PROBLEM OF MOVING SUBSONIC EDGE DISLOCATION NEAR AN INTERFACE SOLVED WITH ONLY DISCRETE IMAGE DISLOCATIONS

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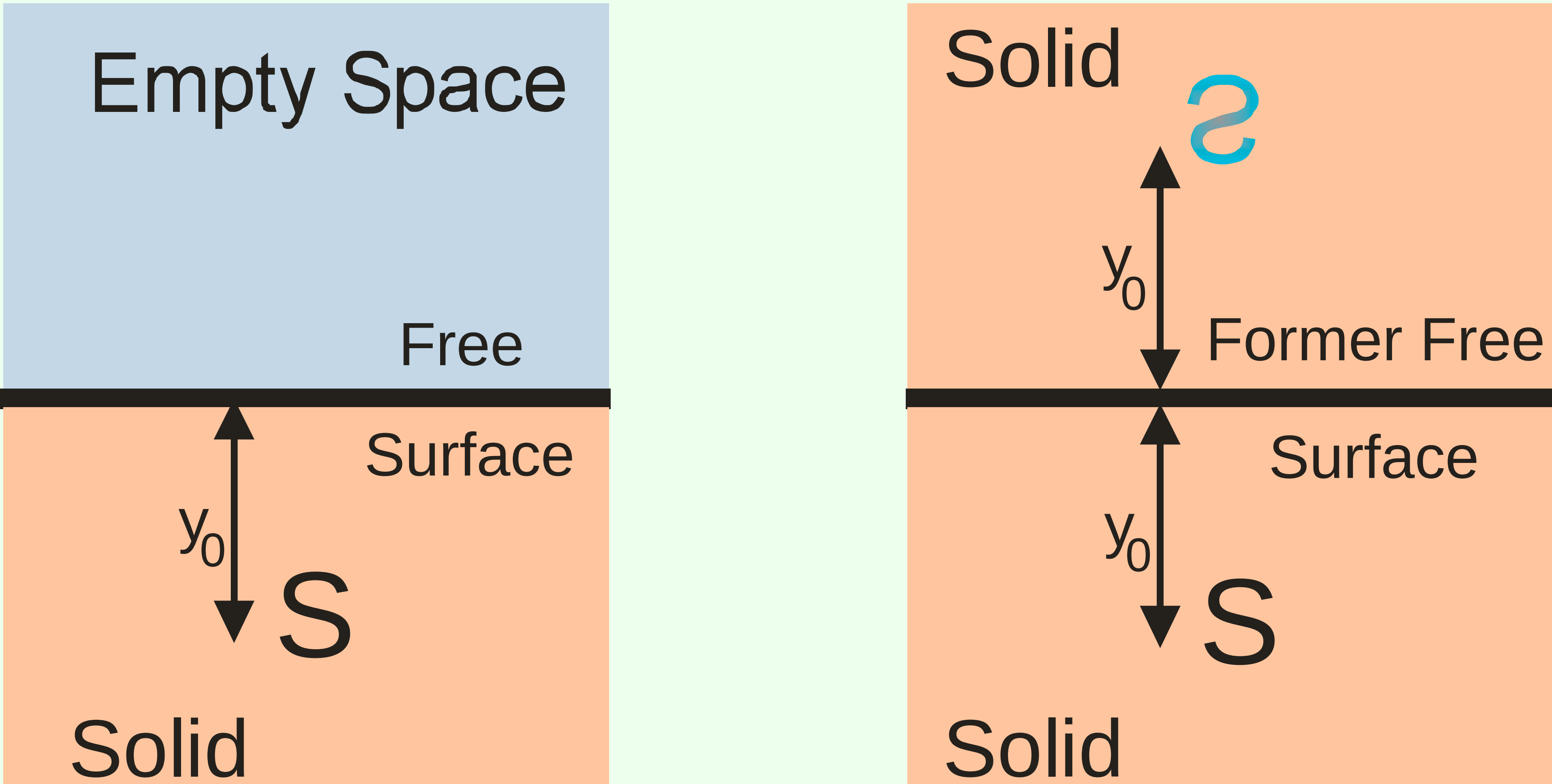
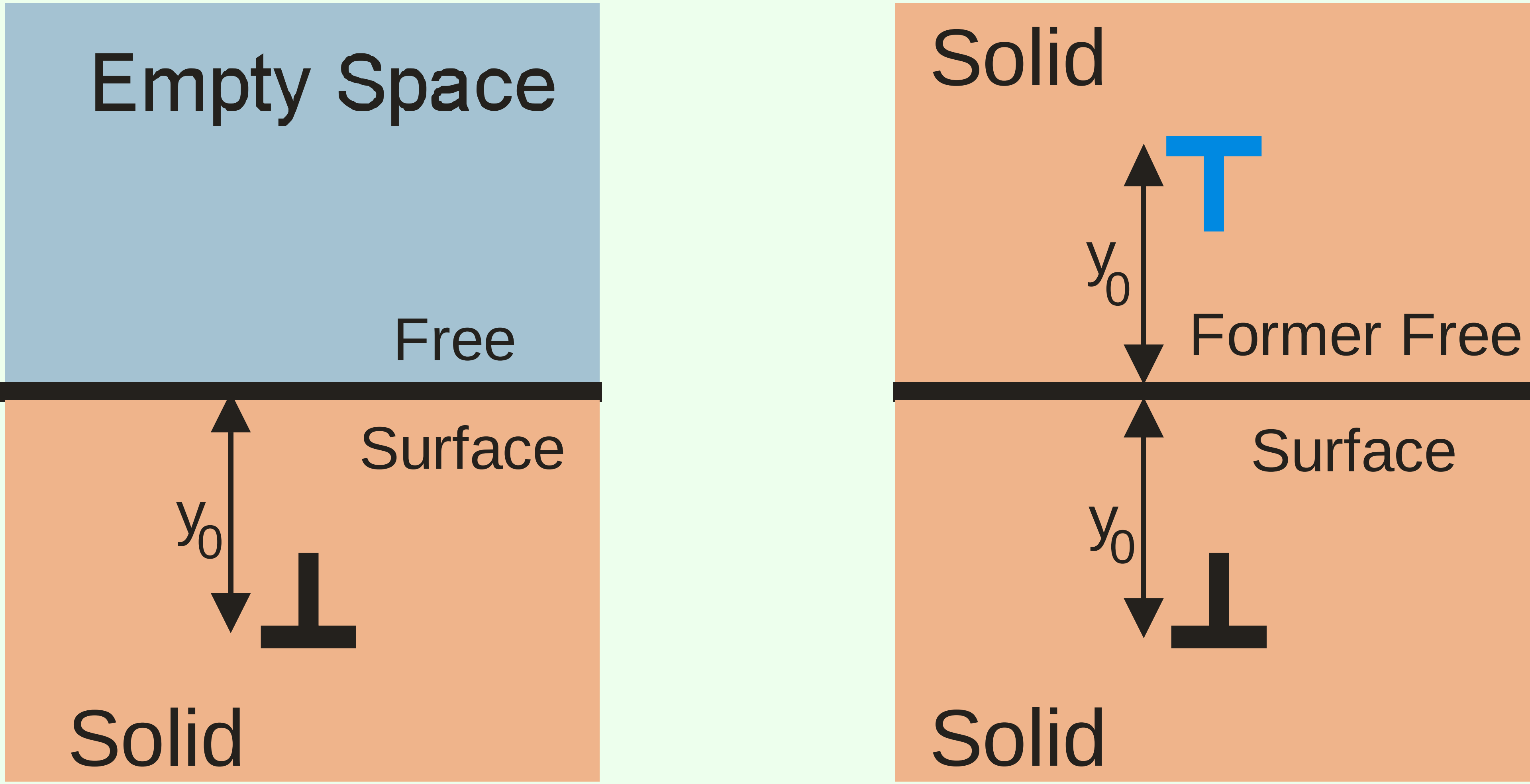
Leon Keer Symposium
Symi, Greece, July 2010



Empty Space
Free
Surface
y0
S
Solid
Solid
y0
Former Free
Surface
y0
S
Solid

$$\left(\sigma_{yz}\right)^{real} = \frac{\mu b}{2\pi} \frac{x}{x^2 + y_0^2}$$

$$\left(\sigma_{yz}\right)^{image} = -\frac{\mu b}{2\pi} \frac{x}{x^2 + y_0^2}$$


Empty Space
Free
Surface
y_0
Solid
Solid
y_0
Former Free
Surface
y_0
Solid

$$\left(\sigma_{xy}\right)^{real} = \frac{\mu b}{2\pi(1-\nu)} \frac{x}{x^2 + y_0^2}$$

$$\left(\sigma_{xy}\right)^{image} = -\frac{\mu b}{2\pi(1-\nu)} \frac{x}{x^2 + y_0^2}$$

$$\left(\sigma_{yy}\right)^{real} = \frac{\mu b}{2\pi(1-\nu)} \frac{y_0}{x^2 + y_0^2}$$

$$\left(\sigma_{yy}\right)^{image} = \frac{\mu b}{2\pi(1-\nu)} \frac{y_0}{x^2 + y_0^2}$$

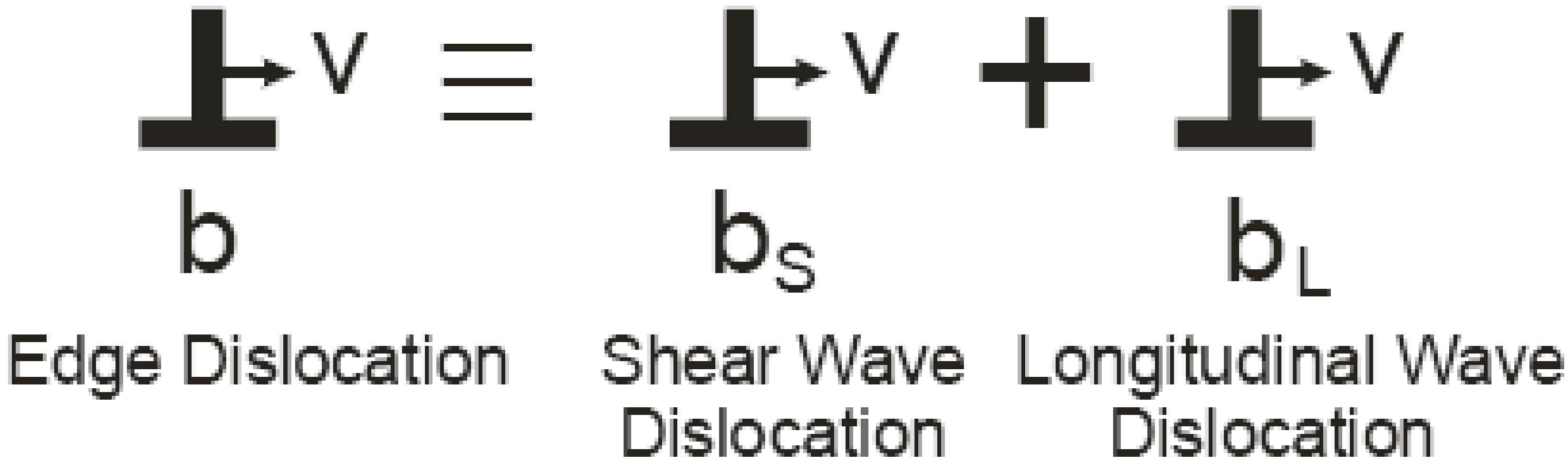


Figure 3. Moving edge dislocation considered to be sum of moving edge shear wave dislocation component and moving edge longitudinal wave dislocation component.

$$\{u_x\}_L = \frac{b_L}{2\pi}\tan^{-1}\frac{\beta_L(y+y_0)}{x}, \qquad \{u_y\}_L = \frac{b_L\beta_L}{2\pi}\ln\sqrt{x^2+\beta_L^2(y+y_0)^2},$$

$$\{\sigma_{xy}\}_L = \frac{\mu b_L}{\pi}\frac{\beta_L x}{x^2+\beta_L^2(y+y_0)^2},$$

$$\{\sigma_{yy}\}_L = \frac{\mu b_L}{\pi}\frac{\alpha^2\beta_L(y+y_0)}{x^2+\beta_L^2(y+y_0)^2},$$

$$\{\sigma_{xx}\}_L = -\frac{\mu b_L}{\pi}\frac{\left(1+\beta_L^2-\alpha^2\right)\beta_L(y+y_0)}{x^2+\beta_L^2(y+y_0)^2}.$$

The Einsteinium terms β_S, β_L and α in these equations are given by

$$\beta_S = \sqrt{1-\frac{V^2}{C_S^2}}, \quad \beta_L = \sqrt{1-\frac{V^2}{C_L^2}}, \quad \alpha = \sqrt{1-\frac{V^2}{2C_S^2}}.$$

$$\{u_x\}_S = \frac{b_S}{2\pi}\tan^{-1}\frac{\beta_S\,(y+y_0)}{x}, \qquad \{u_y\}_S = \frac{b_S}{2\pi\beta_S}\ln\sqrt{x^2+\beta_S^2\,(y+y_0)^2},$$

$$\{\sigma_{xy}\}_S = \frac{\mu b_S}{\pi\beta_S}\frac{\alpha^2 x}{x^2+\beta_S^2\,(y+y_0)^2},$$

$$\{\sigma_{yy}\}_S = \frac{\mu b_S}{\pi}\frac{\beta_S\,(y+y_0)}{x^2+\beta_S^2\,(y+y_0)^2},$$

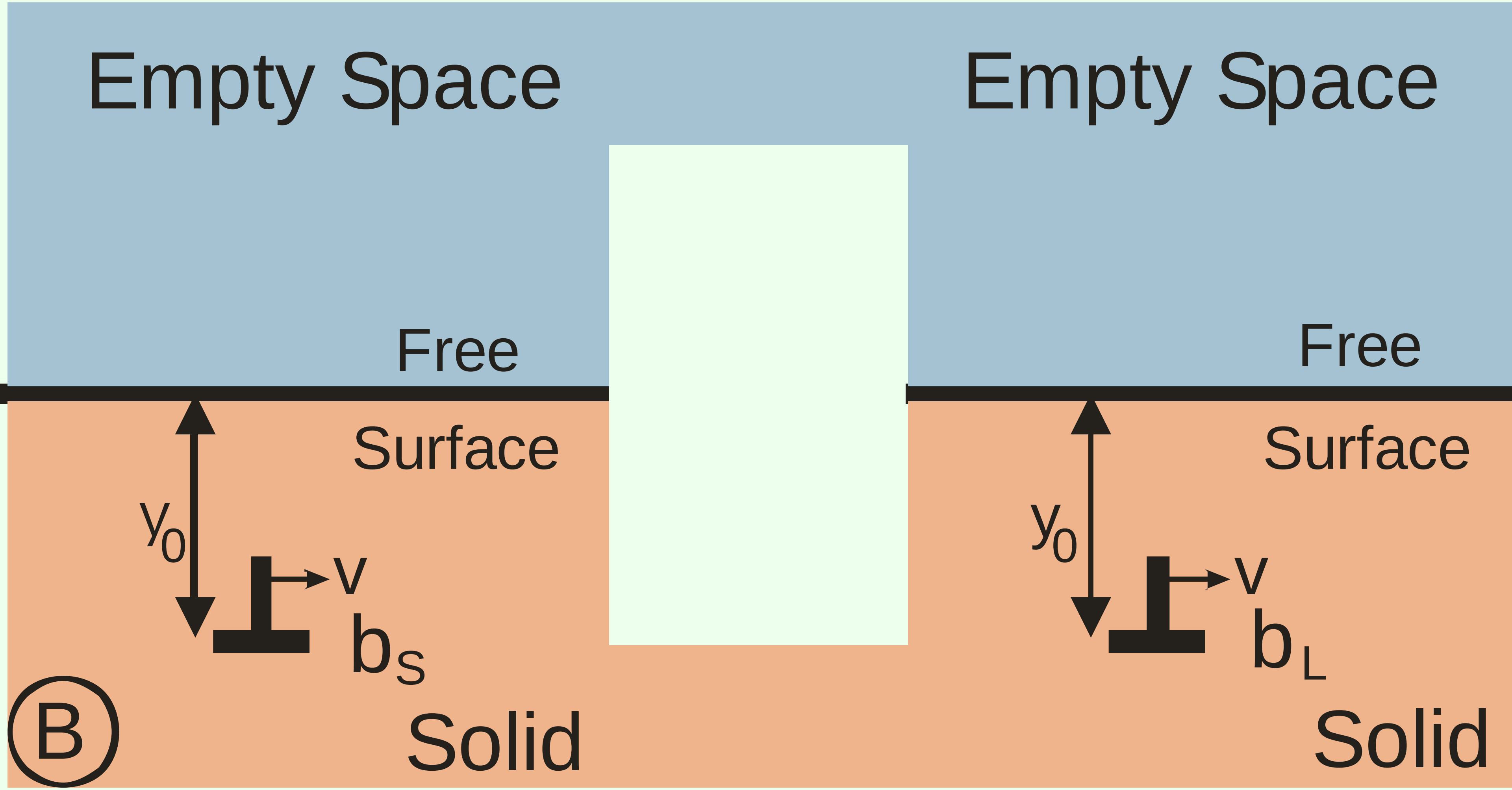
$$\{\sigma_{xx}\}_S = -\frac{\mu b_S}{\pi}\frac{\beta_S\,(y+y_0)}{x^2+\beta_S^2\,(y+y_0)^2}.$$

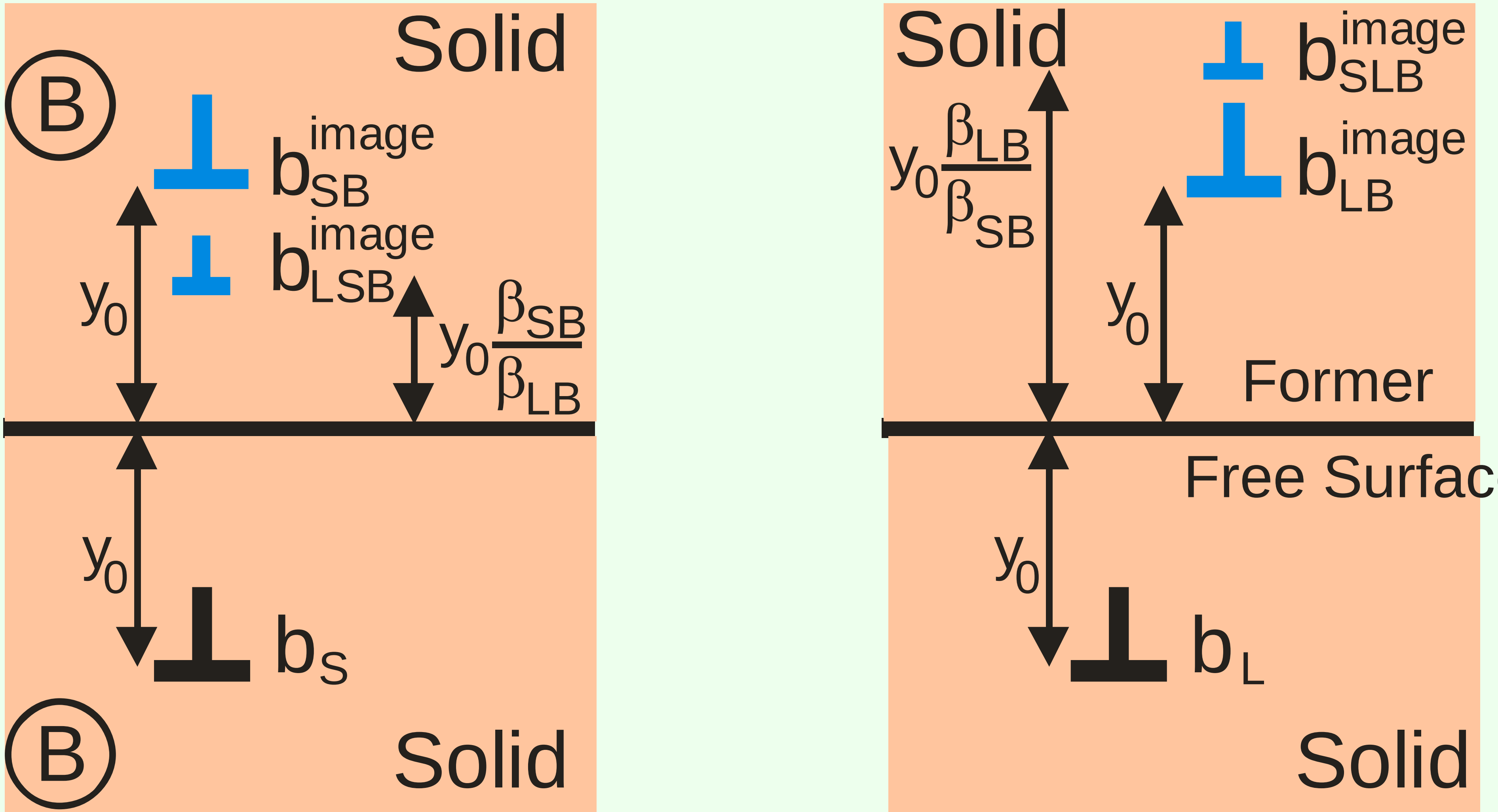
$$b_L = \frac{2C_S^2}{V^2} b, \qquad b_S = -\frac{2\alpha^2 C_S^2}{V^2} b.$$

$$b_S + b_L = b$$

$$\alpha^2 = 1 - \frac{V^2}{2C_S^2}$$

Dislocation moving near a free surface


Empty Space
Empty Space
Free
Free
Surface
Surface
y0
v
bS
y0
v
bL
B
Solid
Solid



Solid
B
b_{SB}^{image}
b_{LSB}^{image}
y_0
$y_0 \frac{\beta_{SB}}{\beta_{LB}}$
y_0
b_S
B
Solid
Solid
b_{SLB}^{image}
$y_0 \frac{\beta_{LB}}{\beta_{SB}}$
b_{LB}^{image}
y_0
Former
Free Surfac
y_0
b_L
Solid

$$\{u_x\}_L = \frac{b_L}{2\pi}\tan^{-1}\frac{\beta_L(y+y_0)}{x}, \qquad \{u_y\}_L = \frac{b_L\beta_L}{2\pi}\ln\sqrt{x^2+\beta_L^2(y+y_0)^2},$$

$$\{\sigma_{xy}\}_L = \frac{\mu b_L}{\pi}\frac{\beta_L x}{x^2+\beta_L^2(y+y_0)^2},$$

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The Einsteinium terms β_S, β_L and α in these equations are given by

$$\beta_S = \sqrt{1-\frac{V^2}{C_S^2}}, \quad \beta_L = \sqrt{1-\frac{V^2}{C_L^2}}, \quad \alpha = \sqrt{1-\frac{V^2}{2C_S^2}}.$$

$$\{u_x\}_S = \frac{b_S}{2\pi}\tan^{-1}\frac{\beta_S\left(y+y_0\right)}{x}, \qquad \{u_y\}_S = \frac{b_S}{2\pi\beta_S}\ln\sqrt{x^2+\beta_S^2\left(y+y_0\right)^2},$$

$$\{\sigma_{xy}\}_S = \frac{\mu b_S}{\pi\beta_S}\frac{\alpha^2 x}{x^2+\beta_S^2\left(y+y_0\right)^2},$$

$$\{\sigma_{yy}\}_S = \frac{\mu b_S}{\pi}\frac{\beta_S\left(y+y_0\right)}{x^2+\beta_S^2\left(y+y_0\right)^2},$$

$$\{\sigma_{xx}\}_S = -\frac{\mu b_S}{\pi}\frac{\beta_S\left(y+y_0\right)}{x^2+\beta_S^2\left(y+y_0\right)^2}.$$

$$\{u_x\}_{LSB}^{image} = \frac{b_{LSB}^{image}}{2\pi} \tan^{-1} \frac{\beta_{LB}\left(y - y_0\beta_{SB}/\beta_{LB}\right)}{x},$$

$$\{u_y\}_{LSB}^{image} = \frac{b_{LSB}^{image}\beta_{LB}}{2\pi} \ln \sqrt{x^2 + \beta_{LB}^2\left(y - y_0\beta_{SB}/\beta_{LB}\right)^2},$$

$$\{\sigma_{xy}\}_{LSB}^{image} = \frac{\mu_B b_{LSB}^{image}}{\pi} \frac{\beta_{LB}x}{x^2 + \beta_{LB}^2\left(y - y_0\beta_{SB}/\beta_{LB}\right)^2},$$

$$\{\sigma_{yy}\}_{LSB}^{image} = \frac{\mu_B b_{LSB}^{image}}{\pi} \frac{\alpha_B^2\beta_{LB}\left(y - y_0\beta_{SB}/\beta_{LB}\right)}{x^2 + \beta_{LB}^2\left(y - y_0\beta_{SB}/\beta_{LB}\right)^2},$$

$$\{\sigma_{xx}\}_{LSB}^{image} = -\frac{\mu_B b_{LSB}^{image}}{\pi} \frac{\left(1 + \beta_{LB}^2 - \alpha_B^2\right)\beta_{LB}\left(y - y_0\beta_{SB}/\beta_{LB}\right)}{x^2 + \beta_{LB}^2\left(y - y_0\beta_{SB}/\beta_{LB}\right)^2}.$$

$$\{u_x\}_{SLB}^{image} = \frac{b_{SLB}^{image}}{2\pi} \tan^{-1} \frac{\beta_{SB}\left(y - y_0\beta_{LB}/\beta_{SB}\right)}{x},$$

$$\{u_y\}_{SLB}^{image} = \frac{b_{SLB}^{image}\beta_{LB}}{2\pi} \ln \sqrt{x^2 + \beta_{SB}^2\left(y - y_0\beta_{LB}/\beta_{SB}\right)^2},$$

$$\{\sigma_{xy}\}_{SLB} = \frac{\mu_B b_{SLB}^{image}}{\pi\beta_{SB}} \frac{\alpha_B^2 x}{x^2 + \beta_{SB}^2\left(y - y_0\beta_{LB}/\beta_{SB}\right)^2},$$

$$\{\sigma_{yy}\}_{SLB}^{image} = \frac{\mu_B b_{SLB}^{image}}{\pi} \frac{\beta_{SB}\left(y - y_0\beta_{LB}/\beta_{SB}\right)}{x^2 + \beta_{SB}^2\left(y - y_0\beta_{LB}/\beta_{SB}\right)^2},$$

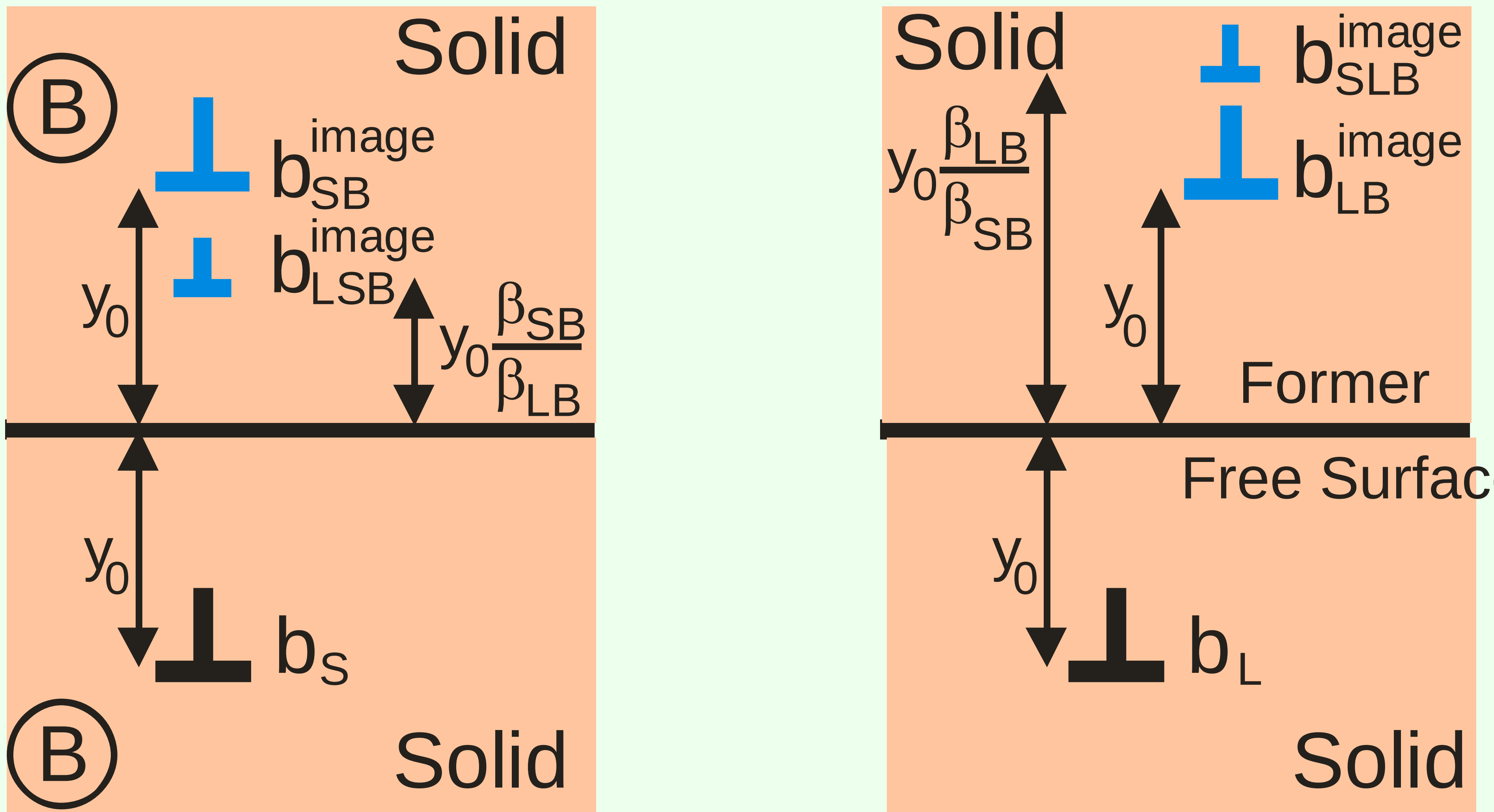
$$\{\sigma_{xx}\}_{SLB}^{image} = -\frac{\mu_B b_{SLB}^{image}}{\pi} \frac{\beta_{SB}\left(y - y_0\beta_{LB}/\beta_{SB}\right)}{x^2 + \beta_{SB}^2\left(y - y_0\beta_{LB}/\beta_{SB}\right)^2}.$$

$$[x^2 + \beta_S^2 (y - y_0\beta_L/\beta_S)^2] = \{[x^2 + \beta_L^2 y_0^2]\}_{y=0}$$

$$[x^2 + \beta_L^2 (y - y_0)^2] = \{[x^2 + \beta_L^2 y_0^2]\}_{y=0}$$

$$[x^2 + \beta_L^2 (y - y_0\beta_S/\beta_L)^2] = \{[x^2 + \beta_S^2 y_0^2]\}_{y=0}$$

$$[x^2 + \beta_S^2 (y - y_0)^2] = \{[x^2 + \beta_S^2 y_0^2]\}_{y=0}$$



Solid
B
b_{SB}^{image}
b_{LSB}^{image}
y_0
$y_0 \frac{\beta_{SB}}{\beta_{LB}}$
y_0
b_S
B
Solid
Solid
b_{SLB}^{image}
$y_0 \frac{\beta_{LB}}{\beta_{SB}}$
b_{LB}^{image}
y_0
Former
Free Surfac
y_0
b_L
Solid

$$\mu_B\left(\alpha_B^2/\beta_{SB}\right)b_S+\mu_B\left(\alpha_B^2/\beta_{SB}\right)b_{SB}^{image}+\mu_B\beta_{LB}b_{LSB}^{image}=0,$$

$$\mu_B\beta_{SB}b_S-\mu_B\beta_{SB}b_{SB}^{image}-\mu_B\alpha_B^2\beta_{SB}b_{LSB}^{image}=0.$$

Solving Equations (7) and (8) gives for the Burgers vectors of the image dislocations

$$b_{LSB}^{image}=\frac{2\alpha_B^2}{\alpha_B^4-\beta_{SB}\beta_{LB}}b_S,$$

$$b_{SB}^{image}=-\frac{\alpha_B^4+\beta_{SB}\beta_{LB}}{\alpha_B^4-\beta_{SB}\beta_{LB}}b_S.$$

$$\mu_B\beta_{LB}b_L+\mu_B\beta_{LB}b_{LB}^{image}+\mu_B\left(\alpha_B^2/\beta_{SB}\right)b_{SLB}^{image}=0, \tag{10}$$

$$\mu_B\alpha_B^2\beta_{LB}b_L-\mu_B\alpha_B^2\beta_{LB}b_{LB}^{image}-\mu_B\beta_{LB}b_{SLB}^{image}=0. \tag{11}$$

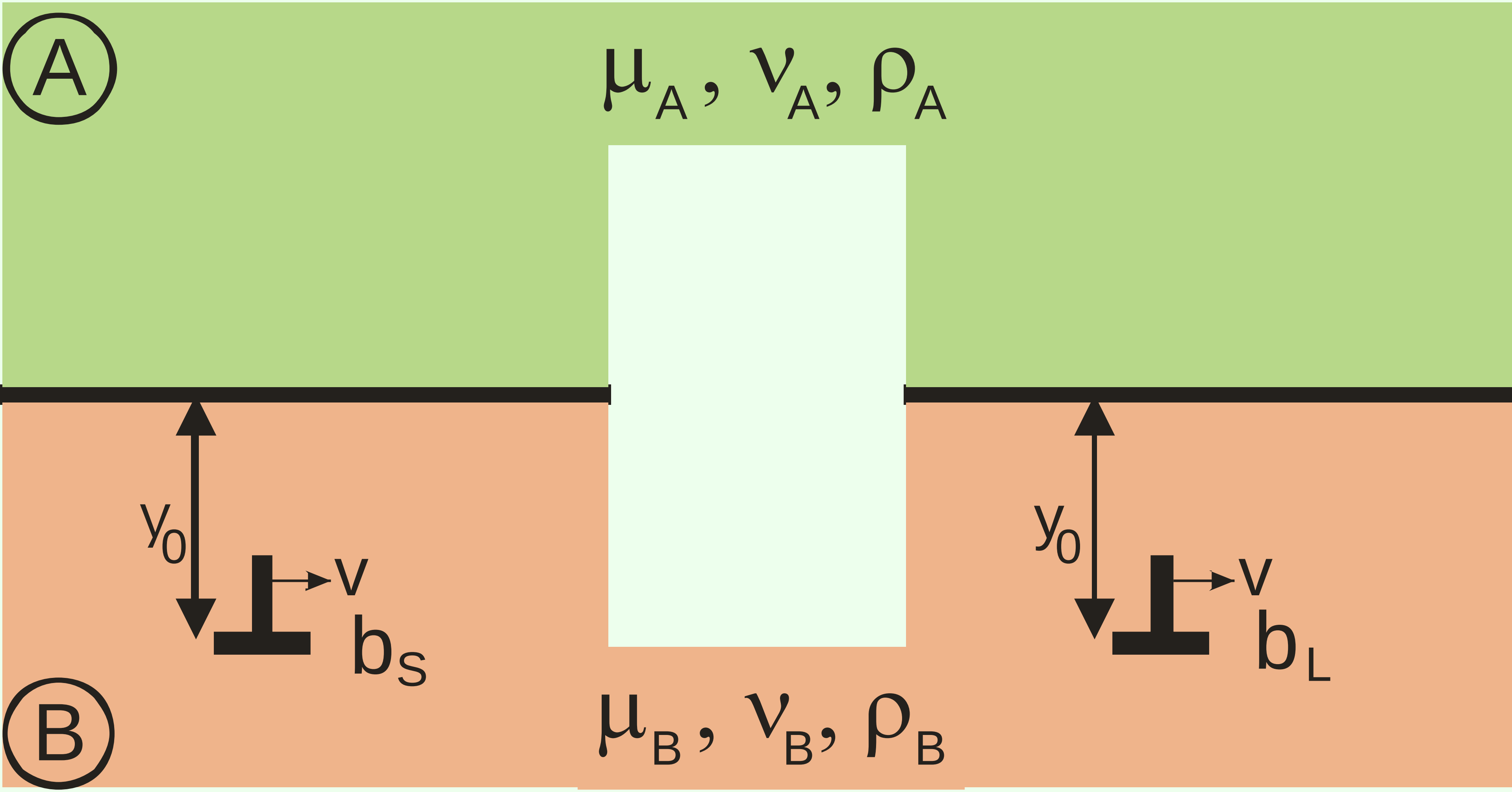
The solution of Equations (10) and (11) gives for the image dislocation Burgers vectors

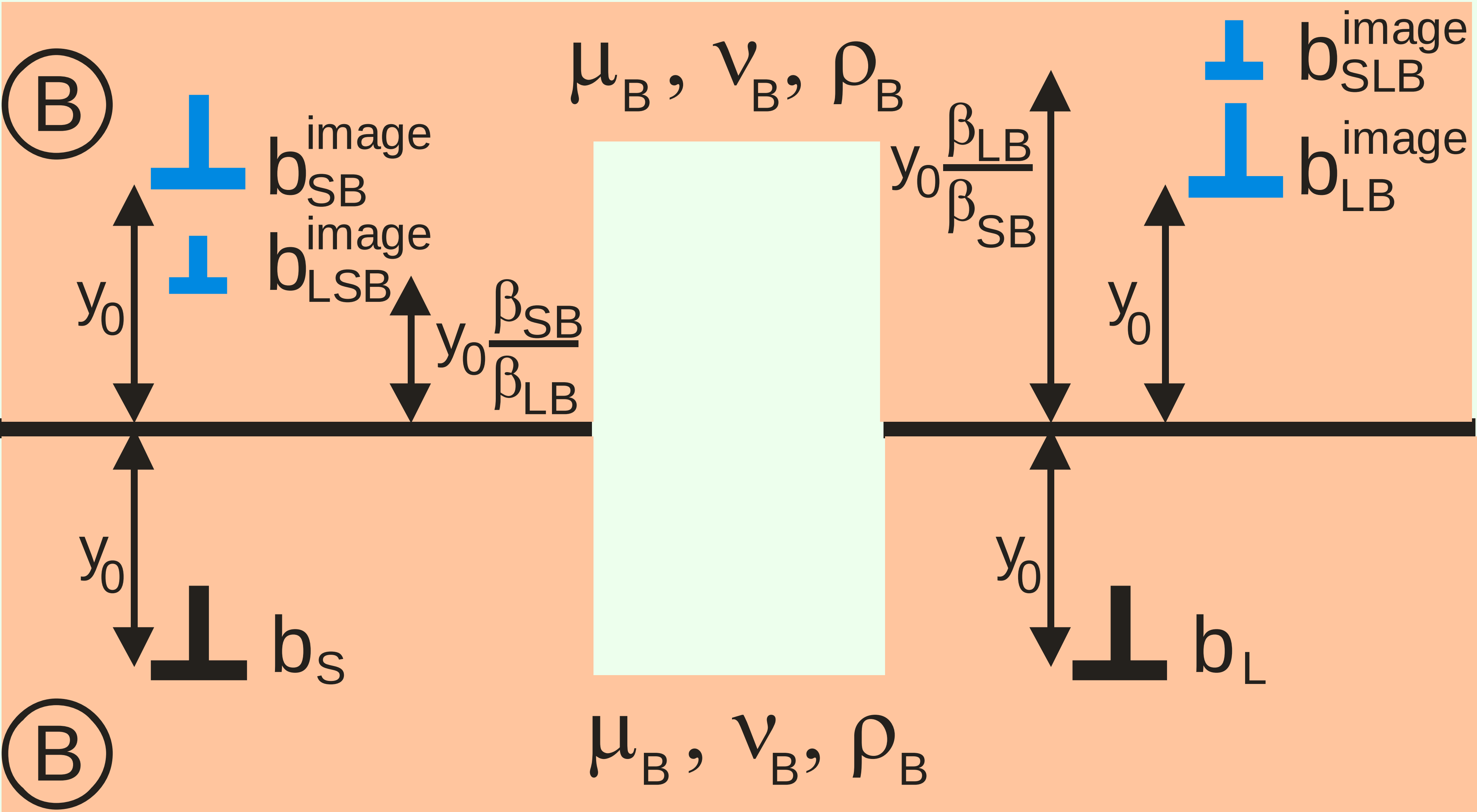
$$b_{SLB}^{image}=-\frac{2\alpha_B^2\beta_{SB}\beta_{LB}}{\alpha_B^4-\beta_{SB}\beta_{LB}}b_L,$$

$$b_{LB}^{image}=\frac{\alpha_B^4+\beta_{SB}\beta_{LB}}{\alpha_B^4-\beta_{SB}\beta_{LB}}b_L. \tag{12}$$

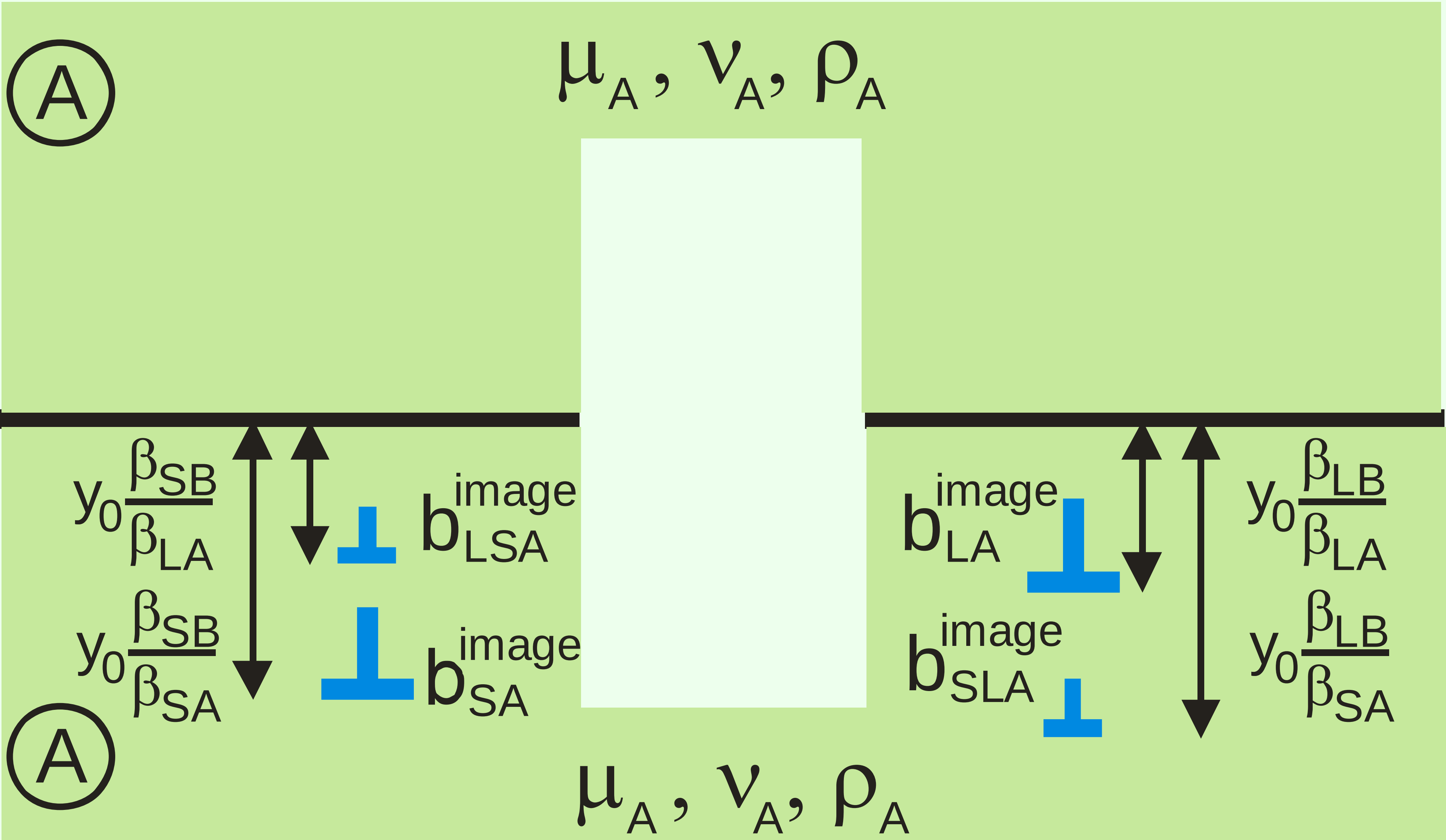
Free Surface Problem Solved

Dislocation moving near an interface


A
μA , νA, ρA
y0
v
bS
y0
v
bL
B
μB , νB, ρB



B
μ_B, ν_B, ρ_B
b_{SLB}^{image}
b_{SB}^{image}
b_{LSB}^{image}
$y_0 \frac{\beta_{LB}}{\beta_{SB}}$
b_{LB}^{image}
y_0
$y_0 \frac{\beta_{SB}}{\beta_{LB}}$
y_0
y_0
y_0
b_S
b_L
B
μ_B, ν_B, ρ_B



A
μ_A, ν_A, ρ_A
$y_0 \frac{\beta_{SB}}{\beta_{LA}}$
b_{LSA}^{image}
b_{LA}^{image}
$y_0 \frac{\beta_{LB}}{\beta_{LA}}$
$y_0 \frac{\beta_{SB}}{\beta_{SA}}$
b_{SA}^{image}
b_{SLA}^{image}
$y_0 \frac{\beta_{LB}}{\beta_{SA}}$
A
μ_A, ν_A, ρ_A

Shear Dislocation

There are four unknowns, b_{SB}^{image}, b_{SA}^{image}, b_{LSB}^{image}, b_{LSA}^{image} that need to be determined. The four equations required for this purpose are the continuity of traction stresses σ_{xy} and σ_{yy} across the interface and the continuity of the displacements u_x and u_y. These four equations, in order, are

$$\mu_B\left(\alpha_B^2/\beta_{SB}\right)b_S + \mu_B\left(\alpha_B^2/\beta_{SB}\right)b_{SB}^{image} + \mu_B\beta_{LB}b_{LSB}^{image} =$$

$$\mu_A\left(\alpha_A^2/\beta_{SA}\right)b_{SA}^{image} + \mu_A\beta_{LA}b_{LSA}^{image}, \tag{15}$$

$$\mu_B b_S - \mu_B b_{SB}^{image} - \mu_B\alpha_B^2 b_{LSB}^{image} = \mu_A b_{SA}^{image} + \mu_A\alpha_A^2 b_{LSA}^{image}, \tag{16}$$

$$b_S - b_{SB}^{image} - b_{LSB}^{image} = b_{SA}^{image} + b_{LSA}^{image}. \tag{17}$$

$$\left(1/\beta_{SB}\right)b_S + \left(1/\beta_{SB}\right)b_{SB}^{image} + \beta_{LB}b_{LSB}^{image} = \left(1/\beta_{SA}\right)b_{SA}^{image} + \beta_{LA}b_{LSA}^{image}. \tag{18}$$

(Similar set of equations for longitudinal dislocation)

Shear Dislocation

$$b_S/b_{SA}^{image} = 1 + b_{LSA}^{image}/b_{SA}^{image} + b_{SB}^{image}/b_{SA}^{image} + b_{LSB}^{image}/b_{SA}^{image},$$

$$b_{LSA}^{image}/b_{SA}^{image} = -\frac{\beta_{LB}[\mu_A - \mu_B] - (1/\beta_{SA})[\mu_A\alpha_A^2 - \mu_B\alpha_B^2]}{\beta_{LB}[\mu_A\alpha_A^2 - \mu_B] - \beta_{LA}[\mu_A - \mu_B\alpha_B^2]},$$

$$b_{SB}^{image}/b_{SA}^{image} = \tfrac{1}{2}[(\beta_{SB}/\beta_{SA}) - 1] -$$

$$\frac{[(\beta_{SB}/\beta_{SA}) - 1][\mu_A - \mu_B]}{2\mu_B[1 - \alpha_B^2]} + \frac{[(\beta_{SB}/\beta_{SA}) - 1]}{2[1 - \alpha_B^2]} \times$$

$$\frac{\beta_{LB}[\mu_A - \mu_B] - (1/\beta_{SA})[\mu_A\alpha_A^2 - \mu_B\alpha_B^2]}{\beta_{LB}[\mu_A\alpha_A^2 - \mu_B] - \beta_{LA}[\mu_A - \mu_B\alpha_B^2]}$$

$$-\frac{[\beta_{SB}\beta_{LA} - 1]\{\beta_{LB}[\mu_A - \mu_B] - (1/\beta_{SA})[\mu_A\alpha_A^2 - \mu_B\alpha_B^2]\}}{2\beta_{LB}[\mu_A\alpha_A^2 - \mu_B] - 2\beta_{LA}[\mu_A - \mu_B\alpha_B^2]},$$

$$b_{LSB}^{image}/b_{SA}^{image} = \frac{[\mu_A - \mu_B]}{\mu_B[1 - \alpha_B^2]} - \frac{1}{[1 - \alpha_B^2]}\frac{\beta_{LB}[\mu_A - \mu_B] - (1/\beta_{SA})[\mu_A\alpha_A^2 - \mu_B\alpha_B^2]}{\beta_{LB}[\mu_A\alpha_A^2 - \mu_B] - \beta_{LA}[\mu_A - \mu_B\alpha_B^2]}.$$

Longitudinal Dislocation

$$b_L/b_{LA}^{image} = 1 + b_{SLA}^{image}/b_{LA}^{image} + b_{LB}^{image}/b_{LA}^{image} + b_{SLB}^{image}/b_{LA}^{image},$$

$$b_{SLA}^{image}/b_{LA}^{image} = -\frac{[\mu_A\alpha_A^2 - \mu_B\alpha_B^2] - \beta_{SB}\beta_{LA}[\mu_A - \mu_B]}{[\mu_A - \mu_B\alpha_B^2] - [\mu_A\alpha_A^2 - \mu_B]},$$

$$b_{LB}^{image}/b_{LA}^{image} = \tfrac{1}{2}[(\beta_{LA}/\beta_{LB}) - 1] +$$

$$+\frac{[1+\beta_{SB}\beta_{LB}][\mu_A\alpha_A^2 - \mu_B\alpha_B^2]}{2\mu_B\beta_{SB}\beta_{LB}[\alpha_B^2 - 1]}$$

$$-\frac{[1+\beta_{SB}\beta_{LB}][\mu_A - \mu_B\alpha_B^2]}{2\mu_B\beta_{SB}\beta_{LB}[\alpha_B^2 - 1]}\frac{[\mu_A\alpha_A^2 - \mu_B\alpha_B^2] - \beta_{SB}\beta_{LA}[\mu_A - \mu_B]}{[\mu_A - \mu_B\alpha_B^2] - [\mu_A\alpha_A^2 - \mu_B]}$$

$$-\frac{[1-\beta_{SA}\beta_{LB}]\{[\mu_A\alpha_A^2 - \mu_B\alpha_B^2] - \beta_{SB}\beta_{LA}[\mu_A - \mu_B]\}}{2\beta_{SA}\beta_{LB}[\mu_A - \mu_B\alpha_B^2] - 2[\mu_A\alpha_A^2 - \mu_B]},$$

$$b_{SLB}^{image}/b_{LA}^{image} = \frac{[\mu_A\alpha_A^2 - \mu_B\alpha_B^2]}{\mu_B[\alpha_B^2 - 1]} - \frac{[\mu_A - \mu_B\alpha_B^2]}{\mu_B[\alpha_B^2 - 1]}\frac{[\mu_A\alpha_A^2 - \mu_B\alpha_B^2] - \beta_{SB}\beta_{LA}[\mu_A - \mu_B]}{[\mu_A - \mu_B\alpha_B^2] - [\mu_A\alpha_A^2 - \mu_B]}.$$

Interface Problem Solved

Reduction to Stationary Dislocation Near a Free Surface

$$\{\sigma_{xy}\}_B = -\frac{\partial^2 \chi_B}{\partial x \partial y}, \quad \{\sigma_{yy}\}_B = \frac{\partial^2 \chi_B}{\partial x^2}, \qquad \{\sigma_{xx}\}_B = \frac{\partial^2 \chi_B}{\partial y^2}.$$

The displacements are found from the relationships [5]

$$2\mu_B \{u_x\}_B = -\frac{\partial \chi_B}{\partial x} + 4(1-\nu_B)\frac{\partial \psi}{\partial y},$$

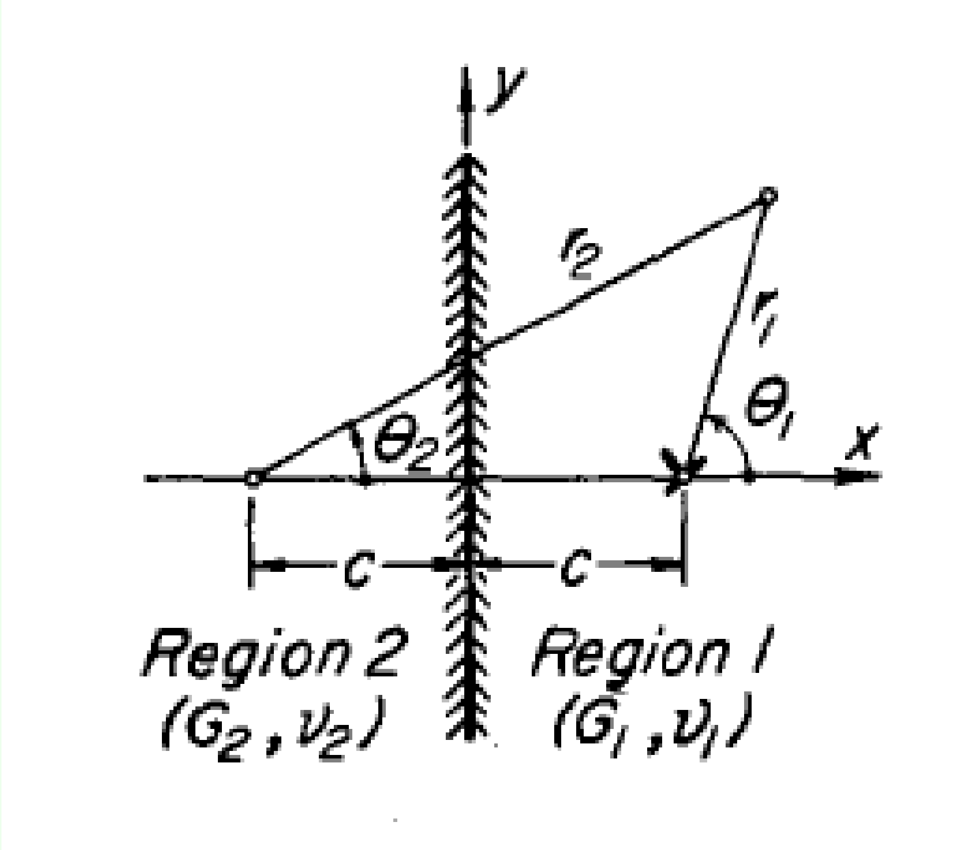
$$2\mu_B \{u_y\}_B = -\frac{\partial \chi_B}{\partial y} + 4(1-\nu_B)\frac{\partial \psi}{\partial x},$$

where the function ψ is determined by the two conditions

$$\frac{\partial^2 \psi}{\partial x \partial y} = \tfrac{1}{4}\nabla^2 \chi_B, \qquad \nabla^2 \psi = 0.$$

[4] J. Dundurs and G. P. Sendeckyj, Behavior of an edge edge dislocation near a bimetallic interface, *J. Appl. Phys.*, *36*, 3353-3354 (1965).

[5] J. Dundurs and T. Mura, Interaction between an edge dislocation and a circular inclusion, *J. Mech. Phys. Solids*, 12, 177-189 (1964).



J. Dundurs and T. Mura, "Interaction between an edge dislocation and a circular inclusion", J. *Mech. Phys. Solids*, 12, 177-189 (1964).
[
J. Dundurs and G. P. Sendeckyj, "Behavior of an edge dislocation near a bimetallic interface", *J. Appl. Phys.*, 36, 3353-3354 (1965).

$$\chi_1 = \frac{G_1 b_y}{\pi(\kappa_1+1)}\Big[2r_1 \log r_1 \cos\theta_1 - (B+A) r_2 \log r_2 \cos\theta_2$$

$$+ (B-A) r_2 \theta_2 \sin\theta_2 + 2Ac\left(2 \log r_2 - \cos 2\theta_2 + 2c\frac{\cos\theta_2}{r_2}\right)\Big], \quad (4)$$

$$\chi_2 = \frac{G_1 b_y}{\pi(\kappa_1+1)}\big[(2-B-A) r_1 \log r_1 \cos\theta_1$$

$$- (B-A)(r_1\theta_1 \sin\theta_1 + 2c \log r_1)\big]. \quad (5)$$

For Free Surface

A=B=1

$\chi_2 = 0$

A = B = 1

1 = space B

2 = empty space A

ndeckyj, “Behavior of an edge dislocation near a bimetallic interface”,

J. Appl. Phys., 36, 3353-3354 (1965).

Appendix

χ	$2Gu_x$	$2Gu_y$	σ_{xx}	σ_{xy}	σ_{yy}
$r \log r \cos\theta$	$\frac{1}{2}(\kappa - 1)\log r - \frac{x^2}{r^2}$	$\frac{1}{2}(\kappa + 1)\theta - \frac{xy}{r^2}$	$-\frac{x}{r^2} + \frac{2x^3}{r^4}$	$-\frac{y}{r^2} + \frac{2x^2 y}{r^4}$	$\frac{3x}{r^2} - \frac{2x^3}{r^4}$
$r\theta \sin\theta$	$\frac{1}{2}(\kappa + 1)\log r - \frac{x^2}{r^2}$	$\frac{1}{2}(\kappa - 1)\theta - \frac{xy}{r^2}$	$\frac{2x^3}{r^4}$	$\frac{2x^2 y}{r^4}$	$\frac{2x}{r^2} - \frac{2x^3}{r^4}$
$\log r$	$-\frac{x}{r^2}$	$-\frac{y}{r^2}$	$\frac{1}{r^2} + \frac{2x^2}{r^4}$	$\frac{2xy}{r^4}$	$\frac{1}{r^2} - \frac{2x^2}{r^4}$
$\cos 2\theta$	$-(3 - \kappa)\frac{x}{r^2} + \frac{4x^3}{r^4}$	$-(\kappa + 1)\frac{y}{r^2} + \frac{4x^2 y}{r^4}$	$\frac{12x^2}{r^4} - \frac{16x^4}{r^6}$	$\frac{8xy}{r^4} - \frac{16x^3 y}{r^6}$	$\frac{4}{r^2} - \frac{20x^2}{r^4} + \frac{16x^4}{r^6}$
$\frac{\cos\theta}{r}$	$-\frac{1}{r^2} + \frac{2x^2}{r^4}$	$\frac{2xy}{r^4}$	$\frac{6x}{r^4} - \frac{8x^3}{r^6}$	$\frac{2y}{r^4} - \frac{8x^2 y}{r^6}$	$-\frac{6x}{r^4} + \frac{8x^3}{r^6}$
$r \log r \sin\theta$	$-\frac{1}{2}(\kappa + 1)\theta - \frac{xy}{r^2}$	$\frac{1}{2}(\kappa - 1)\log r + \frac{x^2}{r^2}$	$\frac{y}{r^2} + \frac{2x^2 y}{r^4}$	$\frac{x}{r^2} - \frac{2x^3}{r^4}$	$\frac{y}{r^2} - \frac{2x^2 y}{r^4}$
$r\theta\cos\theta$	$\frac{1}{2}(\kappa - 1)\theta + \frac{xy}{r^2}$	$-\frac{1}{2}(\kappa + 1)\log r - \frac{x^2}{r^2}$	$-\frac{2x^2 y}{r^4}$	$-\frac{2x}{r^2} + \frac{2x^3}{r^4}$	$-\frac{2y}{r^2} + \frac{2x^2 y}{r^4}$
θ	$\frac{y}{r^2}$	$-\frac{x}{r^2}$	$-\frac{2xy}{r^4}$	$-\frac{1}{r^2} + \frac{2x^2}{r^4}$	$\frac{2xy}{r^4}$
$\sin 2\theta$	$(\kappa - 1)\frac{y}{r^2} + \frac{4x^2 y}{r^4}$	$(3 + \kappa)\frac{x}{r^2} - \frac{4x^3}{r^4}$	$\frac{4xy}{r^4} - \frac{16x^3 y}{r^6}$	$\frac{2}{r^2} - \frac{16x^2}{r^4} + \frac{16x^4}{r^6}$	$-\frac{12xy}{r^4} + \frac{16x^3 y}{r^6}$
$\frac{\sin\theta}{r}$	$\frac{2xy}{r^4}$	$\frac{1}{r^2} - \frac{2x^2}{r^4}$	$\frac{2y}{r^4} - \frac{8x^2 y}{r^6}$	$-\frac{6x}{r^4} + \frac{8x^3}{r^6}$	$-\frac{2y}{r^4} + \frac{8x^2 y}{r^6}$

[4] J. Dundurs and G. P. Sendeckyj, Behavior of an edge edge dislocation near a bimetallic interface, *J. Appl. Phys.*, 36, 3353-3354 (1965).

[5] J. Dundurs and T. Mura, Interaction between an edge dislocation and a circular inclusion, *J. Mech. Phys. Solids*, 12, 177-189 (1964).

Displacement solution constructed from Dundurs et al papers and converted from vertical free surface to horizontal free surface

$$\{u_x\}_B = \frac{b}{4\pi(1-\nu_B)}\left\{2(1-\nu_B)\tan^{-1}\frac{(y+2y_0)}{x} + \frac{x(y+2y_0)}{x^2+(y+2y_0)^2}\right.$$

$$\left. -2(1-\nu_B)\tan^{-1}\frac{y}{x} - \frac{xy}{r_B^2} - 2(1-2\nu_B)\frac{y_0 x}{r_B^2} - \frac{4y_0 x y^2}{r_B^4} - \frac{4y_0^2 xy}{r_B^4}\right\},$$

$$\{u_y\}_B = \frac{b}{4\pi(1-\nu_B)}\left\{-(1-2\nu_B)\ln\sqrt{x^2+(y+2y_0)^2} + \frac{(y+2y_0)^2}{x^2+(y+2y_0)^2}\right.$$

$$\left. +(1-2\nu_B)\ln\sqrt{x^2+y^2} - \frac{y^2}{r_B^2} + 2(1-2\nu_B)\frac{y_0 y}{r_B^2} + \frac{2y_0^2}{r_B^2} - \frac{4y_0 y^3}{r_B^4} - \frac{4y_0^2 y^2}{r_B^4}\right\},$$

where $r_B^2 = x^2 + y^2$. For convenience in what follows the origin of the coordinate
used in Equations (A4) is taken to be a distance $y = y_0$ above the free surface. To ch

Total displacement field origin at y = y_0

$$\{u_x\}_B = \frac{b_{SB}}{2\pi}\tan^{-1}\frac{\beta_{SB}(y+2y_0)}{x} + \frac{b_{LB}}{2\pi}\tan^{-1}\frac{\beta_{LB}(y+2y_0)}{x}$$

$$+\frac{b_{SB}^{image}}{2\pi}\tan^{-1}\frac{\beta_{SB}y}{x} + \frac{b_{LB}^{image}}{2\pi}\tan^{-1}\frac{\beta_{LB}y}{x}$$

$$+\frac{b_{LSB}^{image}}{2\pi}\tan^{-1}\frac{(\beta_{LB}y+\beta_{LB}y_0-\beta_{SB}y_0)}{x} + \frac{b_{SLB}^{image}}{2\pi}\tan^{-1}\frac{(\beta_{SB}y+\beta_{SB}y_0-\beta_{LB}y_0)}{x}, \tag{A5a}$$

$$\{u_y\}_B = \frac{b_{SB}}{2\pi\beta_{SB}}\ln\sqrt{x^2+\beta_{SB}^2(y+2y_0)^2} + \frac{b_{LB}\beta_{LB}}{2\pi}\ln\sqrt{x^2+\beta_{LB}^2(y+2y_0)^2}$$

$$+\frac{b_{SB}^{image}}{2\pi\beta_{SB}}\ln\sqrt{x^2+\beta_{SB}^2y^2} + \frac{b_{LB}^{image}\beta_{LB}}{2\pi}\ln\sqrt{x^2+\beta_{LB}^2y^2}$$

$$+\frac{b_{SLB}^{image}}{2\pi\beta_{SB}}\ln\sqrt{x^2+(\beta_{SB}y+\beta_{SB}y_0-\beta_{LB}y_0)^2}+\frac{b_{LSB}^{image}\beta_{LB}}{2\pi}\ln\sqrt{x^2+(\beta_{LB}y+\beta_{LB}y_0-\beta_{SB}y_0)^2}. \tag{A5b}$$

The Burgers vectors b_{LB} and b_{LB} in these equations are equal to

$$b_{LB} = \frac{2C_{SB}^2}{V^2} b, \qquad b_{SB} = -\alpha^2 \frac{2C_{SB}^2}{V^2} b.$$

In the limit $V \to 0$ the other Burgers vectors in Equations (A5) reduce to

$$b_{LSB}^{image} = \frac{2\alpha_B^2}{\alpha_B^4 - \beta_{SB}\beta_{LB}} b_S \to 4\left(1 - \nu_B\right) \alpha_B^4 \frac{4C_{SB}^4}{V^4} b,$$

$$b_{SB}^{image} = -\frac{\alpha_B^4 + \beta_{SB}\beta_{LB}}{\alpha_B^4 - \beta_{SB}\beta_{LB}} b_S \to -4\left(1 - \nu_B\right) \left(1 - \frac{3V^2}{4C_{SB}^2} - \frac{V^2}{4C_{LB}^2}\right) \alpha_B^2 \frac{4C_{SB}^4}{V^4} b,$$

$$b_{SLB}^{image} = -\frac{2\alpha_B^2 \beta_{SB}\beta_{LB}}{\alpha_B^4 - \beta_{SB}\beta_{LB}} b_L \to 4\left(1 - \nu_B\right) \alpha_B^2 \beta_{SB}\beta_{LB} \frac{4C_{SB}^4}{V^4} b,$$

$$b_{LB}^{image} = \frac{\alpha_B^4 + \beta_{SB}\beta_{LB}}{\alpha_B^4 - \beta_{SB}\beta_{LB}} b_L \to -4\left(1 - \nu_B\right) \left(1 - \frac{3V^2}{4C_{SB}^2} - \frac{V^2}{4C_{LB}^2}\right) \frac{4C_{SB}^4}{V^4} b.$$

Note that $\left(\alpha_B^4 - \beta_{SB}\beta_{LB}\right)^{-1} \to -2\left(1 - \nu_B\right)\left(2C_{SB}^2/V^2\right)$

$$\tan^{-1}\frac{\beta_{SB}y}{x} = \tan^{-1}\frac{y}{x} - \frac{xy}{r_B^2}\left(\frac{V^2}{2C_{SB}^2} + \frac{V^4}{8C_{SB}^4}\right) - \frac{2xy^3}{r_B^4}\frac{V^4}{4C_{SB}^4},$$

$$\tan^{-1}\frac{\beta_{LB}y}{x} = \tan^{-1}\frac{y}{x} - \frac{xy}{r_B^2}\left(\frac{V^2}{2C_{LB}^2} + \frac{V^4}{8C_{LB}^4}\right) - \frac{2xy^3}{r_B^4}\frac{V^4}{4C_{LB}^4},$$

$$\begin{aligned}\tan^{-1}\frac{(\beta_{SB}y + (\beta_{SB} - \beta_{LB})\, y_0)}{x} &\to \tan^{-1}\frac{y}{x}\\ &-\frac{x}{r_B^2}\left\{\left(\frac{V^2}{2C_{SB}^2} + \frac{V^4}{8C_{SB}^4}\right)y - \left[\left(\frac{V^2}{2C_{LB}^2} - \frac{V^2}{2C_{SB}^2}\right) + \left(\frac{V^4}{8C_{LB}^4} - \frac{V^4}{8C_{SB}^4}\right)\right]y_0\right\}\\ &-\frac{2xy}{r_B^4}\left\{\frac{V^4}{4C_{SB}^4}y^2 + \left(\frac{V^2}{2C_{LB}^2} - \frac{V^2}{2C_{SB}^2}\right)^2 y_0^2 - 2\frac{V^2}{2C_{SB}^2}\left(\frac{V^2}{2C_{LB}^2} - \frac{V^2}{2C_{SB}^2}\right)yy_0\right\},\end{aligned}$$

$$\begin{aligned}\tan^{-1}\frac{(\beta_{LB}y + (\beta_{LB} - \beta_{SB})\, y_0)}{x} &\to \tan^{-1}\frac{y}{x}\\ &-\frac{x}{r_B^2}\left\{\left(\frac{V^2}{2C_{LB}^2} + \frac{V^4}{8C_{LB}^4}\right)y - \left[\left(\frac{V^2}{2C_{SB}^2} - \frac{V^2}{2C_{LB}^2}\right) + \left(\frac{V^4}{8C_{SB}^4} - \frac{V^4}{8C_{LB}^4}\right)\right]y_0\right\}\\ &-\frac{2xy}{r_B^4}\left\{\frac{V^4}{4C_{LB}^4}y^2 + \left(\frac{V^2}{2C_{SB}^2} - \frac{V^2}{2C_{LB}^2}\right)^2 y_0^2 - 2\frac{V^2}{2C_{LB}^2}\left(\frac{V^2}{2C_{SB}^2} - \frac{V^2}{2C_{LB}^2}\right)yy_0\right\}. \qquad \text{(A7a)}\end{aligned}$$

$$\ln\sqrt{x^2+\beta_{SB}^2 y^2} \to \ln\sqrt{x^2+y^2} - \frac{y^2}{2r_B^2}\frac{V^2}{C_{SB}^2} - \frac{y^4}{4r_B^4}\frac{V^4}{C_{SB}^4},$$

$$\ln\sqrt{x^2+\beta_{LB}^2 y^2} \to \ln\sqrt{x^2+y^2} - \frac{y^2}{2r_B^2}\frac{V^2}{C_{LB}^2} - \frac{y^4}{4r_B^4}\frac{V^4}{C_{LB}^4},$$

$$\ln\sqrt{x^2+\left(\beta_{LB}y+\left(\beta_{LB}-\beta_{SB}\right)y_0\right)^2} \to \ln\sqrt{x^2+y^2}$$

$$-\frac{1}{2r_B^2}\left(\frac{V^2}{C_{LB}^2}y^2+\frac{V^2}{C_{LB}^2}y_0 y-\frac{V^2}{C_{SB}^2}y_0 y\right)$$

$$-\frac{1}{2r_B^2}\left(-\frac{V^2}{2C_{LB}^2}+\frac{V^2}{2C_{SB}^2}\right)^2\left(y_0^2+y_0 y\right)$$

$$-\frac{1}{4r_B^4}\left(\frac{V^2}{C_{LB}^2}y^2+\frac{V^2}{C_{LB}^2}y_0 y-\frac{V^2}{C_{SB}^2}y_0 y\right)^2,$$

$$\ln\sqrt{x^2+\left(\beta_{SB}y+\left(\beta_{SB}-\beta_{LB}\right)y_0\right)^2}\rightarrow\ln\sqrt{x^2+y^2}$$

$$-\frac{1}{2r_B^2}\left(\frac{V^2}{C_{SB}^2}y^2+\frac{V^2}{C_{SB}^2}y_0y-\frac{V^2}{C_{LB}^2}y_0y\right)$$

$$-\frac{1}{2r_B^2}\left(-\frac{V^2}{2C_{LB}^2}+\frac{V^2}{2C_{SB}^2}\right)^2\left(y_0^2+y_0y\right)$$

$$-\frac{1}{4r_B^4}\left(\frac{V^2}{C_{SB}^2}y^2+\frac{V^2}{C_{SB}^2}y_0y-\frac{V^2}{C_{LB}^2}y_0y\right)^2.$$

Displacement solution constructed from Dundurs et al papers and converted from vertical free surface to horizontal free surface

$$\{u_x\}_B = \frac{b}{4\pi(1-\nu_B)}\left\{2(1-\nu_B)\tan^{-1}\frac{(y+2y_0)}{x} + \frac{x(y+2y_0)}{x^2+(y+2y_0)^2}\right.$$
$$\left. -2(1-\nu_B)\tan^{-1}\frac{y}{x} - \frac{xy}{r_B^2} - 2(1-2\nu_B)\frac{y_0 x}{r_B^2} - \frac{4y_0xy^2}{r_B^4} - \frac{4y_0^2xy}{r_B^4}\right\},$$

$$\{u_y\}_B = \frac{b}{4\pi(1-\nu_B)}\left\{-(1-2\nu_B)\ln\sqrt{x^2+(y+2y_0)^2} + \frac{(y+2y_0)^2}{x^2+(y+2y_0)^2}\right.$$
$$\left. +(1-2\nu_B)\ln\sqrt{x^2+y^2} - \frac{y^2}{r_B^2} + 2(1-2\nu_B)\frac{y_0 y}{r_B^2} + \frac{2y_0^2}{r_B^2} - \frac{4y_0y^3}{r_B^4} - \frac{4y_0^2y^2}{r_B^4}\right\},$$

where $r_B^2 = x^2 + y^2$. For convenience in what follows the origin of the coordinate
used in Equations (A4) is taken to be a distance $y = y_0$ above the free surface. To ch

SUMMARY

The problem of a moving edge dislocation gliding near an interface or free surface can be solved with image dislocations if the dislocations first are separated into shear wave and longitudinal wave dependent components.

Thank you for listening to this elementary dislocation theory talk

SUMMARY

The analysis just presented demonstrates how the problem of a uniformly moving edge dislocation near a free surface or a welded interface that separates two material of different elastic constants can be solved with use of discrete image edge dislocations. The discrete image dislocations are of either a shear wave dependent type or a longitudinal wave dependent type. Each image dislocation has stress-displacement fields analogous to those of Equations (1) and (2). The total stress-displacement fields in the lower half space **B** of either Figure 4 or Figure 6 are the sum of the fields of the real shear and longitudinal dislocations b_S and b_L and of the image dislocations shown in Figure 5 or 7. In the upper half space **A** of Figure 6 the total field is the sum of the fields of the image dislocations of Figure 8.

The solution for the stationary dislocation near an interface is found on setting the limit $V \to 0$. In the Appendix this limit is taken for the case of a moving dislocation near a free surface and shown to agree with the solution that can be obtained from the papers of Dundurs et al [4,5].