RESEARCH AREAS

1) **The dynamic Eshelby problem solves the deep earthquakes**
(publications # A. 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142)

The self-similarly expanding Eshelby ellipsoidal inclusion constitutes the dynamic generalization of the Eshelby problem, where the interior strain/stress is constant, the interior particle velocity is zero (lacuna), and the Dynamic Eshelby tensor is obtained. The static Eshelby inclusion is obtained as a particular limit, as well as the elliptically expanding crack of Burridge and Willis (1969), in which limit Rayleigh waves are obtained as a particular limit of the dynamic Eshelby problem. The Dynamic Eshelby Tensor allows the solution of the solution of a self-similarly expanding region with phase change (change in density (volume collapse) and moduli) under prestress, and this solution has been applied to solve the problem of deep earthquakes, considered a deep mystery, as to why a phase transformation with substantial change in density (up to 10%) under pressure would produce the radiation from a shear wave source with zero or no volumetric components. It is shown that the energetics of the self-similarly expanding ellipsoid through Noether’s theorem of the calculus of variations in a variable domain (dyn J and M integral) produce instabilities (in the shape and growth) under high pressure, so that the inclusion can assume a flattened ellipsoidal shape that can grow large at less energy expense than the sphere. The asymptotic limit of the Eshelby inclusion in the penny-shape limit produces shear deformation to accommodate a large volume into a very thin inclusion to avoid openings or overlaps. By a successive instability the symmetric center of shear (CLVD) decomposes into two anti-symmetric centers of shear (Double Couple) with the one radiating through the perimeter without interaction energy losses. Thus, a symmetric input (volume collapse in isotropy under pressure) results in an anti-symmetric output, consistent with observations, allowing the radiation of the deep earthquakes unhindered by the pressure. It is also shown that at a critical pressure (nucleation pressure) an arbitrarily small densified inclusion (anticrack, solved here) can grow unstably at constant potential energy. At the value of the pressure pre-stress the tensile mean stress due to the volume collapse is cancelled while for small deviatoric prestress the remaining strain energy density is distortional resulting in a shear seismic source, explaining the puzzle. The phenomenon is ubiquitous in dynamic phase transformations under high pressure in nature, with the dynamic Eshelby problem being valid also in Newtonian fluids, with the instabilities providing insight into planetary impacts, amorphization, failure waves, and also to an avalanche model of plasticity.

**Dislocation dynamics with inertia effects**
(Publications # A.14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 36, 38, 81, 83, 85, 105, 107, 108, 109, 110, 111, 116, 120)

Dislocation dynamics with inertia effects, screw, edge dislocations in general motion, generally expanding dislocation loops, also screw and edge dislocations in anisotropic solids in general motion, detailed treatment of the singularities for the near field (singular asymptotics of integrals for the logarithmic singularities). The transition from subsonic to supersonic /transonic motion of a Volterra dislocation was analyzed, and the “effective mass” of a moving Volterra dislocation was also obtained; both being classical long-standing problems in dislocation theory.
Expanding Eshelby inclusions/ inhomogeneities with transformation strain.
(Publications # A.112, 113, 114, 117, 118, 122, 124, 127, 130, 131)

Solutions were obtained for generally dynamically (with inertia) expanding spherical Eshelby inclusions and plane phase boundaries with transformation strain, as well as for expanding inhomogeneities with transformation strain (i.e. when the material properties change as the inclusion expands). The “driving forces,” i.e the energy-release rate required to create (dynamically) an incremental region of eigenstrain, or of inhomogeneity with eigenstrain, were obtained.

It is shown that self-similarly expanding Eshelby ellipsoidal inclusions preserve the constant stress Eshelby property in the interior domain where the particle velocity vanishes. The Dynamic Eshelby Tensor for the spherical expanding inclusion under general eigenstrain is obtained. By a limiting procedure the static Eshelby Tensor is obtained. Three papers are under preparation for submission in the summer of 2015.

Asymptotic homogenization (publications # A.123, A.125)
At the unit cell level the microcrack interaction and growth is governed by the J integral (energy dissipation) and the microhole interaction and growth by the M integral. This energy dissipation at the microlevel is carried to the macro-level as macroscopic damage by asymptotic homogenization. Currently, by dynamic asymptotic homogenization the dynamic damage evolution is accounted by the energy loss through the dynamic J and M integrals for the early motion of defects (cracks, inclusions) in the unit cell. This extends the early work of Clifton and Markenscoff (1981) on elastic precursor decay and radiation from moving dislocations.

Inverse problem of Eshelby inclusions:
(Publications # A.61, 62, 63, 70, 94, also: 69, 70)

Proved that the ellipsoid is the only shape for which the Eshelby property of constant stress is maintained after perturbation of the inclusion domain, and also that polyhedra are excluded from having the Eshelby property.

2) Configurational forces

Configurational forces on moving defects
(Publications # A. 108, 112,113,114,115, 116,117,120,124, 125,129, 133,141)

The self-forces for moving defects, dislocations (“effective mass”) and phase boundaries with transformation strain (“driving force”) have been obtained. On the basis of Noether’s theorem, an interpretation of the dynamic J integral was given as necessary and sufficient condition for linear momentum to be preserved in the domain for any perturbation of the inhomogeneity position, which settles the open issue of how loading and phase boundary velocity are related (evolution equation).

Conservation laws and integrals:
(Publications# A.74, 86, 92, 95, 100, 101, 102, 103, 104, 106, 12)
(a) Conservation laws for incompatibility:
Based on Noether’s theorem for a positive definite functional that has as Euler-Lagrange
equations the Beltrami-Michell compatibility equations for the stress, new
conservation laws were obtained that allow from surface data to determine the
incompatibility content in the volume.
(b) Conserved integrals for couple-stress and micropolar elasticity (based on Noether’s
theorem) and interpretation as energy-release rates.
(c) Dual integrals based on complementary energies for elasticity and micropolar elasticity.

3) Singular asymptotics for thin ligaments
(Publications # A.30, 37, 39, 52, 56, 65, 75, 76, 99, C4)

The singular amplification of the stress as a function of the ligament thickness (for thin
ligaments) has been obtained analytically for different geometries of ligaments (two
holes, two cracks, etc.) and loadings, by newly developed singular asymptotics of series.
The singular dependence of the stress is also found by matched inner and outer
expansions. This amplification can account for the acceleration of the damage at the
macroscale due to interaction of microcracks/microholes, in the framework of an
asymptotic homogeneization model.

(Publications # A.49, 51, 58, 59, 64, 65, 66, 67, 68, 72, 73, 89, 90, 91, 93)

The spectral theory of elasticity, initiated by the Cosserat brothers and mathematically
developed by Mikhlin (1970) has been further developed both theoretically and by
obtaining the eigenfuctions for the spherical shell, and applying the theory to problems in
elasticity, thermoelasticity, viscoelasticity, and poroelasticity. The spectral theory allows
for the unique representation (due to the completeness of the eigenfuctions) of the
solution in terms of the geometry, loading, and elastic properties.

Necessary and sufficient conditions for the Poisson’s ratio dependence of the stress in
multiply connected domains in the presence of body force loading have been also
obtained generalizing the classical Michell conditions.

5) Theory of Elasticity.
(Publications# A.32, 33, 40, 41, 42, 45, 46, 47, 48, 50, 53, 54, 96)

The wedge paradox (and Saint Venant’s principle) was viewed from the point of interaction
of a load induced singularity (concentrated moment, dislocation dipole, etc.) and a
geometric singularity (wedge vertex, crack tip), and a new interpretation was given to the
paradox.
Rigid line inclusions (coined anticracks) were considered as dual to cracks, and their
interaction with load induced singularities was analyzed. Green’s functions were given
for point forces and dislocations in an infinite solid containing a rigid line inclusion.

Interface conditions in elasticity: expressing continuity of displacement in terms of strains
through the continuity of curvature. Jump conditions and Cesaro integrals for slipping
interfaces.
General conditions for the reduction in the number of elastic constants in the stress
dependence of multi-phase composites with bonded and slipping inclusions under body
force and boundary traction loadings.

6) Robotics
Publication# A.35 (The Geometry of Grasping) provides the solution to the problem of the
number of fingers required to hold an object of arbitrary geometry in any position (that
had been open for over a hundred years): (12 fingers for general geometry, 7 fingers for
polyhedra, 4 with friction. We have five fingers, why?); it is a classic, of permanent value
to robotics with 486 citations (according to Google Scholar), while the application of it
A.31 has 236 citations.

7) Other topics
1. Asymptotic homogenization (publications # A.123, A.125)
2. Hadamard instability analysis for coupled mechano-thermo-chemical systems. Conditions
for “negative creep.”
3. Publications # A.77, 78, 82, 84, 128)
4. Third and fourth order elastic constants (crystal symmetries)
(Publications # A.6, 9, 13, 11, 12, 13)
4. High Frequency Vibrations of crystal plates under large initial deformation
(Publications# A.1, 3, 5, 7)
Miscellaneous topics

PUBLICATIONS IN REFEREED JOURNALS:


2. X. Markenscoff, “Distribution of Shearing Stresses in an Orthotropic Centilever,” ASME. J.

3. P.C.Y Lee, Y.S. Wang and X. Markenscoff, “Nonlinear Effects of Initial Bending on the

Applied Geophysics, 114, No. 5, 805-810, (Special issue on “Seismic Surface Waves and Free
Oscillations of the Earth”), 1976.


143. X. Markenscoff “Phase transformation under high pressure radiates out as a Double Couple Deep Earthquake” *J. Geoph. Res. Solid Earth*, (in press)