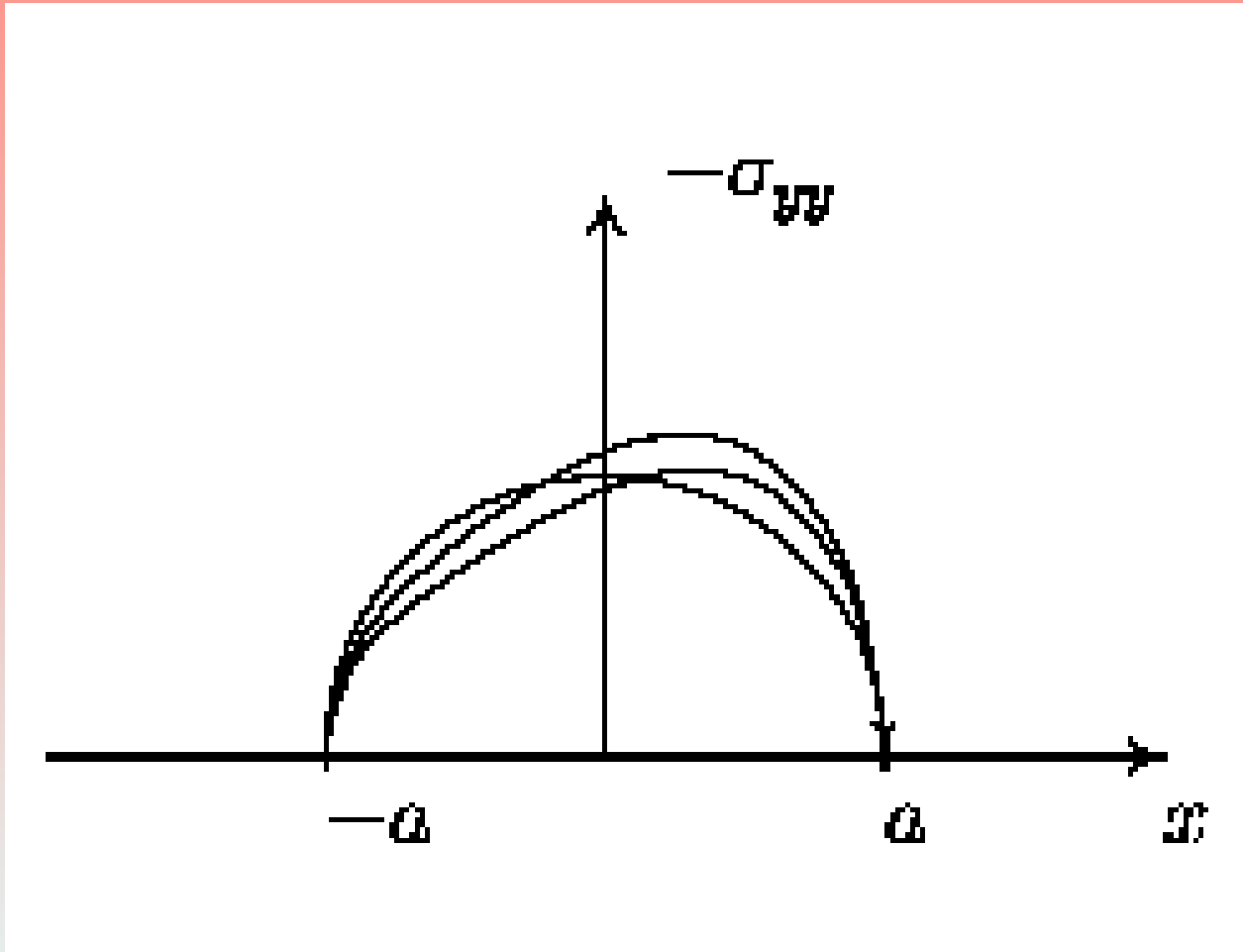
$$c_1 = \frac{E}{2(1-\nu^2)}, \quad c_2 = \frac{(1-2\nu)}{2(1-\nu)}$$

Asymptotic solution for small concentration of particles c

Law $\eta(c)$, $\kappa(c)$.

- order 0 : Hertz contact solution
- order 1 : depends upon criterion of wear : $\phi = \lambda \sigma_{yy}^2$

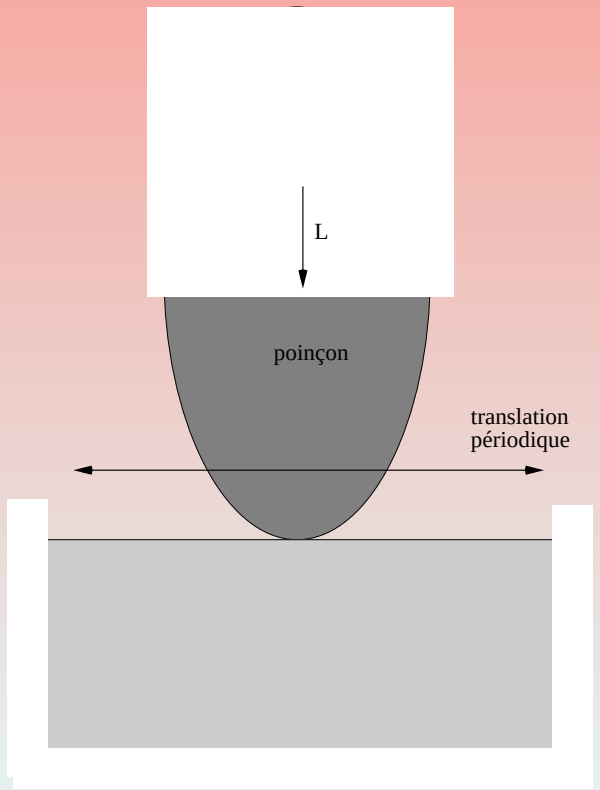
Effect of viscosity in shear



Equivalent to answer with friction law.

One more example

Cyclic accommodation for a Cyclic Horizontal motion of a rigid punch



The solution of an half space with a boundary $\eta(x)$

In Galin's Equation, the kernel is changed at order 1 in η.

$$\underline{u}_1(x) = \int_A N_o(x,y)T_1(y)dy + \int_A N_1(x,y)T_o(y)dy$$

The applied loading follows the boundary, using of $D_\phi(\boldsymbol{\sigma}.\underline{n})$ leads to

$$D_\phi(\boldsymbol{\sigma}.\underline{n}) = \dot{\boldsymbol{\sigma}}.\underline{n} + \frac{\mathrm{d}}{\mathrm{d}s}(\eta\boldsymbol{\sigma}.\tau) + \gamma\phi\boldsymbol{\sigma}.\underline{n}$$

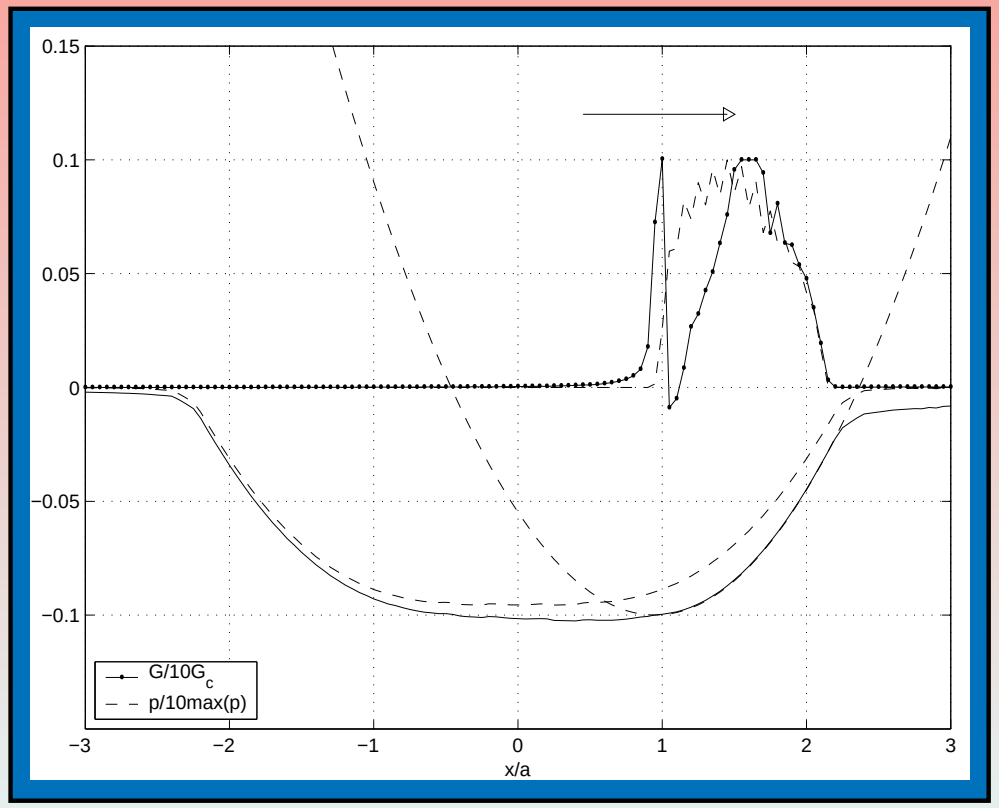
1 Asymptotic Respons

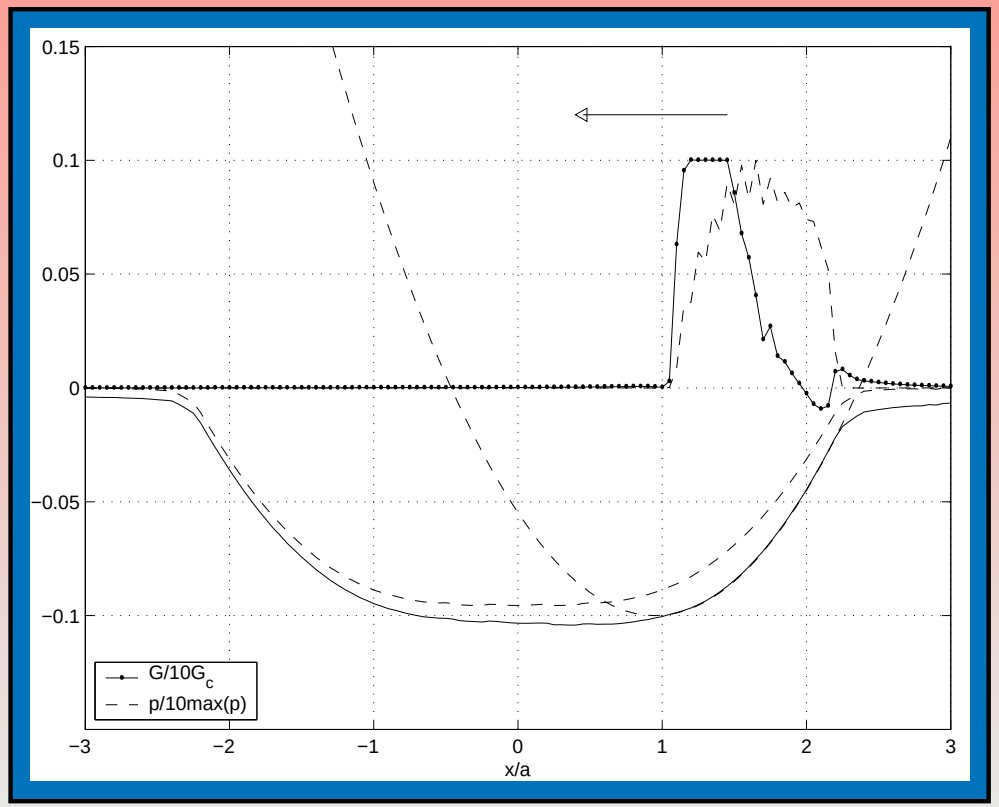
- Final State : $\eta(x)$
- Control Variable η
- Cost function (dissipation) J

$$J = \int_{\Gamma} |\eta| dx + a \int_{T} \int_{\Gamma} < G(\eta, x, t) - G_c >_{+}^{2} \, dx dt$$

Results

Lost of matter : $\int_\Gamma |\eta(x)| dx$




0.15
0.1
0.05
0
−0.05
−0.1
−3
−2
−1
0
1
2
3
x/a
G/10G_c
p/10max(p)

Possibility of using homogeneization technics to describe wear contact and to propose wear laws.

The mass density in the interface is a important internal parameter.

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Bui H.D, D-L, C. S : On Prand'l lifting equation arising in wear mechanics Arch. Mech., 2000, 52, 547-567.

Galin, Goryatcheva, Mroz, Zmitrowicz, Stupkiewicz,...