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On the structure of salt fingers

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Abstract—The structure of fingers in a heat–solute system is investigated in the laboratory. In particular, the velocity of the fluid in the fingers and the horizontal and vertical length scales of the fingers, which form at the interface between two layers with the upper layer having an excess of the heat and solute but still less dense than the lower layer, are measured and related to the temperature and solute concentration differences across the interface. Dimensional analysis is used to predict the way the details of the finger motions depend upon the properties of the well-mixed layers on either side of the interface. The experimental results are found to be in agreement with these predictions. The laboratory data are then extrapolated in order to provide estimates for the size and velocity of salt fingers in the ocean, using data appropriate to the layered microstructure found beneath the Mediterranean outflow by HOWE and TAIT (1970).

1. INTRODUCTION

A FLUID containing opposing gradients of temperature and a solute can convect even though the overall density stratification is statically stable. When the solute concentration is destabilizing and the temperature stabilizing, the convection takes the form of long, thin vertical cells called fingers. Although the basic physics of the convection was first discussed by STOMMEL, ARONS and BLANCHARD (1956), and the possible role fingers may play in the vertical transport of heat and salt in certain oceanographic contexts has been recognised for some time (TURNER, 1967), very little is known about their structure or properties.

The most comprehensive quantitative experiments on heat–salt fingers reported in the literature are those of TURNER (1967) who measured the vertical fluxes of heat and salt across an interface containing fingers. He related the magnitudes of the fluxes of heat and salt to the temperature and salinity of the layers on either side of the interface, and demonstrated that the vertical transport of salt was considerably increased by the presence of the fingers. LINDEN (1971a) examined the manner in which these fluxes and some gross properties of the fingers were affected by turbulent motions imposed on the interface. He found that the fingers tended to be disrupted by the turbulent motions, but concluded that fingers could still be an important mechanism for transporting heat and salt vertically in certain regions of the ocean. It has also been found that a steady shear changes the fingers into two-dimensional sheets aligned downshear: these sheets (analogous to the two-dimensional rolls found in thermal convection) are very effective in transporting heat and salt vertically (LINDEN, 1971b).

It has been demonstrated that fingers will form when other stratifying components with different molecular diffusivities are suitably distributed in the fluid (HUPPERT and MANINS, 1973). STERN and TURNER (1969) have investigated some properties of fingers in a sugar–salt system, and they found that whilst certain features of the motions show similarities to heat–salt fingers, there appears to be some significant differences between the two systems. However, no satisfactory theory of the finger

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motions has yet been provided which would allow a comparison of experiments using different pairs of stratifying components to produce the fingers. Clearly then, it is necessary to have detailed measurements of different systems in order to determine the effects of varying the values of the molecular diffusion coefficients. Further, in order to make extrapolations from laboratory to oceanic situations, it is desirable to have more detailed data on laboratory heat–solute fingers.

In this paper, measurements of the detailed structure of laboratory heat–solute fingers are presented. These measurements extend the work of TURNER (1967) by considering the vertical and horizontal length scales of the fingers, and the velocity of the fluid in the fingers. The results of a scale analysis of the relevant equations is presented in the next section, and the laboratory data are compared with this analysis. Finally, some comparison with sugar–salt fingers are given, and some extrapolations to the ocean are made.

2. A SCALE ANALYSIS OF THE EQUATIONS

Before describing the experiments, it is helpful, in order to prepare for a discussion of the experimental results, to consider the implications of a scale analysis of the finger motions. The situation in which fingers are most readily studied in the laboratory is when they are allowed to form at the interface between two well-mixed layers. This configuration appears to be relevant to some oceanographic situations where the temperature and salinity microstructure consists of well-mixed layers separated by relatively sharp interfaces (TAIT and HOWE, 1971).

Assume for the moment that the fingers are infinitely long and that the motion in the fingers is both steady and vertical. Then, if L denotes the horizontal length scale of the fingers, w is the vertical velocity and T and S are the temperature and solute concentration differences between adjacent fingers, the equations of conservation of momentum, heat and solute (to the Boussinesq approximation) are*

$$\frac{\nu w}{L^2} \sim g(\alpha T - \beta S), \tag{2.1}$$

$$w\bar{T}_z \sim \kappa_T T/L^2, \tag{2.2}$$

$$w\bar{S}_z \sim \kappa_S S/L^2. \tag{2.3}$$

Here $\bar{T}_z$ and $\bar{S}_z$ are the mean vertical gradients of temperature and solute, ν, κ_T and κ_S are the molecular diffusivities of momentum, heat and solute, respectively, and g is the acceleration due to gravity. An equation of state of the form

$$\rho = \rho_0 (1 - \alpha T + \beta S),$$

has been assumed, where α and β are the coefficients of expansion of heat and solute, respectively.

The buoyancy fluxes of heat and solute produced by the fingers are

$$\begin{aligned} F_T &\sim w\alpha T, \\ F_S &\sim w\beta S, \end{aligned} \tag{2.4}$$

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*For details of the derivation of these equations see HUPPERT and MANINS (1973).

respectively. The ratio of the buoyancy fluxes $r = F_T/F_S$ is given by

$$\begin{aligned} r &\sim \alpha T/\beta S. \\ &\sim \tau\alpha \bar{T}_z/\beta \bar{S}_z, \end{aligned} \tag{2.5}$$

where $\tau = \kappa_S/\kappa_T$.† Substitution of (2.3)–(2.5) into (2.2) shows that

$$L \sim \left[\frac{-g\alpha \bar{T}_z}{\nu\kappa_T} \left(1 - \frac{1}{r} \right) \right]^{-1/4}. \tag{2.6}$$

At this point it is convenient to recognise that the fingers are bounded at the top and bottom by convecting layers of depth H. It was shown by TURNER (1967) that if the flux across the interface is assumed independent of H, then on dimensional grounds the solute flux may be written in the form

$$F_S = f_1 \left(\frac{\alpha \Delta T}{\beta \Delta S}, \tau \right) (\beta \Delta S)^{4/3}, \tag{2.7}$$

where Δ denotes the difference between the well-mixed layers.
Equivalently,

$$\frac{F_S}{F_S{}^*} = f \left(\frac{\alpha \Delta T}{\beta \Delta S}, \tau \right), \tag{2.8}$$

where $F_S{}^*$ is the "solid-plane value" of the solute flux; i.e. the value of F_S if the interface were replaced by a solid membrane permeable to the solute. TURNER (1967) found that his measurements were consistent with (2.8), and showed that $F_S/F_S{}^*$ could be as large as 50, a result confirmed by the measurements reported here. It is now assumed that the fingers and the convecting region are in equilibrium. Then the work done by the buoyancy forces $gF_S(1 - \tau)$ is balanced by the dissipation in the well-mixed layers: an estimate for the latter is given by $\nu\zeta^2$ where ζ is the vorticity in the fluid. This vorticity is produced by the buoyancy gradient across the interface. This implies

$$\begin{aligned} F_S &\sim \frac{\nu}{1 - r} \beta \Delta S/h , \\ &= D\beta \Delta S/h. \end{aligned} \tag{2.9}$$

[The argument given above is essentially the same as that given by STERN (1969); see his equation (5.3).]

Consider now explicitly the case of heat–solute fingers; i.e. $0 < \tau \ll 1$. The length of the fingers h will be defined by the relation

$$h\bar{T}_z = \Delta T. \tag{2.10}$$

In order to proceed further, it has been found necessary to make a further assumption. In the limit of very small τ the solute must be advected along the fingers; consequently it is assumed that the solute difference between adjacent fingers is a constant fraction of the solute difference between the layers: i.e.

$$S = \Gamma \Delta S, \tag{2.11}$$

where $\Gamma \leq 1$ is a constant.

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†Note that this equation indicates that the flux of a particular component increases when the value of its molecular diffusion coefficient decreases.

Then use of (2.9)–(2.11), with (2.1) and (2.6) in (2.4) gives

$$f_1\left(\frac{\alpha\Delta T}{\beta\Delta S}, 0 < \tau \ll 1\right) \sim \left(\frac{g\kappa_T}{\nu}\right)^{1/3} D^{1/3}\Gamma^{4/3} r^{1/3}(1-r)^{1/3}\left(\frac{\alpha\Delta T}{\beta\Delta S}\right)^{-1/3}. \qquad (2.12)$$

Thus, on the basis of these dimensional arguments an expression for $f_1(\alpha\Delta T/\beta\Delta S, \tau \ll 1)$ has been obtained. It now remains to test the form of this relationship experimentally. Clearly the above arguments are a gross over-simplification of the real flow; these simplifications result in the presence of the undetermined parameters r and Γ. In order to determine these parameters it would appear to be necessary to resolve the details of the transition region between the fingers and the large scale convection. In the following sections of this paper r, D and Γ will be assumed to be constant. The justification for this assumption comes from an examination of the experimental data; r has been measured for heat–salt fingers by TURNER (1967),* and for sugar–salt fingers by STERN and TURNER (1969) and has been found to be constant over a wide range of conditions (but different for the two systems). D is also found to be constant over a wide range of conditions for heat–solute fingers (see Fig. 7 supporting this assumption).

Although the above analysis assumes that the motions are steady the experimental configuration of fingers existing between well-mixed layers is unsteady as the temperature and solute concentrations of the layers change with time due to transport across the interface. STERN and TURNER (1969) have shown that sugar–salt fingers run down through a series of equilibrium states, in each of which the motion can be considered as quasi-steady. Between any two such states the solute flux is given by

$$F_S = \frac{H}{2}\frac{d(\beta\Delta S)}{dt}.$$

From (2.9) it follows that

$$\frac{dh}{dt} = \text{constant}. \qquad (2.13)$$

Further, from (2.7) provided $f_1(\alpha\Delta T/\beta\Delta S, \tau)$ is a slowly varying function of $\alpha\Delta T/\beta\Delta S$ one finds that

$$(\beta\Delta S)^{-1/3} \propto t. \qquad (2.14)$$

The experiments will now be described and then the results will be discussed on the basis of this analysis.

3. THE EXPERIMENTAL METHOD.

The majority of experiments described in this paper were carried out using a heat–sugar system. It was found to be convenient to use sugar and heat as they have molecular properties similar to the oceanographically interesting heat–salt fingers.† The main reason for using sugar instead of salt was that the technique used for measuring the fluid velocities required that the fluid be only weakly conducting, thereby making it impossible to use salt. It is believed that heat–salt and heat–sugar

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*The value of τ has been estimated for heat–sugar fingers and indications are that it is smaller ($\sim$ 0·1) than the value of 0·56 found by TURNER (1967) for heat–salt fingers. It seems as though Turner's value may be an overestimate as the fingers were artificially sharpened by the turbulence and the motions were not in an equilibrium state.

†The values of the molecular diffusion coefficients are $1{\cdot}5 \times 10^{-5}$ cm^2 sec^{-1} for salt and $0{\cdot}5 \times 10^{-5}$ cm^2 sec^{-1} for sugar. Thus $\tau = 0{\cdot}01$ for heat and salt, and $\tau = 0{\cdot}003$ for sugar and heat.

fingers are equivalent in that they are both cases in which the limit $\tau = 0$ is applicable. Indeed, observations of flux magnitudes and scales indicate that there is very little difference between the two systems (see Section 4).

The tank in which the experiments were performed was a thermally insulated rectangular perspex box measuring 25·4 × 25·4 cm in cross-section and 40 cm deep. Two layers of water were set up in the tank, the upper layer being warmer than the lower. Sugar, dissolved in hot water, was added to the upper layer which was stirred slowly to mix the sugar uniformly. The stirring was then stopped, and as soon as the slopping motions of the interface had ceased, measurements of velocity, temperature and sugar profiles were made until the system had run down.

Sugar profiles were obtained by withdrawing samples at known depths using a syringe with a long vertical needle the end of which was bent horizontal. Such a sample is an average over several fingers at a given depth. The depths of the samples are thought to be known to within ± 0·3 cm. The refractive index of the samples was determined using a hand refractometer (American Optical Corp.); use of this refractometer allowed very small samples (~ 1 ml) to be taken. The density was then determined using data from the International Critical Tables: the sugar concentration obtained by this method is accurate to ± 0·03‰.

Temperature–depth profiles were obtained by traversing one junction of a thermocouple vertically through the fluid. The other junction was kept in a constant temperature bath, the temperature of which remained constant to within 0·02°C during a run. The output from the thermocouple was amplified and the amplified signal was used to drive the Y-axis of an X–Y plotter. The thermocouple was traversed vertically by a rack and pinion drive, and the depth of the junction was determined by imposing a known voltage across a linear potentiometer connected to the pinion shaft. The output from the potentiometer was used to drive the X-axis of the plotter, and was calibrated directly against the depth of the junction. This method gave direct instantaneous profiles, with linear representations of the depth and temperature.

In order to determine the velocity and horizontal length scale of the fingers the Thymol Blue technique described by Baker (1966) was used. Thymol Blue is a pH indicator, which is orange for pH ~ 8; increasing the pH of the fluid changes the indicator to blue. Consequently, the use of suitable electrodes allows the introduction of local blue colourations in the fluid, as the pH increases in the neighbourhood of the cathode. It was convenient to use platinum wires as electrodes, and wires of 0·002 cm in diameter were used. As the dominant scale in the fingers was approximately 0·2 cm it is assumed that the presence of the wires did not seriously effect the motions on this scale. A potential difference of approximately 8 V was applied between electrodes, 2·5 cm apart, stretched horizontally across the tank. The dye lines so produced were photographed at known intervals of time, usually about 5 sec., and the velocity and length scale were measured from the photographs. The details of the averaging procedure used is given in the Appendix. In order to minimize the effect of the wires on the flow, the dye traces were photographed after the dye had been advected approximately 100 radii downstream.

A major advantage of using the Thymol Blue technique to mark the fluid compared with, say, simply adding dye to one of the layers, is that the former produces a neutrally buoyant marker which can have no affect on the motions in the fingers. (This fact is important as the results of Huppert and Manins (1973) show that a very small

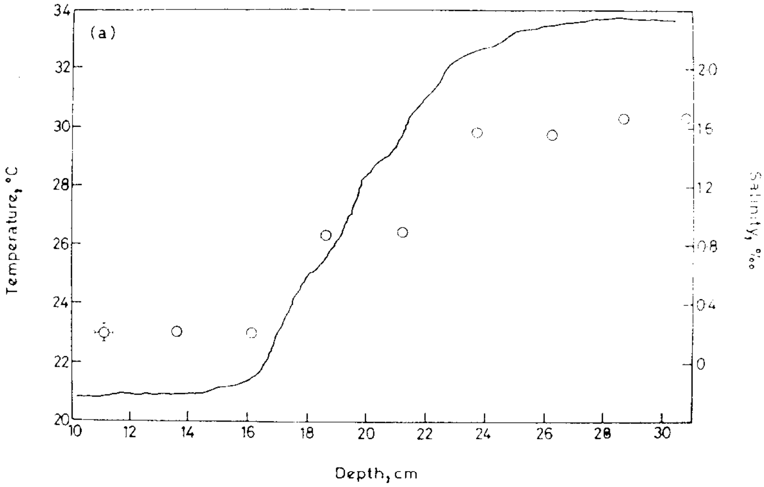
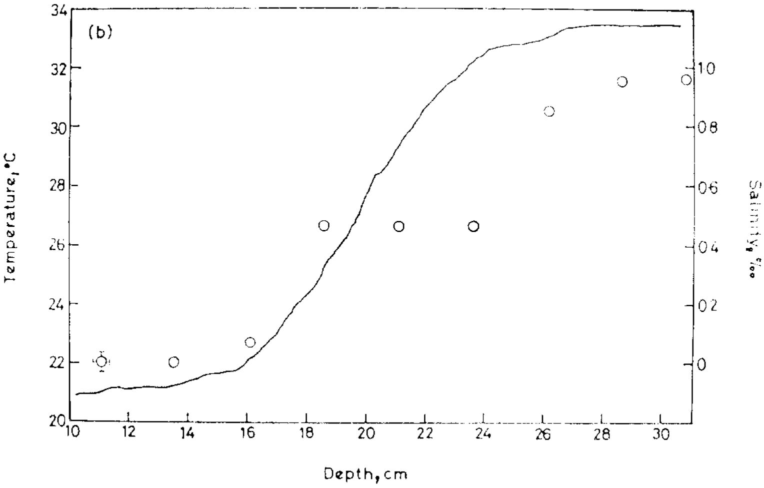

(a)
34
32
30
28
26
24
22
20
Temperature, °C
10
12
14
16
18
20
22
24
26
28
30
Depth, cm
2.0
1.6
1.2
0.8
0.4
0
Salinity, ‰

Fig. 1(a)


(b)
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Temperature, °C
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Depth, cm
1.0
0.8
0.6
0.4
0.2
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Salinity, ‰

Fig. 1(b)

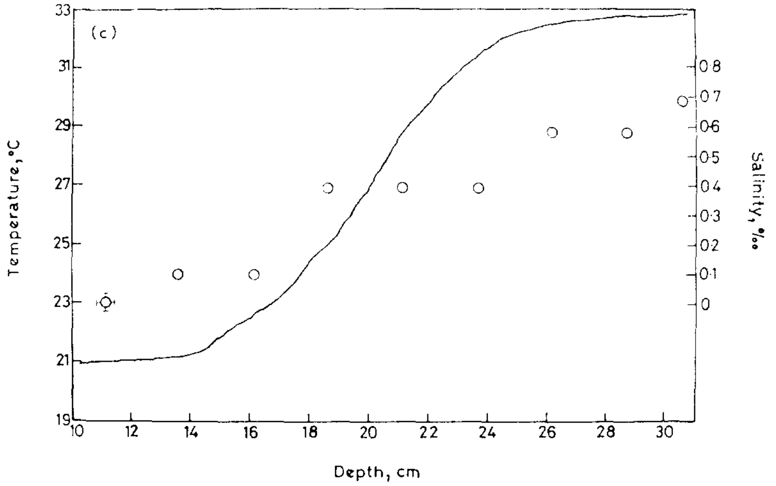

Fig (1c)

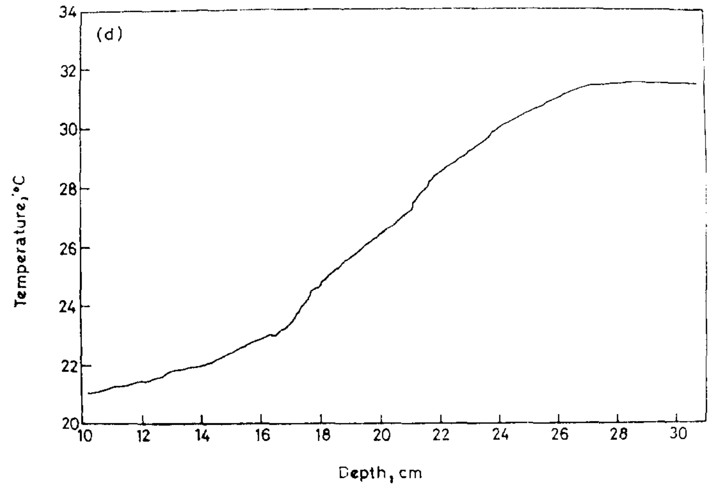

Fig 1(d)

Fig. 1. Profiles of temperature —— and sugar concentration o with depth. (a) $\alpha\Delta T = 4{\cdot}16‰$, $\beta\Delta S = 1{\cdot}25‰$ (b) $\alpha\Delta T = 4{\cdot}02‰$ $\beta\Delta S = 0{\cdot}95‰$ (c) $\alpha\Delta T = 3{\cdot}78‰$, $\beta\Delta S = 0{\cdot}68‰$ (d) $\alpha\Delta T = 3{\cdot}3‰$, $\beta\Delta S \sim 0{\cdot}1‰$. Note the regions of linear temperature gradient in (a), (b) and (c). (d) is a profile taken when the fingers had run down; sugar concentrations were not taken for this profile.

amount of solute can produce fingers in a heat–solute system.) In order to produce this neutral buoyancy it is necessary to make up all solutions used in a particular run from a single stock which has the Thymol Blue already added: this was done for the experiments reported here. Further, the lack of a significant concentration gradient of the Thymol Blue implies that the dye diffuses very slowly and ensures that the distortions of the dye lines are due to advection.

4. THE EXPERIMENTAL RESULTS

The results of the experiments will be presented in two parts. In the first part an overall picture of the convection, as determined from the profiles of temperature and sugar concentration and from the velocity fields, will be described. In the second part the more detailed measurements will be discussed, with particular attention paid to the features of the motion described in Section 2.

Some examples of the vertical profiles of temperature and sugar concentration are shown on Fig. 1. It is possible to distinguish three regions in these profiles: viz. a gradient region with essentially a constant temperature gradient, two well-mixed layers on either side of this interface and two transition layers separating the gradient region from the well-mixed layers. The gradient region was identified on a shadowgraph (see Fig. 2) as the region containing fingers. In general, the mean temperature gradient was, to a good approximation, constant, although when the system had run down it was not always possible to identify a region of constant gradient; Fig. 1(d) shows one such case. The sugar profiles in this region were less well-defined as the change in the sugar concentration from the top to the bottom of the fingers was usually within the accuracy of the sampling, i.e. ± 0·03‰. The fact that the variation in sugar concentration with distance along the fingers is much smaller than the corresponding change in temperature is consistent with (2.5) and the fact that $r = 0(1)$. Examples of dye traces in the gradient region are shown in Fig. 3. They show that the motion is vertical, and consisted of alternate up and down cells. Typically, $w \sim 0{\cdot}05$ cm/sec and $L \sim 0{\cdot}23$ cm.* In general, the velocity increased with increasing values of $\beta \Delta S$ (see Fig. 4).

The well-mixed layers were uniform in both temperature and sugar concentration to within 0·05°C and 0·03‰, respectively. The motion in these layers was more random than that observed in the fingers and generally of a larger scale, although some motion on the scale of the fingers was observed. These are represented by the distortions in the dye traces at the wires at 16 cm and 31 cm shown on Fig. 3(a).

The transition regions between the well-mixed layers and the region of linear temperature variation are clearly defined in the temperature profiles; the sugar transition was not resolved due to the inaccuracy of the sampling technique. From the temperature profiles (Fig. 1) it is seen that the transition region consisted of a region of decreasing temperature gradient. The temperature step across the region was roughly 5–10% of the temperature difference between the layers; the width of the region was of the order of 2–3 cm. The edge of the finger region as shown on Fig. 2 was identified with the inner part of the transition region, although this correspondence was not exact. In no case were any temperature inversions found and the fluid was stably stratified at all points where measurements were taken.

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*For comparison some runs with glucose–sucrose fingers were made; it was found that typically $w \sim 0{\cdot}003$ cm/sec and $L \sim 0{\cdot}006$ cm.

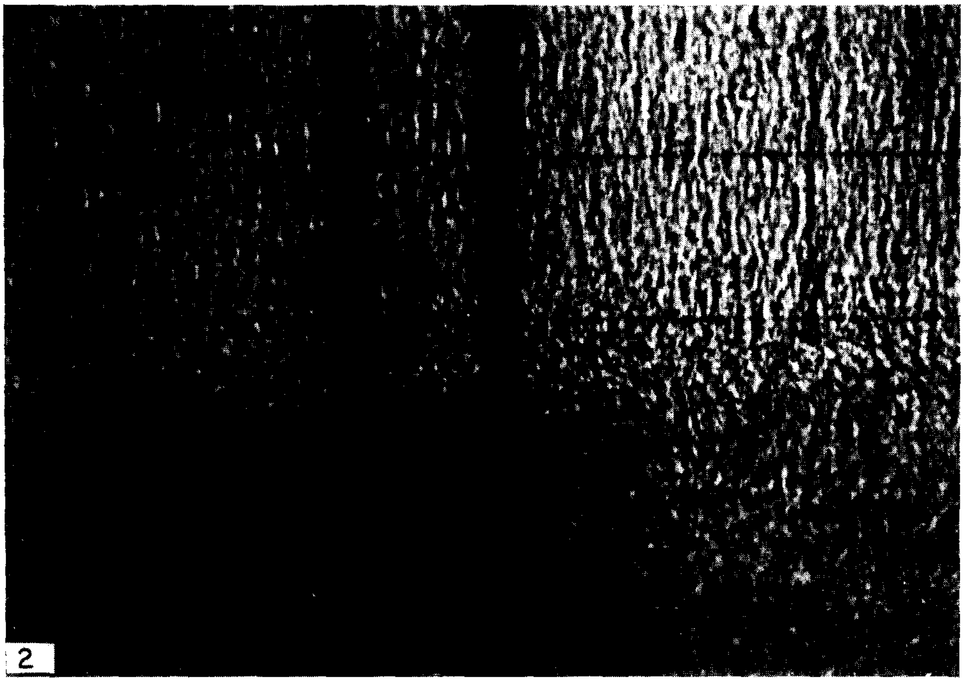

Fig. 2. A shadowgraph of heat–sugar fingers showing a close-up of the transition region and the break-up of the fingers. Note the organised fingers in the upper half of the picture compared with the larger scale, more random motions below.

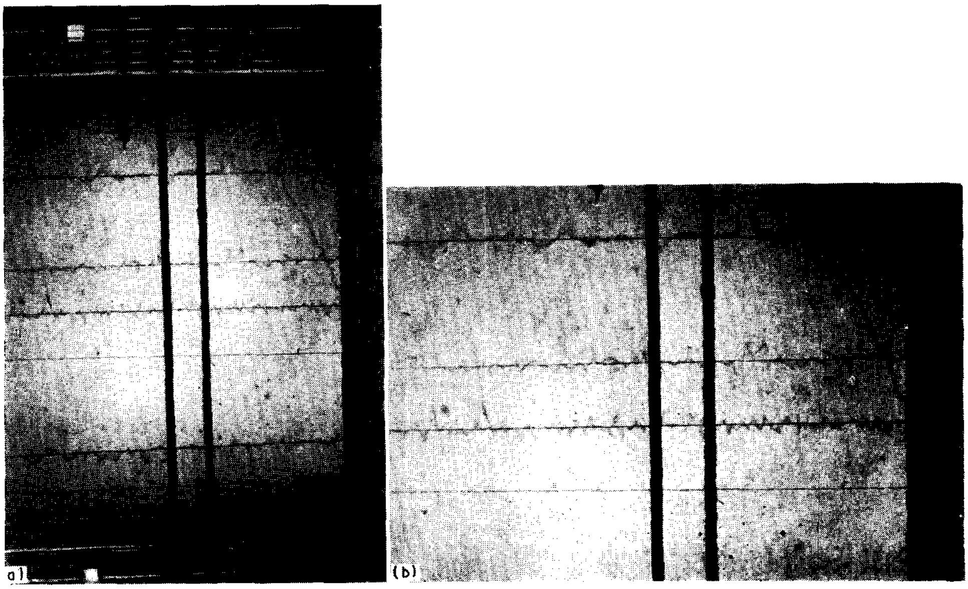

Fig. 3. The flow as shown by the distortions of the dye streaks in heat–sugar fingers. The interface is centred at approximately the 24 cm mark on (a). (b) is an enlargement of the three central wires shown on (a). This clearly shows that the fingers are cells of vertical motion with alternate fingers moving in opposite directions.

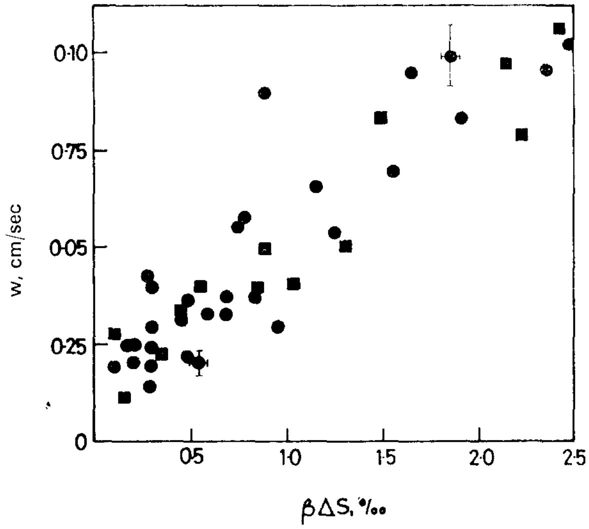

Fig. 4. The velocity of the fluid in the fingers w, plotted against the difference in sugar concentration between the well-mixed layers $\beta\Delta S$. ● Measurements taken in the tank described in Section 3. ■ Measurements taken in a smaller tank (10 × 5 × 25 cm).

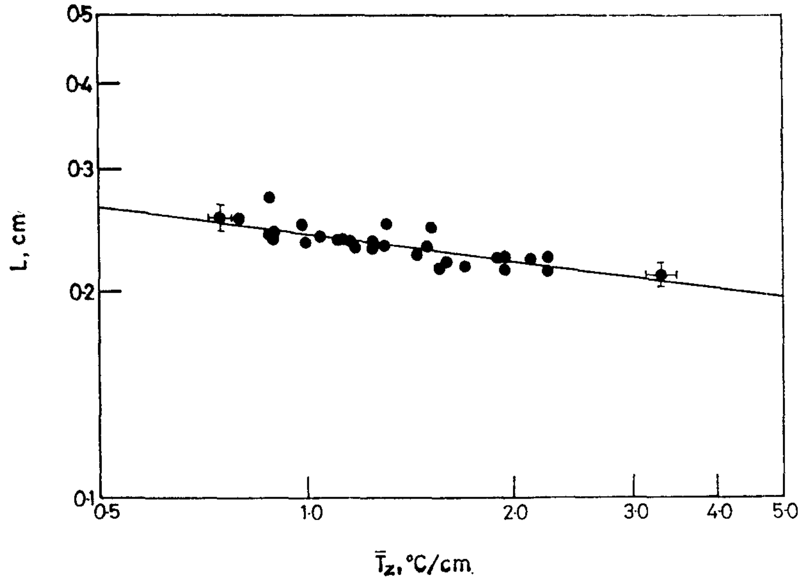

Fig. 5. The width of the fingers L plotted against the temperature gradient $\bar{T}_z$ in the fingers. The line is a fit to the data with slope −1/4.

It is now convenient to look at some other properties of the convection with a view to examining the relations derived in Section 2. Bearing in mind that r and Γ are assumed constant, then the finite amplitude balance for steady, infinitely long fingers (2.6) implies that $L \propto \bar{T}_z^{-1/4}$. These two quantities are plotted on Fig. 5, and the straight line represents a fit to the data with slope -1/4. The three points which lie above the other data are thought to be due to a freak alignment of the fingers with respect to the dye trace (see the Appendix). The length and width of the fingers are shown on Fig. 6. Whilst it is seen that L increases as h increases, the results do not reveal a simple relationship between the two. It should be recalled that for sugar–salt fingers Shirtcliffe and Turner (1970) found that $h^{1/4} \propto L$; although there is some evidence for this relationship in the results shown on Fig. 6, particularly for individual runs, the scatter among the results makes it difficult to make a reliable statement regarding an equivalent relationship for heat–sugar fingers.

The sugar flux F_S was determined by measuring the sugar concentration in the well-mixed layers at different times and using the formula

$$F_S = \frac{2H-h}{4}\,\frac{\mathrm{d}(\beta\Delta S)}{\mathrm{d}t},$$

where $2H$ is the depth of the water in the tank (40 cm). Figure 7 shows F_S plotted against $\beta\Delta S/h$: the data are consistent with (2·9) with $D = 0{\cdot}04 \pm 0{\cdot}002$ cm^2 sec.$^{-1}$ Equation (2.9) relates the flux of sugar to the mean gradient of sugar across the interface; the value of the diffusion coefficient D is much larger than the molecular diffusivity of sugar, with $\kappa_S/D \sim 10^{-4}$. The increased value of the diffusion coefficient is reflected in the ratio $F_S/F_S{}^*$ i.e. the increase in the flux over its 'solid-plane value'. A graph of $F_S/F_S{}^*$ against $\alpha\Delta T/\beta\Delta S$ is shown on Fig. 8. Turner (1967) results for heat–salt fingers are included for comparison; it is seen that the magnitudes of the

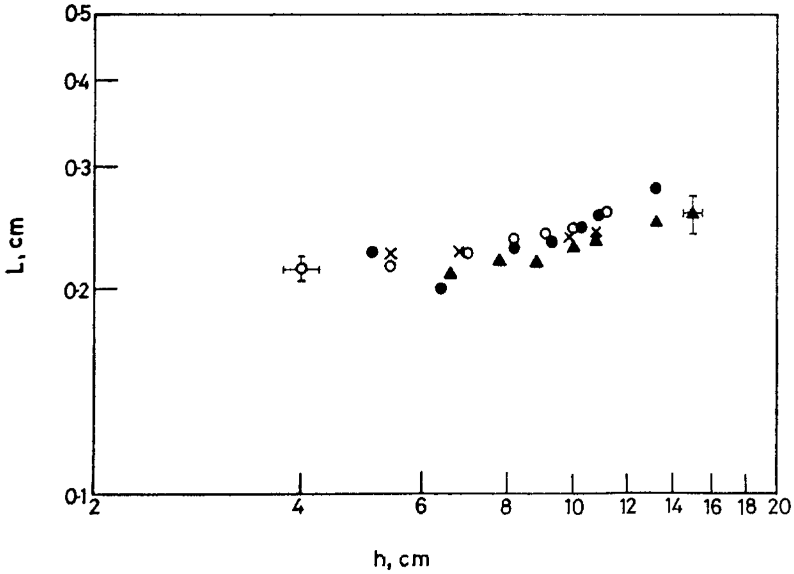

Fig. 6. The width of the fingers L plotted against their length h; the different symbols refer to different runs.

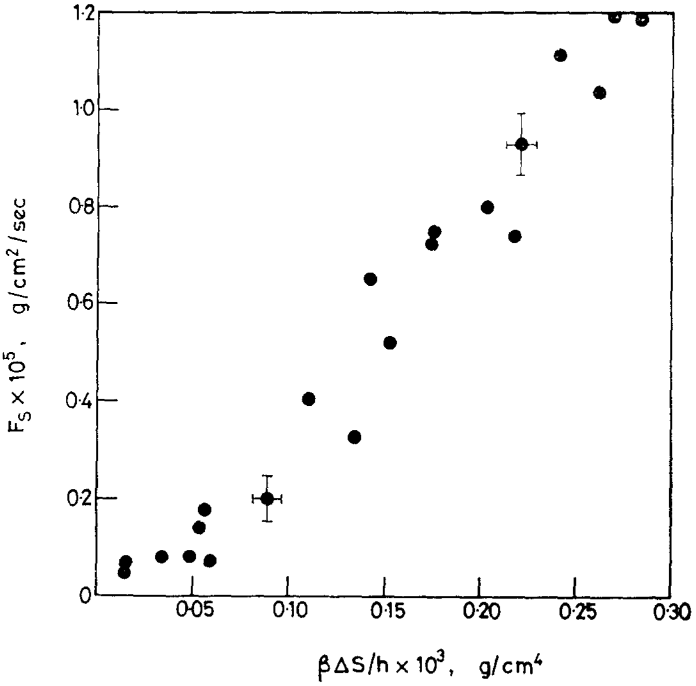

Fig. 7. The flux of sugar F_s plotted against $\beta\Delta S/h$.

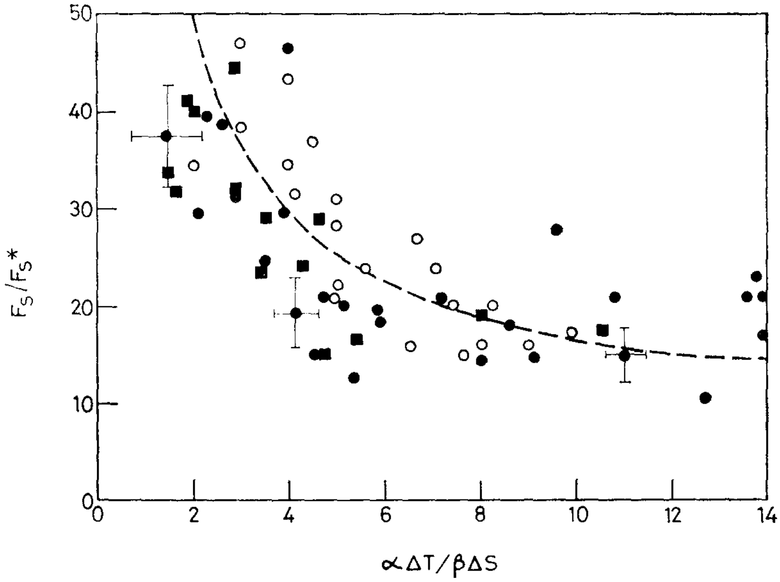

Fig. 8. The sugar flux F_s normalised with respect to its 'solid plane value' F_s^* plotted against $\alpha\Delta T/\beta\Delta S$. The symbols have the same meaning as on Fig. 4; the open circles are TURNER's (1967) data obtained with heat-salt fingers included here for comparison. The broken line is a curve $\propto (\alpha\Delta T/\beta\Delta S)^{-1/3}$.

fluxes are very similar to those for heat–sugar fingers. The curve $\propto (\alpha\Delta T/\beta\Delta S)^{-1/3}$ is fitted to the data, showing that the form of the results is consistent with (2.12).

The time-dependent features of the convection are shown on Figs. 9 and 10. These results are consistent with the predictions of (2.13) and (2.14). The increase in length of the fingers with time was found to occur at the constant rate of 2·3 × 10^{-3} cm sec^{-1}.

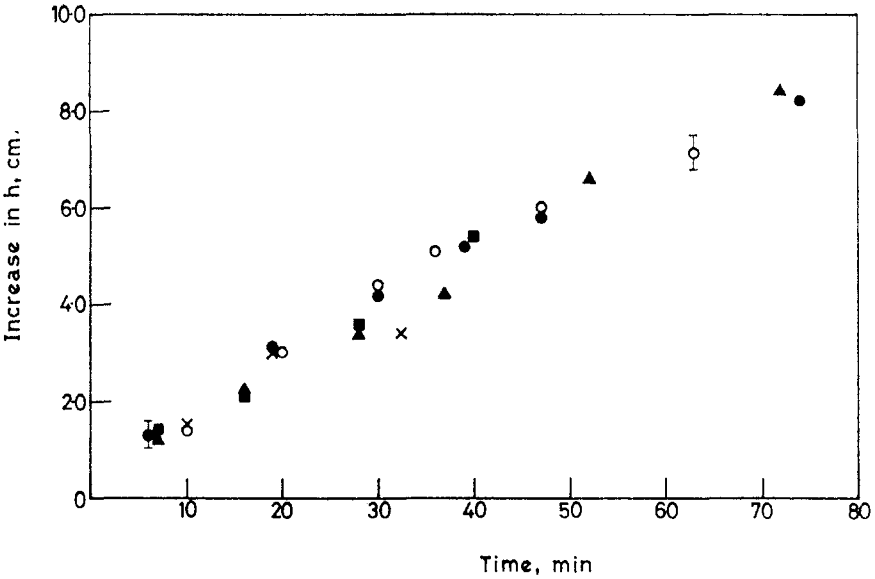

Fig. 9. The increase in length of the fingers plotted against time; the different symbols refer to separate runs.

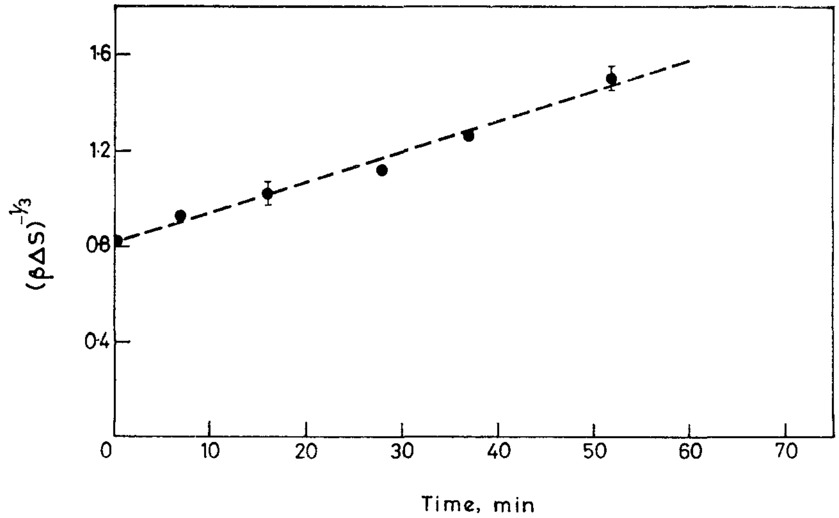

Fig. 10. $(\beta\Delta S)^{-1/3}$ plotted against time.

5. DISCUSSION AND SUMMARY

The experimental results presented above show good agreement with the dimensional arguments given in Section 2. Consequently, the basic picture given there of the convection appears to be a valid one, at least for these laboratory experiments. The properties of the fingers were calculated by assuming that the flow was steady and that the ends of the fingers did not seriously affect the motion. It was then assumed that the fingers were in equilibrium with deeper convecting layers on either side of the interface, the convection in these layers being driven by the unstable buoyancy flux through the fingers. Although the laboratory experiments were unsteady, the idea, proposed by STERN and TURNER (1969), that the system passed through a series of quasi-steady equilibrium states, is adequate to account for the time-dependent features of the motion.

The fluid velocity in the fingers was much larger than the rate of increase of the length of the fingers. Therefore, the fingers must advect the sugar vertically and produce anomalies at the ends of the fingers which provide the driving force for the convection in the well-mixed layers. However, the motion at the ends of the fingers and in the transition regions is not well understood. STERN (1969) proposed a model for the break-up of the fingers based on an energy transfer from a group of fingers to a large-scale internal wave. His analysis shows that when the sugar flux exceeded a critical value given by

$$\frac{F_S}{\nu\alpha \bar{T}_t} = A, \tag{5.1}$$

where A is an 0(1) constant, the fingers would become unstable. The values of A for 18 experiments in which all the necessary information is recorded is shown on Table 1. In general, the values are slightly smaller than the value predicted by Stern and they

Table 1. *The values of* A *defined by* (5.1) *as determined from the experimental results.*

Run number	$\nu\alpha \bar{T}_z$ (g cm^{-2} sec^{-1} × 10^{-6})	F_s (g cm^{-2} sec^{-1} × 10^{-4})	$F_s/\nu\alpha \bar{T}_Z$
D2	4·66	0·075	1·6
D3	3·36	0·045	1·3
D4	2·62	0·008	0·3
E2	6·24	0·119	1·9
E3	4·01	0·018	0·5
E4	3·11	0·011	0·4
E5	2·65	0·006	0·2
E6	2·33	0·008	0·3
F2	4·73	0·093	1·9
F3	3·93	0·033	0·9
F4	3·39	0·020	0·6
F5	2·85	0·018	0·6
F6	2·53	0·008	0·3
F7	2·08	0·005	0·2
G2	4·85	0·073	1·5
G3	3·88	0·012	0·3
G4	2·42	0·007	0·3
G5	2·92	0·008	0·3

seem to decrease as the sugar flux decreases. Further, it has been shown that $F_S \propto \beta\Delta S/h$ (see Fig. 7), which is only consistent with (5.1) when $\alpha\Delta T/\beta\Delta S \rightarrow 1$. In that limit though Stern's analysis does seem to adequately describe the criterion for the break-up of the fingers.

The data shown on Fig. 8 provide further support for TURNER's (1967) assumption that the flux may be written in the form $F_S = f(\alpha\Delta T/\beta\Delta S,\ \tau)\ (\beta\Delta S)^{4/3}$. The fact that the magnitudes of the fluxes of sugar and salt are approximately the same for heat–sugar and heat–salt fingers indicates that the two systems are both described by (2.1)–(2.3) in the limit $\kappa_S = 0$. Observations of heat–salt fingers using a shadowgraph also support this hypothesis, as the structure of the convection was indistinguishable from heat–sugar fingers. Consequently, it seems justified to extrapolate the results of these experiments to obtain predictions for the properties of salt fingers in the ocean. This is particularly important as salt fingers themselves have not yet been observed in the ocean. Therefore, predictions of their scale, etc. are necessary in order to determine what type of instrumentation would be needed to observe them.

In the ocean the temperature and salinity steps across interfaces are much smaller than those used in these experiments; typically $\beta\Delta S \sim 0{\cdot}05$‰ and $\alpha\Delta T \sim 0{\cdot}1$‰ (see HOWE and TAIT, 1970). Use of (2.10) and (2.9) in (2.4) and (2.6) shows that

$$h \sim \left(\frac{\nu}{g\kappa_T}\right)^{1/3} \frac{D^{2/3}}{\Gamma^{4/3} r^{1/3} (1-r)^{1/3}}\ (\alpha\Delta T)^{1/3}\ (\beta\Delta S)^{4/3}.$$

Using laboratory values for D and r and assuming $\Gamma \sim 0{\cdot}5$, one finds that h would typically be about 20 cm. The experiments showed that $L \propto (\bar{T}_z)^{-1/4}$; oceanic data from HOWE and TAIT (1970) indicates that the fingers would be approximately 0·5 cm in width. The velocity of the fluid in the fingers would be $\sim 0{\cdot}02$ cm sec.$^{-1}$[*]

In order to determine the depth of the convecting layers H we consider a series of layers and interfaces in equilibrium. Then following STERN (1969), the assumption that the mean flux is independent of z, implies that

$$\hat{F}_S \frac{\mathrm{d}\beta\hat{S}}{\mathrm{d}z} = \kappa|_S\beta\nabla S|^2,$$

where $\mathrm{d}\hat{S}/\mathrm{d}z$ is the mean vertical gradient of salinity. From (2.9), and identifying $|\beta\nabla S| \sim \beta\Delta S/h$ then

$$|\nabla S| \sim D/\kappa_S \frac{\mathrm{d}\hat{S}}{\mathrm{d}z}.$$

The fact that the convecting layers are homogeneous (to a good approximation) implies that the depth of the layer is given by

$$H \sim D/\kappa_S h.$$

The value of D determined from the experiments and the estimate that $h \sim 20$ cm gives $H \sim 40$ m, which is in agreement with the observations of layered structure found beneath the Mediterranean outflow in the Northeast Atlantic (HOWE and TAIT, 1970), although it is larger than layer depths found near Bermuda by COOPER and STOMMEL (1968). On the other hand the observed gradient regions found by HOWE

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[*] An upper limit on the length of the fingers is 10 m as salt diffusion would wipe out the lateral salt differences between fingers 0·5 cm wide moving with velocity 0·02 cm sec^{-1} in that distance.

and TAIT (1970) were about 7 m thick. This is much thicker than would be expected on the basis of extrapolations of the laboratory data. It is possible that instruments with better depth resolution may reveal that the gradient regions are thinner than they are presently believed to be, or measurements taken over a longer time scale may show the oceanic layers to be in a time-dependent, non-equilibrium state. However, at the moment this discrepancy between the predicted and the observed scales remains a real problem yet to be resolved.

In conclusion, it remains to comment that these experiments are the first in which the detailed structure of heat–solute fingers has been examined. Much work still remains to be done before the dynamics of the convection can be claimed to be completely understood. In particular, continuous profiles of the solute concentration and a study of the flow in the transition region needs to be made, in order to come to grips with some of the details which have been ignored in this paper.

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APPENDIX

Measurements of the length scale and velocity of the fingers were made by sampling along a horizontal line passing through the field of fingers. The field is to some extent randomly oriented with respect to this line and so it is necessary to examine the statistics of the measurements. Observations by SHIRTCLIFFE and TURNER (1970) of sugar–salt fingers show that the fingers are predominantly square in plan form, and they found some correlation between the orientation of a finger and its immediate neighbours. The extent of this correlation is such that the influence of a finger is probably lost after about 5 fingers widths.

(1) *Length scale measurements*

The correlation between the orientation of adjacent fingers makes a precise determination of the length scale statistics complicated. As the dye traces indicated that there were approximately 70 fingers across the tank, a correlation between 5 fingers implies that any sample trace consists of about 10 arbitrarily oriented clusters. Such a cluster will be defined to consist of those fingers adjacent to one where the dye line passes through a corner of the finger. Through the cluster the dye trace is straight (in plan) and an estimate of the finger width in such a finger cluster varies from L to $L\sqrt{2}$. If it is assumed that each cluster is arbitrarily oriented with respected to the dye trace, then the mean value is given by

$$L_M = \frac{4}{\pi}\int_0^{\pi/4} \frac{L}{\cos\theta}\, d\theta\,,$$
$$= 1{\cdot}12\, L.$$

The experimental results presented above assume $L_M = L$; this approximation is made as the definition of L_M is to some extent arbitrary and in any case only differs from L by about 10%.

(2) *Velocity measurements*

Suppose that the fingers are square in plan form and that

$$w = w \sin\frac{\pi x}{L} \sin\frac{\pi y}{L}\ . \tag{A1}$$

For the sake of simplicity assume that an arbitrary line $y = x \tan\theta + c$ passes through a finger with boundaries at $x = 0$, L and $y = 0$, L. From dye traces the average value of the maximum velocity in the fingers w_M is measured; i.e. the maximum value of

$$w = \sin\frac{\pi x}{L} \sin\frac{\pi}{L}(x\tan\theta + c)\ .$$

On the assumption that the finger is randomly positioned with respect to the dye trace

$$w_M = \frac{w}{\pi}\int_{-\pi/4}^{\pi/4}\frac{d\theta}{1+\tan\theta}\int_{-1-\tan\theta}^{1+\tan\theta}\sin\frac{\pi x}{2}\sin\frac{\pi}{2}(x\tan\theta + c)\,dx\,,$$

where

$$\frac{\partial w}{\partial x} = \frac{\pi}{2}\left[\cos\frac{\pi x}{2}\sin\frac{\pi}{2}(x\tan\theta + c) + \tan\theta\sin\frac{\pi x}{2}\cos\frac{\pi}{2}(x\tan\theta + c)\right] = 0\,.$$

The integration was carried out numerically and it was found that

$$w_M = 0{\cdot}60\, w. \tag{A2}$$

Determining w_M by averaging overall the peaks in a dye trace was a very tedious process. Consequently, in most cases w was determined by averaging over the fastest 25% of the peaks on a particular trace. It was found that the value of w determined in this way was consistent with that found by (A2) after averaging over all the peaks; the two estimates being the same to within 15% of w. The similarity of the two values is an indication that (A1) is an adequate representation of the velocity field in the fingers.

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